Alpha clustering in the modern shell model approach

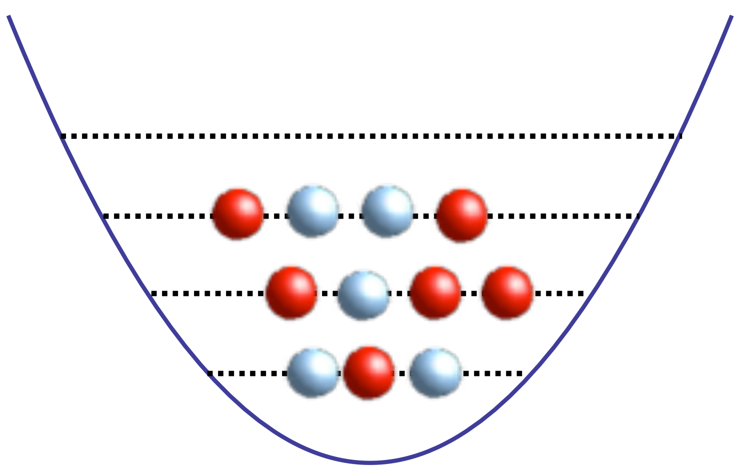
Alexander Volya

Florida State University

Cluster-nucleon configuration interaction approach

Traditional shell model configuration
m-scheme

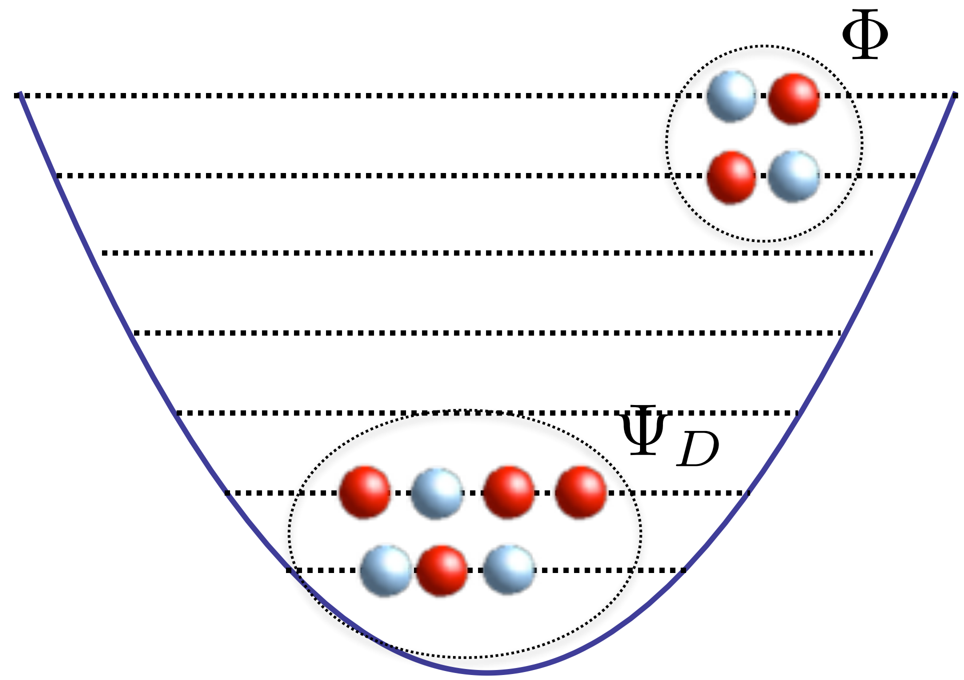
$$|\Psi\rangle = \Psi^\dagger|0\rangle \sim a_1^\dagger a_2^\dagger \ldots a_A^\dagger|0\rangle$$



+

Cluster configuration
SU(3)-symmetry basis

$$|\mathrm{channel}\rangle = |\mathcal{A}\{\Phi\,\Psi_D\} \equiv \Phi^\dagger\Psi^\dagger|0\rangle \equiv \Phi^\dagger|\Psi_D\rangle$$



$$|\Psi\rangle \quad + \quad \Phi^\dagger|\Psi_D\rangle$$

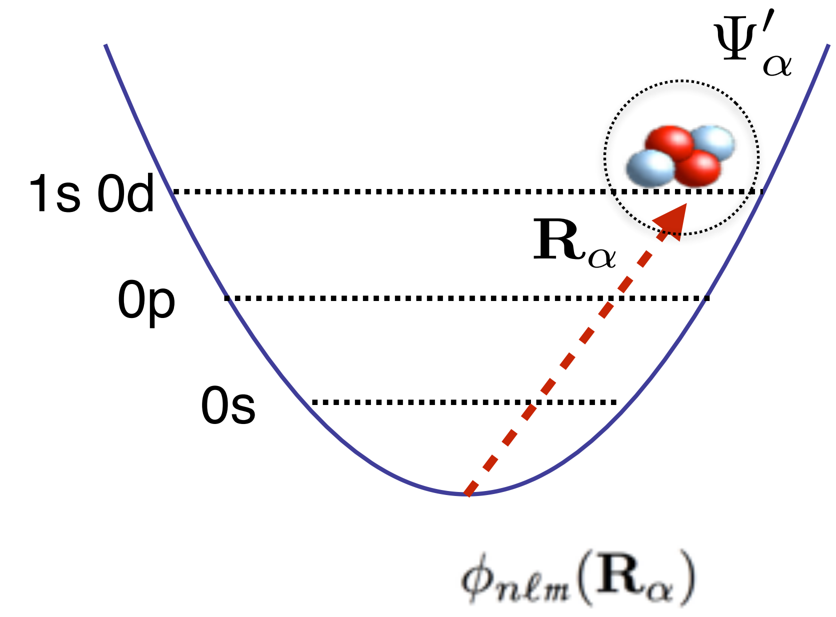
- m-scheme and SU(3) basis
- Construction and classification of cluster configurations
- Center of mass and translational invariance
- Non-orthogonality and bosonic principle

Cluster configurations

Example: alpha decay with ℓ=0 from sd shell
21 way to make L=0 T=0 4-nucleon combination
Each nucleon has 2 oscillator quanta, 8 quanta total
In oscillator basis excitation quanta are conserved
We model alpha as 4-nucleons on s-shell $(0s)^4$

Make single SU(3) operator with quantum numbers (8,0) $\Phi^{\eta}_{(8,0):\ell m}$

Cluster coefficient is known analytically $X^{\eta}_{n'\ell}$



$$\underbrace{\phi_{n\ell m}(1)\phi_{n\ell m}(2)\phi_{n\ell m}(3)\phi_{n\ell m}(4)}_{4\times 2=8\,\text{quanta}} \leftrightarrow \sum_{\eta} X^{\eta}_{n'\ell}\Phi^{\eta}_{(8,0):\ell m} = \underbrace{\phi_{n'\ell' m'}(\mathbf{R}_\alpha)}_{8\ \text{quanta}}\ \underbrace{\Psi'_\alpha}_{0\ \text{quanta}}$$

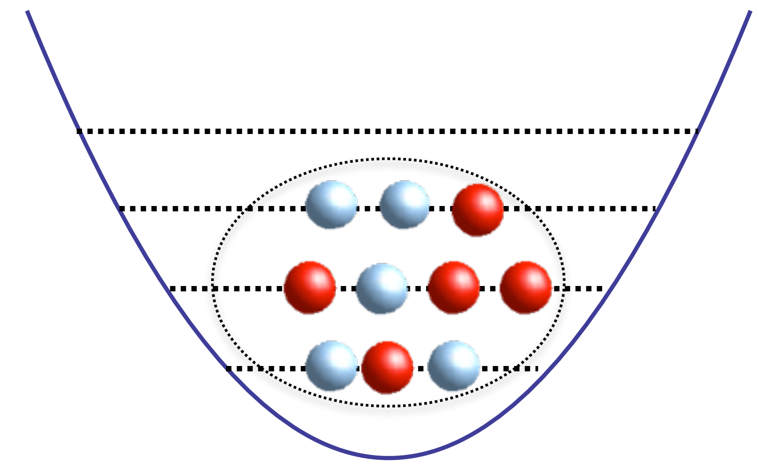
m-scheme state | SU(3) symmetry state | motion of alpha

Methods

- Direct diagonalization Casimir operators of SU(3), J^2 , T^2 …
- Coupling and U(N) Clebsh-Gordan coefficients (via diagonalization)
- Casimir projection techniques. Generators of algebra.

