DAMPING OF SIMPLE MODES OF HIGH-ENERGY NUCLEAR EXCITATIONS: DISPERSIVE OPTICAL MODELS AND THEIR IMPLEMENTATIONS

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Outline

I. Introduction

The resonance-line structures, corresponding to simple modes of nuclear excitations (single-quasiparticle- and particle-hole-type) are observed in nuclear reactions at high excitation energies (up to a few tens of MeV). Therefore, for these energies the mean-field concept does work, or (that is the same) nuclei are "grey" (not "black") for the mentioned degrees of freedom.

To describe coupling of these high-energy modes to many-quasiparticle (chaotic) states (the spreading effect) only a phenomenological way seems to be realistic and related to the corresponding optical models. Microscopically-based transition to these models is based on many-body Green function method introduced in nuclear physics by Migdal.

The main topic of this presentation is the description of the recently developed particle-hole dispersive optical model (PHDOM) and some implementations of this model. The "traditional" single-quasiparticle dispersive optical model (SQDOM) is also discussed together with a description of deep-hole states.

II. PHDOM

1. General description

1.1. The particle–hole dispersive optical model is developed recently (U., PAN'11, PRC'13) to describe in a semimicroscopic way the main properties of a great variety of high-energy (p-h)-type nuclear excitations (including giant resonances) in "hard" medium-heavy mass spherical nuclei.

1.2. Within the model, the main relaxation modes of the above-mentioned excitations are commonly taken into account.

These modes are:

(i) distribution of the p-h strength, or Landau damping ,~ the result of shell structure of nuclei;

(ii) coupling of (p-h)-type states to the s.p. continuum ~ nuclei are the open Fermi-systems;

(iii) coupling of (p-h)-type states to many quasi-particle (chaotic) configurations, or the spreading effect ~ high excitation energies.

1.3. Within the PHDOM, which is a semi-microscopic model, Landau damping and coupling to the s.p. continuum are described microscopically (in terms of a mean field and p-h interaction), while the spreading effect is treated phenomenologically and in average over the energy (in terms of the imaginary part of an effective optical-model potential).

1.4. Microscopically based transition to the PHDOM (as well as to the single-quasiparticle DOM) is performed with the use of the many-body Green function method introduced in nuclear physics by Migdal (“Nauka”,’67,’83).

Actually, the PHDOM is an extension of the standard and non-standard continuum-RPA (cRPA) versions on phenomenological account for the spreading effect. The imaginary part of the effective optical-model potential determines also the corresponding real part via a proper dispersive relationship (Tulupov, U., PAN’09).

1.5. The unique feature of the PHDOM is its ability to describe:

(i) direct-nucleon-decay properties of the (p-h)-type states, including the so-called direct+semidirect (DSD) reactions induced by a s.p. external field;

(ii) the energy-averaged double p-h transition density and, therefore, various strength functions at arbitrary (but high-enough) excitation energies, including giant resonances.

1.6. Ingredients of the model:

(i) Landau-Migdal p-h interaction and a phenomenological partially self-consistent mean field. In the description of photonuclear reactions the isovector velocity-dependent forces taken in the simplest (separable) form are also used;

(ii) the energy-dependent phenomenological imaginary part of an effective optical-model potential

2. Basic relationships (schematically)

2.1. The expressions for the main energy-averaged quantities look similarly to the corresponding expressions of the cRPA standard and non-standard versions (i.e. obtained without taking the spreading effect into account).

(i) The strength function $S_{V_0}(\omega)$ corresponding to a s.p. external field V_0 (ω is the excitation energy):

$$S_{V_0}(\omega) = -\frac{1}{\pi}\mathrm{Im}\int V_0^+(x)A_0(x,x',\omega)V(x',\omega)dxdx'.$$

(ii) The effective field $V(x, \omega)$ is different from $V_0(x)$ due to a p-h interaction $F(x, x')$, which is responsible for long-range correlations:

$$V(x, \omega) = V_0(x) + \int F(x, x_1)A_0(x_1, x_2, \omega)V(x_2, \omega)dx_1dx_2,$$

Where the key PHDOM quantity A_0 is the “free” p-h propagator corresponding to the model of independent and damping quasiparticles.