Yu. F. Smirnov and Yu. M. Tchuvil'sky, Phys. Rev. C 15, 84 (1977).
M. Ichimura, A. Arima, E. C. Halbert, and T. Terasawa, Nucl. Phys. A 204, 225 (1973).
O. F. Nemetz, V. G. Neudatchin, A. T. Rudchik, Yu. F. Smirnov, and Yu. M. Tchuvil'sky, Nucleon Clusters in Atomic Nuclei and Multi-Nucleon Transfer Reactions (Naukova Dumka, Kiev, 1988), p. 295.

Translational invariance



Shell model, Glockner-Lawson procedure

$$\Psi_D = \phi_{000}(\mathbf{R}_D)\,\Psi'_D$$

SM state — Center-of-mass vibration — Intrinsic state

Factorizing center of mass in overlap integral

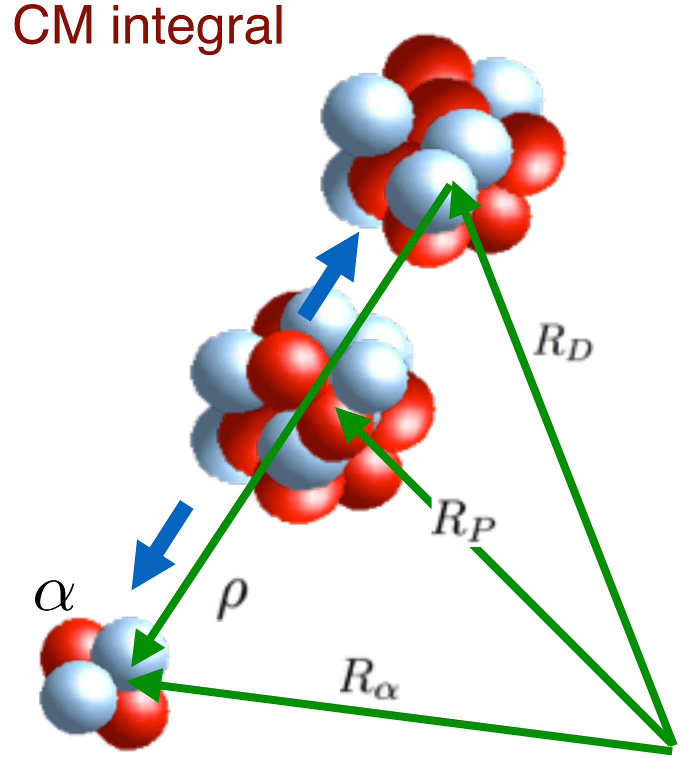
$$\langle \Psi_P | \hat{\mathcal{A}}\{\phi_{n\ell m}(\mathbf{R}_\alpha)\Psi'_\alpha\,\Psi_D\}\rangle = \langle \Psi'_P | \hat{\mathcal{A}}\{\phi_{n\ell m}(\boldsymbol{\rho})\Psi'_\alpha\,\Psi'_D\}\rangle \times \langle \phi_{000}(\mathbf{R}_P)\,\phi_{n\ell m}(\boldsymbol{\rho}) | \phi_{n\ell m}(\mathbf{R}_\alpha)\,\phi_{000}(\mathbf{R}_D)\rangle$$

SM overlap integral (FPC) — Translationally invariant part — Spurious CM integral

Recoil factor (inverse of Talmi-Moshinsky coefficient)

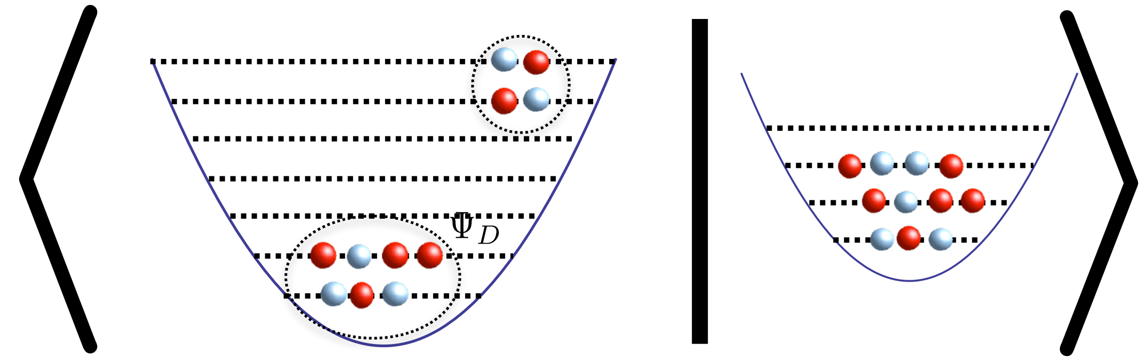
$$\mathbf{R}_P = \frac{m_D\mathbf{R}_D + m_\alpha \mathbf{R}_\alpha}{m_D + m_\alpha}, \quad \boldsymbol{\rho} = \mathbf{R}_D - \mathbf{R}_\alpha$$

$$\mathcal{R}_{n\ell} \equiv \left(\langle 00, n\ell : \ell | \{n\ell\}_{m_\alpha}, \{00\}_{m_D} : \ell\rangle\right)^{-1} = (-1)^n \left(\frac{m_D + m_\alpha}{m_D}\right)^{n/2}$$



Traditional Cluster Spectroscopic Characteristics

$$\langle \phi_{n\ell} | \varphi_\ell \rangle = \langle \hat{\mathcal{A}} \{ \phi_{n\ell m}(\boldsymbol{\rho}) \, \Psi'_\alpha \, \Psi'_D \} | \Psi'_P \rangle \; =$$



$$\langle \phi_{n\ell} | \varphi_\ell \rangle = \mathcal{R}_{n\ell} \sum_\eta X^\eta_{n\ell} \, \mathcal{F}^\eta_{n\ell}$$

Recoil Factor   Cluster Coefficient   Fractional Parentage Coefficient

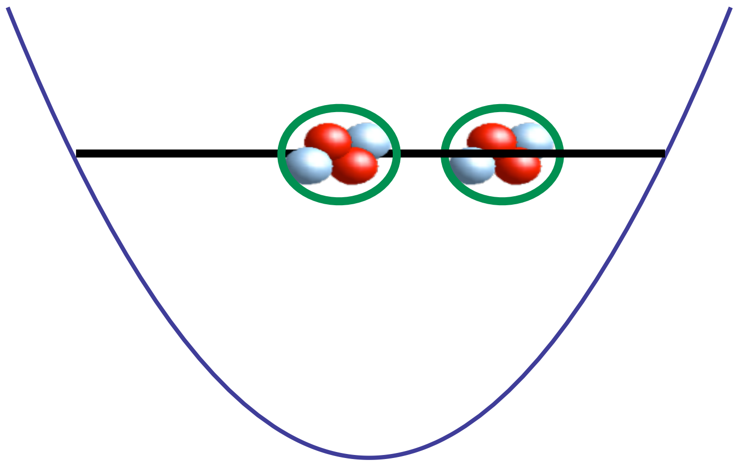
Traditional "old" spectroscopic factors

$$\varphi_\ell(\rho) = \sum_n \langle \phi_{n\ell} | \varphi_\ell \rangle \, \phi_{n\ell}(\rho)$$

Expand radial motion in HO wave functions

$$\mathcal{S}^{(\mathrm{old})}_\ell = \langle \varphi_\ell | \varphi_\ell \rangle = \int \rho^2 d\rho \, |\varphi_\ell(\rho)|^2 = \sum_n |\langle \phi_{n\ell} | \varphi_\ell \rangle|^2$$

Bosonic nature of 4-nucleon operators non-orgothogonality



If $\Phi^\dagger$ is thought of as being a boson then $\Phi\Phi^\dagger = 1 + N_b$

$$|\Psi_D\rangle = |\Phi\rangle \quad \langle\Phi_D|\hat{\Phi}\hat{\Phi}^\dagger|\Psi_D\rangle = \langle 0|\hat{\Phi}\hat{\Phi}\hat{\Phi}^\dagger\hat{\Phi}^\dagger|0\rangle = 2$$

$$L = S = T = 0$$

Φ	Ψ_P	$\left\lvert\langle\Psi_P\lvert\hat{\Phi}^\dagger\rvert\Psi_D\rangle\right\rvert^2$	$\langle 0\lvert\hat{\Phi}\hat{\Phi}\hat{\Phi}^\dagger\hat{\Phi}^\dagger\rvert 0\rangle$
$(p)^4\,(4,0)$	$(p)^8\,(0,4)$	1.42222*	1.42222
$(sd)^4\,(8,0)$	$(sd)^8\,(8,4)$	0.487903	1.20213
$(fp)^4\,(12,0)$	$(fp)^8\,(16,4)$	0.292411	1.41503
$(sdg)^4\,(16,0)$	$(sdg)^8\,(24,4)$	0.209525	1.5278

* For p-shell the result is known analytically 64/45

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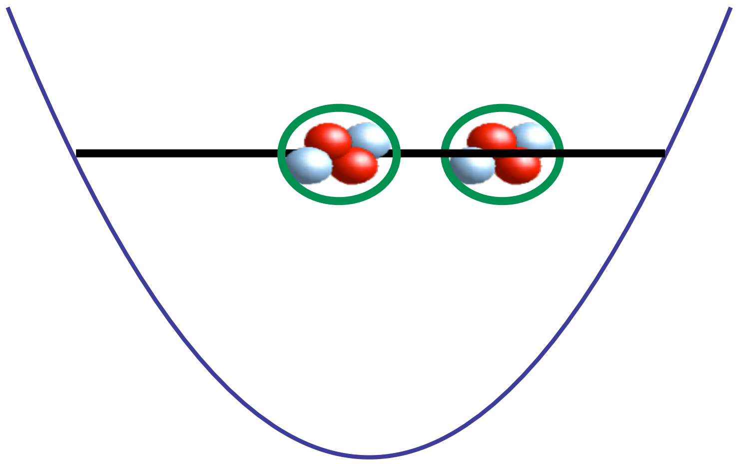

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Effective operators (alphas) are not ideal bosons
Cluster configurations are not orthogonal and not normalized

Orthogonality condition model, new SF

- Non-orthogonal set of channels (over-complete set of configurations)
- Pauli exclusion principle
- Matching procedure, asymptotic normalization, connection to observables
- No agreement with experiment on absolute scale

Resonating group method

$$\hat{\mathcal{H}}_\ell\, f_\ell(\rho) = E\hat{\mathcal{N}}_\ell\, f_\ell(\rho) \qquad \hat{\mathcal{N}}_\ell^{-1/2}\hat{\mathcal{H}}_\ell\,\hat{\mathcal{N}}_\ell^{-1/2} F_\ell(\rho) = E\, F_\ell(\rho)$$

New spectroscopic factor

$$\psi_\ell(\rho) \equiv \hat{\mathcal{N}}_\ell^{-1/2}\varphi_\ell(\rho)$$

$$S_\ell^{(\mathrm{new})} \equiv \langle\psi_\ell|\psi_\ell\rangle = \int \rho^2 d\rho\, |\psi_\ell(\rho)|^2$$

Sum of all new SF from all parent states to a given final state equals to the number of channels

R. Id Betan and W. Nazarewicz Phys. Rev. C 86, 034338 (2012)
S. G. Kadmenskya, S. D. Kurgalina, and Yu. M. Tchuvil'sky Phys. Part. Nucl., 38, 699–742 (2007).
R. Lovas et al. Phys. Rep. 294, No. 5 (1998) 265 – 362.
T. Fliessbach and H. J. Mang, Nucl. Phys. A **263**, 75–85 (1976).
H. Feschbach et al. Ann. Phys. 41 (1967) 230 – 286

Alpha clustering in sd-shell nuclei

$A_P - A_D$	$S_0^{(exp)}$ [1]	$S_0^{(exp)}$ [2]	$S_0^{(exp)}$ [3]	$\mathcal{S}_0^{(old)}$ [4]	$\mathcal{S}_0^{(old)}$ this work	$S_0^{(new)}$ this work
^{20}Ne-^{16}O	1.0	0.54	1	0.18	0.173	0.755
^{22}Ne-^{18}O			0.37	0.099	0.085	0.481
^{24}Mg-^{20}Ne	0.76	0.42	0.66	0.11	0.091	0.411
^{26}Mg-^{22}Ne			0.20	0.077	0.068	0.439
^{28}Si-^{24}Mg	0.37	0.20	0.33	0.076	0.080	0.526
^{30}Si-^{26}Mg			0.55	0.067	0.061	0.555
^{32}S-^{28}Si	1.05	0.55	0.45	0.090	0.082	0.911
^{34}S-^{30}Si				0.065	0.062	0.974
^{36}Ar-^{32}S				0.070	0.061	0.986
^{38}Ar-^{34}S			1.30	0.034	0.030	0.997
^{40}Ca-^{36}Ar	1.56	0.86	1.18	0.043	0.037	1

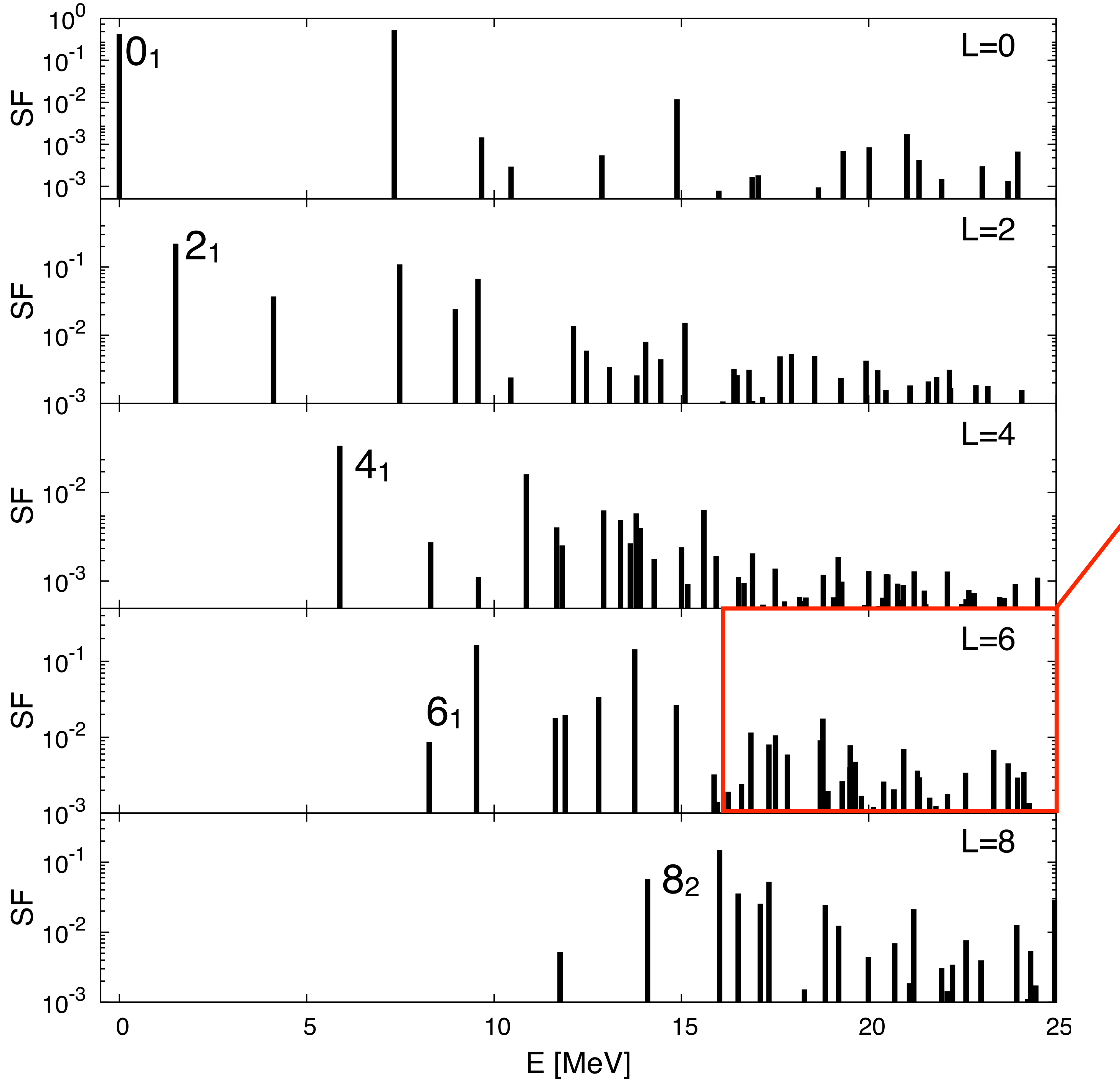
USDB interaction [5]
(8,0) configuration

- Old SF are small
- Old SF decrease with A

[1] T. Carey, P. Roos, N. Chant, A. Nadasen, and H. L. Chen, Phys. Rev. C 23, 576(R) (1981).
[2] T. Carey, P. Roos, N. Chant, A. Nadasen, and H. L. Chen, Phys. Rev. C 29, 1273 (1984).
[3] N. Anantaraman and et al., Phys. Rev. Lett. 35, 1131 (1975).
[4] W. Chung, J. van Hienen, B. H. Wildenthal, and C. L. Bennett, Phys. Lett. B 79, 381 (1978).
[5] B. A. Brown and W. A. Richter, Phys. Rev. C 74, 034315 (2006)

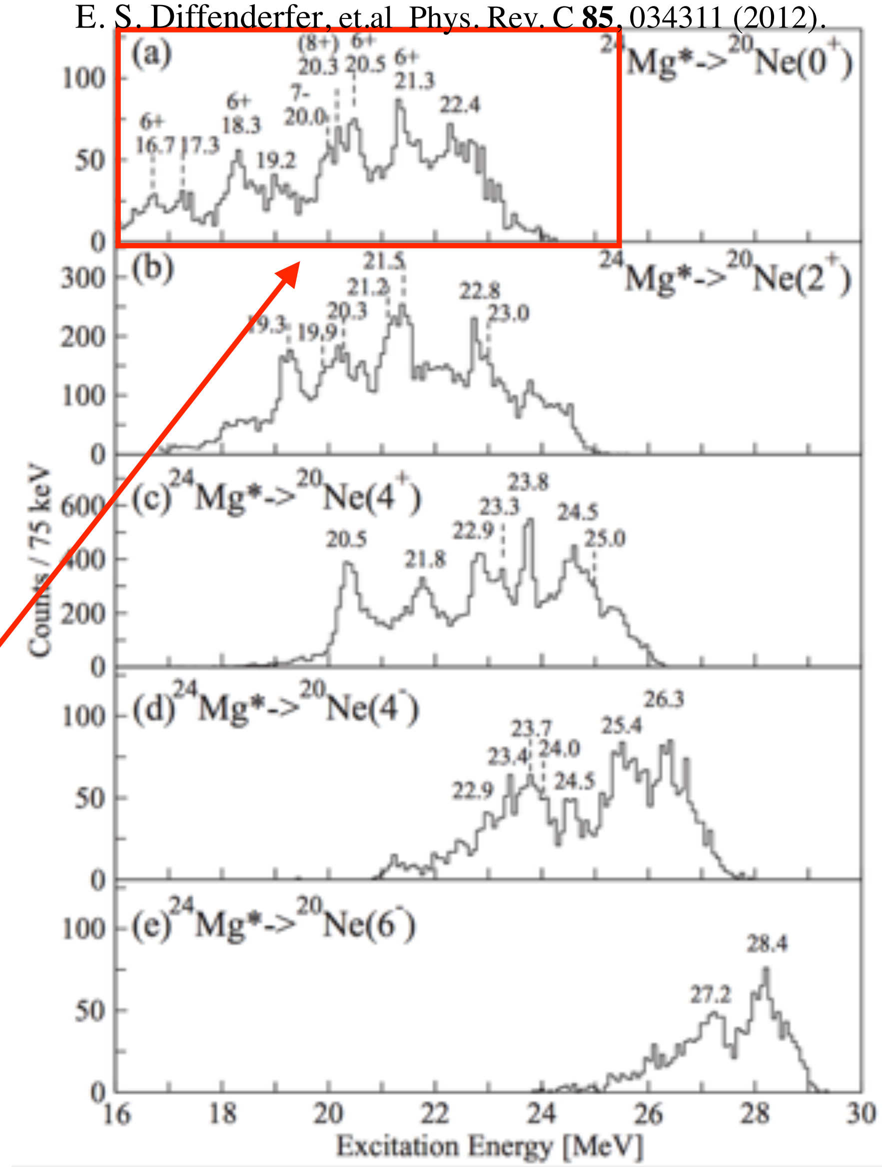
Alpha cluster spectroscopic factors in ^{24}Mg

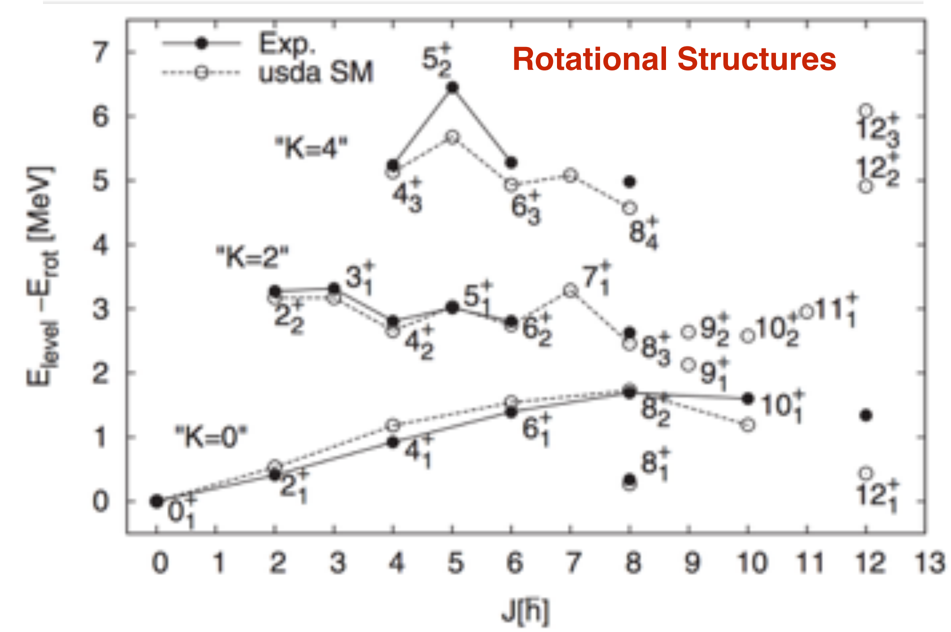
Theoretical calculations in SD shell



Experimental results

E. S. Diffenderfer, et.al Phys. Rev. C **85**, 034311 (2012).

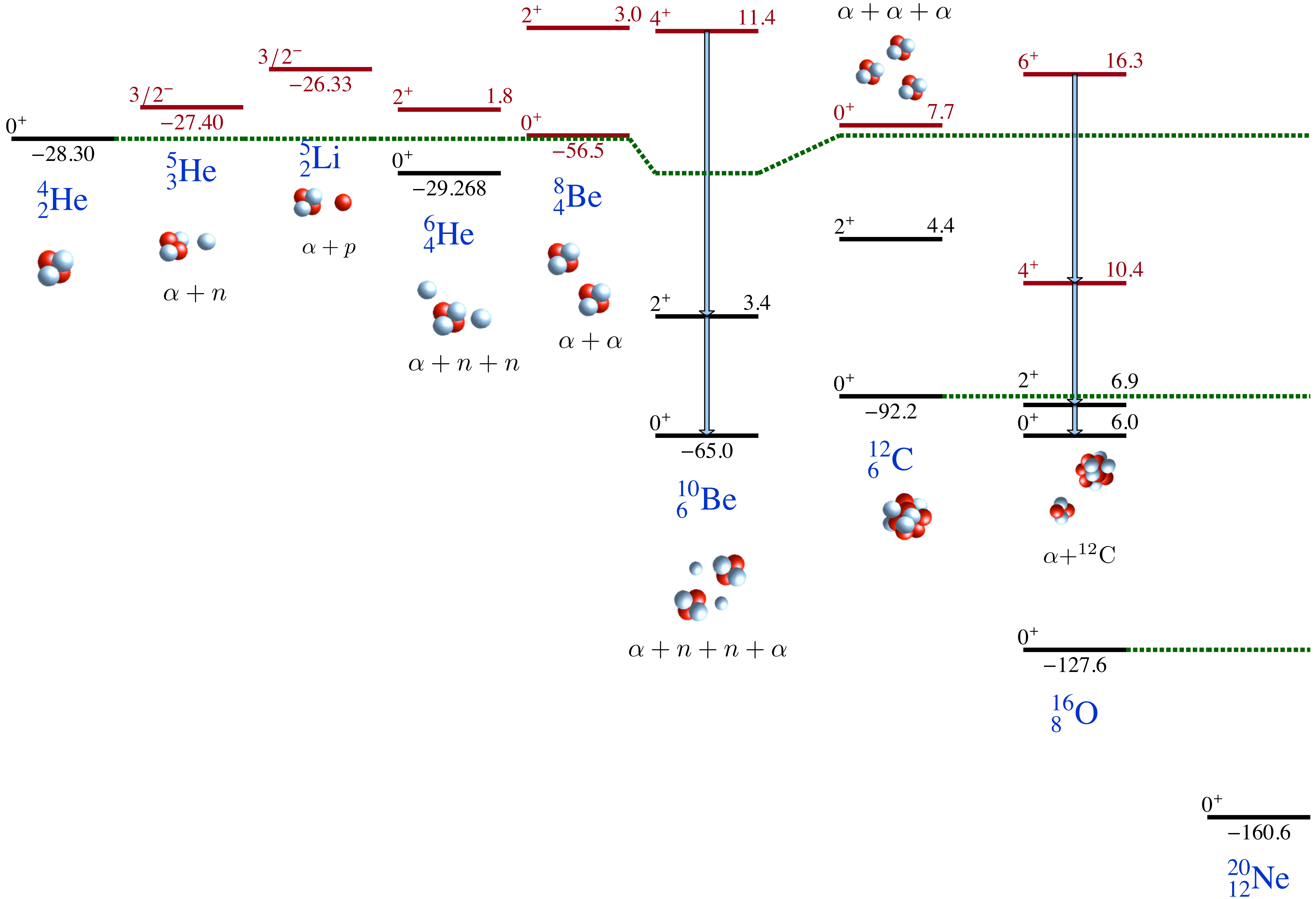




The sd valence space is considered with USDB interaction the operator is

$$|\Phi_{(8,0):L}\rangle = |(sd)^4[4]\,(8,0),:\,L\,S = T = 0\rangle$$

Clustering in light nuclei



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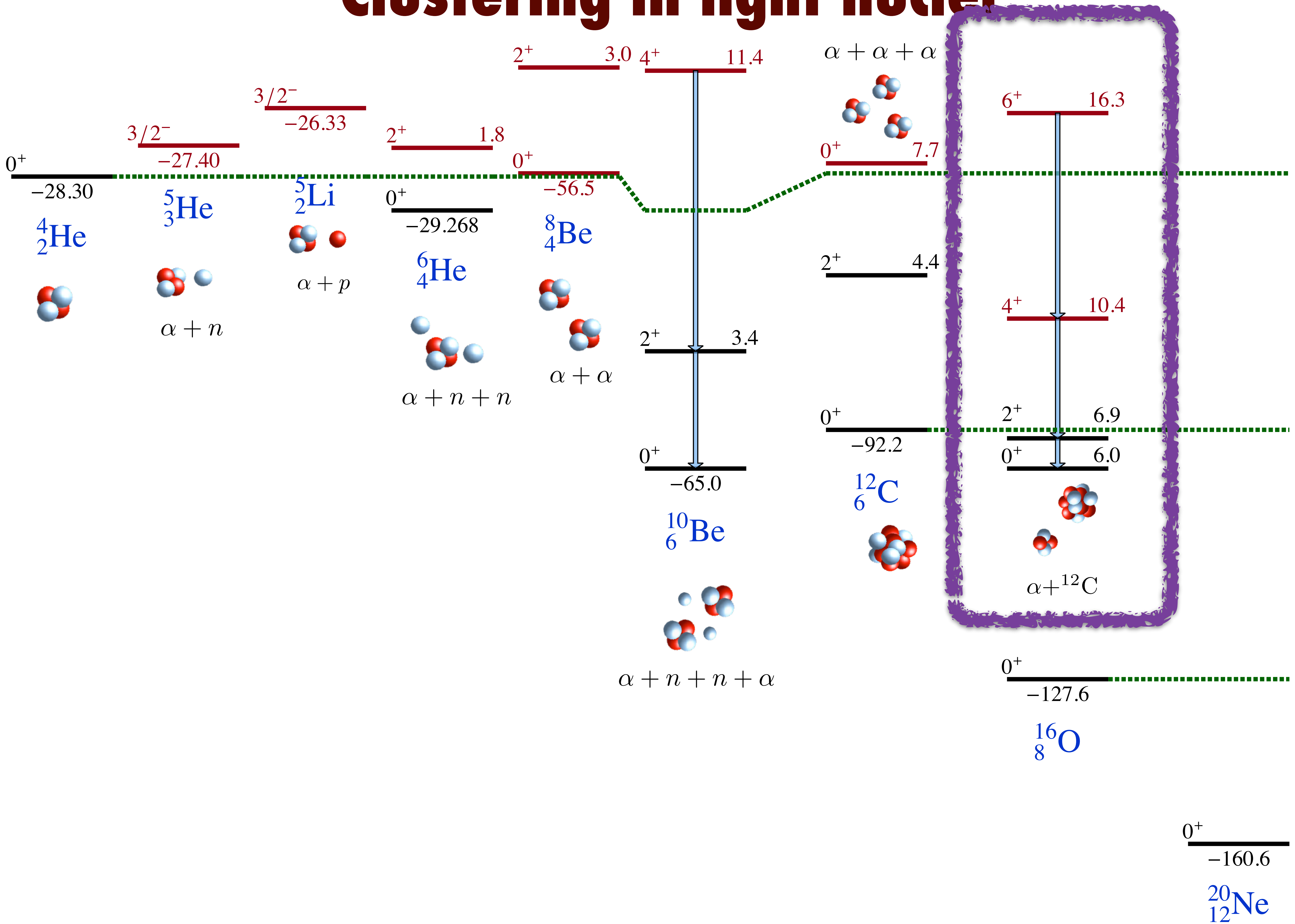
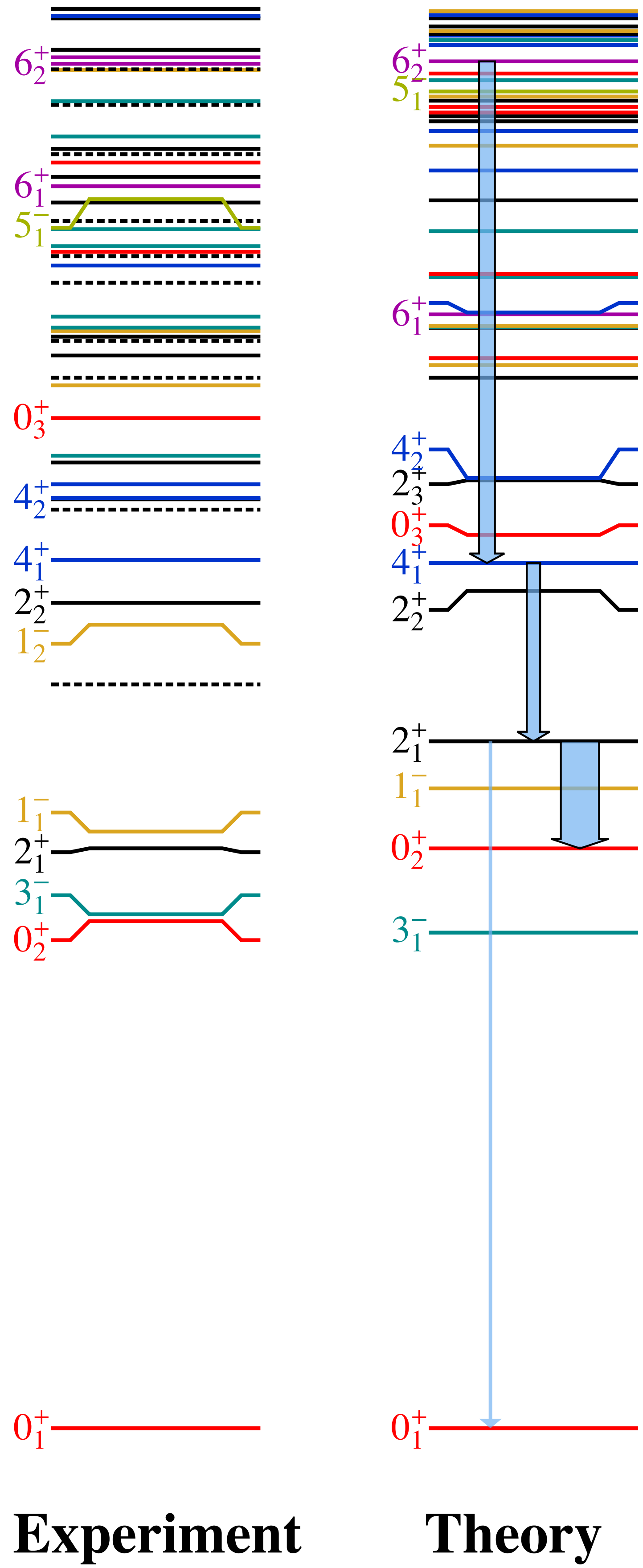
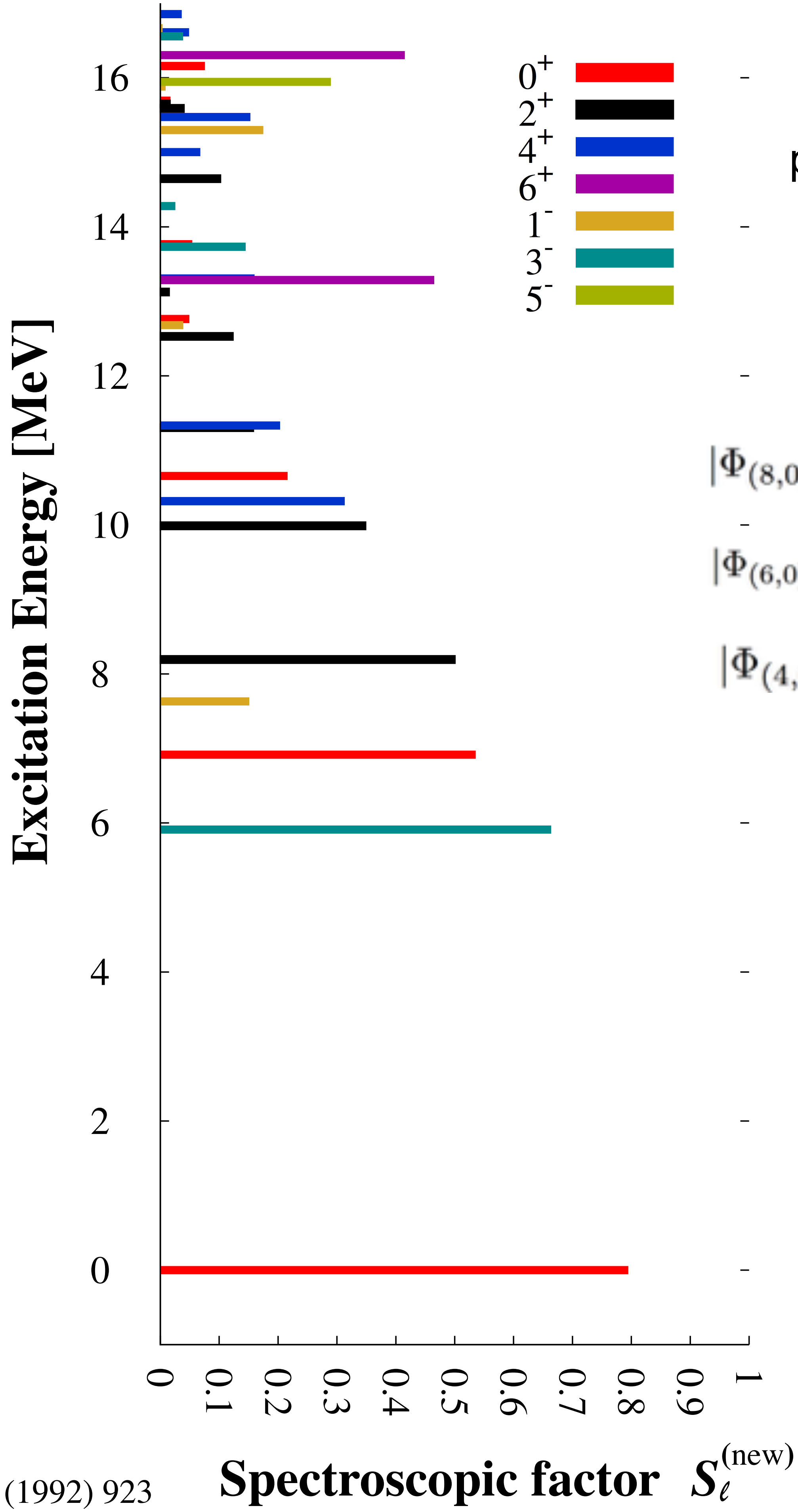






p-sd shell model

SU(3) configurations

For positive parity

$$|\Phi_{(8,0):L}\rangle = |(sd)^4[4]\,(8,0),:L\,S=T=0\rangle$$

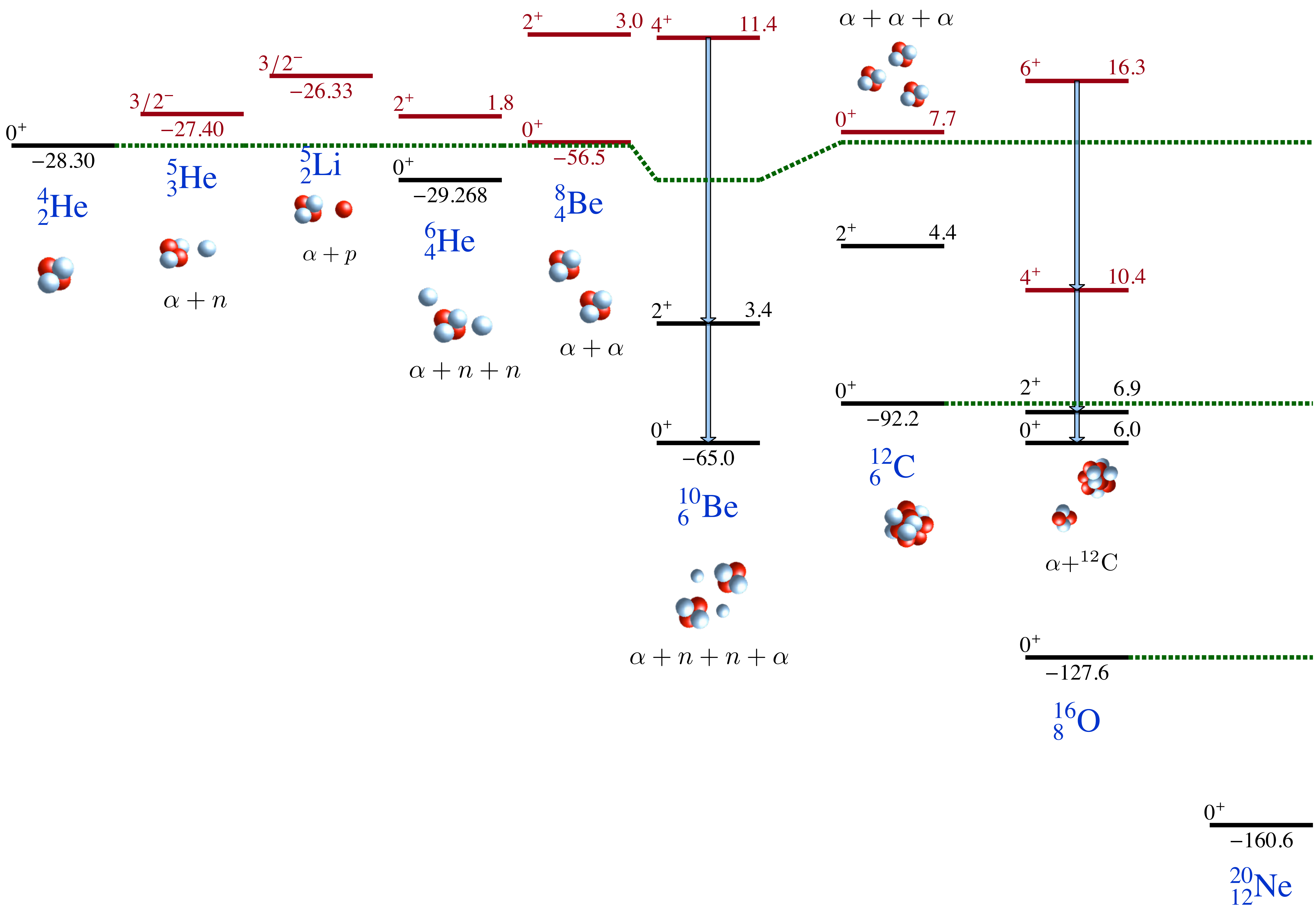
$$|\Phi_{(6,0):L}\rangle = |p^2(sd)^4[4]\,(6,0),:L\,S=T=0\rangle$$

$$|\Phi_{(4,0):L}\rangle = |p^4[4]\,(4,0),:LS=T=0\rangle$$

transition	B(E2) $e^2\mathrm{fm}^4$
$2^+(1) \to 0^+(1)$	4.1
$2^+(1) \to 0^+(2)$	44.2
$4^+(1) \to 2^+(1)$	15.4
$4^+(1) \to 2^+(2)$	4.2
$6^+(1) \to 4^+(1)$	0.2
$6^+(2) \to 4^+(1)$	18.3
$8^+(1) \to 6^+(1)$	0.1
$8^+(1) \to 6^+(2)$	31.7

Hamiltonian from
E. K. Warburton and B. A. Brown, Phys. Rev. C 46 (1992) 923
Y. Utsuno and S. Chiba, Phys. Rev. C83 021301(R) (2011)

Clustering in light nuclei



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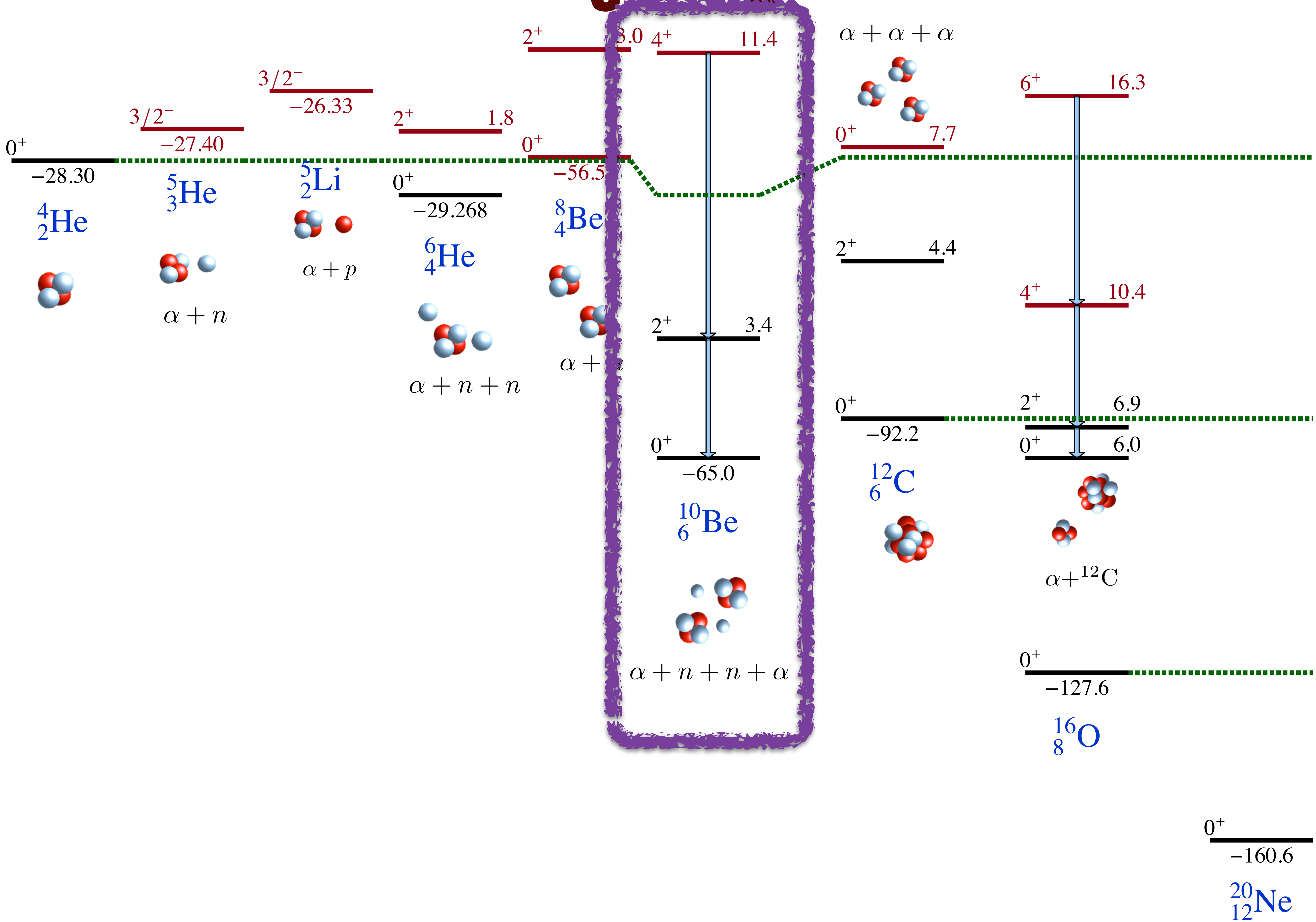

Detailed shell model analysis of ^{10}Be (^{10}Be $\rightarrow ^{6}$He+α) and experimental data

J^π_x	S_I	E_x^{th}	Γ_α^{th}	E_x^{exp}	Γ_α^{exp}	$\theta^2_\alpha(r_1)$	$\theta^2_\alpha(r_2)$
0^+_1	0.686	0.000		0			
2^+_1	0.563	3.330		3.368			
0^+_2	0.095	4.244		6.197			
2^+_2	0.049	5.741		5.958			
2^+_3	0.052	6.123		(a)			
1^-_1	0.027	6.290		5.96			
3^-_1	0.098	6.926		7.371		$0.42^{(b,c)}$	
2^+_4	0.116	7.650	$3\cdot10^{-4}$	7.542	$5\cdot10^{-4}$	$1.1^{(b,c)}$	0.19
0^+_3	0.023	8.068	17				
4^+_1	0.049	8.933	4.7				
1^-_2	0.045	9.755	180	10.57			
3^-_2	0.046	9.897	61				
2^+_5	0.027	10.819	50	9.56	$141^{(e)}$		0.074
2^+_6	0.023	11.295	43				
0^+	0.153	11.403	800				
4^+_2	0.370	11.426	180	10.15	$185^{(c)}$	$1.5^{(c)}$	0.38
5^-_1	0.148	11.440	150	11.93	200		0.20
1^-_5	0.013	12.650	76				
6^+_1	0.013	13.134	24	$13.54^{(c,f)}$	99	$1.0^{(c)}$	0.051
5^-_2	0.128	13.545	250				
2^+_{10}	0.040	13.789	240				
4^+_3	0.011	13.992	20	11.76	121		0.066
4^+_4	0.022	14.233	40				
0^+_6	0.018	14.252	120				
3^-_7	0.014	14.468	77				
5^-_3	0.059	14.992	180				
4^+_5	0.161	15.071	800	$15.3(6^-)^{(d)}$	$800^{(e)}$		0.16

(a) The existence of this state is suggested by the existence of 8.070 MeV isobaric analog state in ^{10}B, see analogous discussion in Ref. [20];
(b) Widths deduced from the isobaric analog channel ^{10}B $\rightarrow ^{6}$Li(0^+)+α [21,22];
(c) results from Ref. [22];
(d) results from Ref. [23].
(e) Total width Γ^{tot}.
(f) In Ref. [22] the state was assigned spin-parity 6^+.

r_1=4.77 r_2=6.0 fm

[21]A. N. Kuchera et al.: Phys. Rev. C 84 (2011) 054615.
[22] A. N. Kuchera http://diginole.lib.fsu.edu/etd/8585/
[23] D. R. Tilley et al.: Nucl. Phys. A 745 (2004) 155.

No-core shell model studies with JISP16 Hamiltonian, clustering in $^{10}Be \rightarrow {}^{6}He+\alpha$

	psd	N_{max}=0	N_{max}=2	N_{max}=4	N_{max}=4	Exp
SF	0.686	0.713	0.622	0.609	0.687	0.55 [2]
operators	3	1	7	7	20	
Radius [fm]		3.4	4.0	4.0	4.5	4.7-6

$$r \approx \sqrt{\frac{\hbar}{m\omega}\left(n_{\max} + \frac{3}{2}\right)} \qquad \hbar\omega = 20 \text{ MeV}$$

In order to get to r=6 fm, n_{max}=16 is needed, relative to core this is N_{max}=10, 14

[1] P. Maris and J. Vary, Int. J. Mod. Phys. E 22, 1330016 (2013); J. Vary private communication
[2] W. Oelert, Few-body Problems, Volume 3; Volume 1985 By E. Hadjimichael, W. Oelert, see page 252

Acknowledgements:
Thanks to: K. Kravvaris, Yu. Tchuvil’sky, J. Vary, T Dytrych, G. Rogachev, V. Goldberg, V. Zelevinsky
Funding: U.S. DOE contract DE-SC0009883.

Publications:

A. Volya and Y. M. Tchuvil'sky, Phys.Rev.C 91, 044319 (2015); J. Phys. Conf. Ser. 569, 012054 (2014); (World Scientific, 2014), p. 215.
M. L. Avila, G. V. Rogachev, V. Z. Goldberg, E. D. Johnson, K. W. Kemper, Y. M. Tchuvil'sky, and A. Volya, Phys. Rev. C 90, 024327 (2014).