(iii) The amplitude $M_{V,\mu}$ of the DSD reaction induced by the external field $V_0(x)$ and accompanied by population of one-hole state μ^{-1} of the product nucleus:

$$M_{V,\mu}(\omega) = n_\mu^{1/2} \int \varphi_{\varepsilon>0}^{(+)}(x) V(x,\omega) \varphi_\mu^*(x) dx,$$

$$b_\mu(\delta) = \int_{(\delta)} \left| M_{V,\mu}(\omega) \right|^2 d\omega \Big/ \int_{(\delta)} S_{V_0}(\omega) d\omega$$

Here, n_μ is the occupation factor, $\varphi_{\varepsilon>0}^{(+)}(x)$ and $\varphi_\mu(x)$ are the s.p. continuum-state and bound-state wave functions, $\varepsilon = \omega + \varepsilon_\mu > 0$ is the kinetic energy of the escaped nucleon, $b_\mu(\delta)$ is the partial branching ratio for direct nucleon decay from an energy interval δ.

(iv) The Bethe-Goldstone-type equation is actually the basic PHDOM equation

$$A(x,x',\omega) = A_0(x,x',\omega) + \int A_0(x,x_1,\omega)\, F\,(x_1,x_2) A(x_2,x',\omega) dx_1 dx_2$$

for the energy-averaged p-h effective propagator, which determines the p-h double transition density

$$\rho(x,x',\omega) = -\frac{1}{\pi} \mathrm{Im}\, A(x,x',\omega)$$

and also the above-mentioned strength function

$$S_{V_0}(\omega) = -\frac{1}{\pi} \mathrm{Im} \int {V_0}^{+}\,(x) A(x,x',\omega) V_0(x') dx dx'.$$

2.2 The key PHDOM quantity $A_0(x, x', \omega)$ is the energy-averaged “free” p-h propagator. Being derived with taking a statistical assumption into account, the expression for A_0 is:

$$A_0(x, x', \omega) = \sum_{\lambda\mu} \varphi_\lambda(x)\varphi_\mu^*(x)\varphi_\lambda^*(x')\varphi_\mu(x')A_{\lambda\mu}(\omega),$$

$$A_{\lambda\mu} = \frac{n_\lambda - n_\mu}{\varepsilon_\lambda - \varepsilon_\mu - \omega + (n_\lambda - n_\mu)[iW(\omega) - P(\omega)]f_\lambda f_\mu},$$

where $[-iW(\omega) + P(\omega)]$ is the intensity of a specific p-h interaction, which appears due to the spreading effect, $f_\lambda = \int f_{WS}(x)|\varphi_\lambda(x)|^2 dx$, $f_{WS}(r, R, a)$ – the Woods-Saxon function.

2.3. The PHDOM continuum version follows from the approximate transformation of the above-given expression for $A_0(x, x', \omega)$ to the form, which contains also the optical-model Green functions $g(x, x', \varepsilon_\mu \pm \omega)$. The non-gomogenious equation for these functions contains the optical-model-like addition to the mean field: $[-iW(\omega) +$

Actually, the dispersive relationship follows from the 2p-2h Green function spectral expansion.

The simplest version of this relationship

$$P(\omega) = \frac{2}{\pi} P.V. \int W(\omega') \left\{ \frac{\omega'}{\omega^2 - \omega'^2} + \frac{1}{\omega'} \right\} d\omega$$

is adopted to satisfy the condition: $P(\omega \to 0) \to 0$.

2.4 A weak violation of the PHDOM unitarity takes place due to an energy dependence of $P(\omega)$ and $W(\omega)$ and also to the use of the approximate spectral expansion for the OM Green functions. This violation can be eliminated (Gorelik et al., NSRT-2015, Dubna).

Restoration of the model unitarity can be done as follows:

(i) Within the realistic approximation $(dW/d\omega)^2 \ll dP/d\omega$, the key PHDOM quantity is renormalized:

$$A_{\lambda\mu} \to A^R_{\lambda\mu} = \frac{(n_\lambda - n_\mu)R_{\lambda\mu}(\omega)}{\varepsilon_\lambda - \varepsilon_\mu - \omega + (n_\lambda - n_\mu)\left[iW^R_{\lambda\mu}(\omega) - P(\omega)f_\lambda f_\mu\right]},$$

$$R_{\lambda\mu}(\omega) = 1 - f_\lambda f_\mu \frac{dP(\omega)}{d\omega}; W^R_{\lambda\mu}(\omega) = W(\omega) f_\lambda f_\mu R_{\lambda\mu}(\omega).$$

As a result, the renormalized p-h strength is restored:

$$S^R_{\lambda\mu} = -\frac{1}{\pi}\int \mathrm{Im}A^R_{\lambda\mu}(\omega)d\omega = (1 - n_\lambda)n_\mu$$

(ii) In the description of the isoscalar monopole (ISM) strength functions corresponding to an ISM external field $V_0(\mathbf{r}) = V_0(r)Y_{00}$, the radial part $V_0(r)$ should be modified:

$$V_0 \to V_0(r) - < V_0 >,$$

Where averaging is performed on the ground-state matter density.

Improvement of description of the EWSR for ISM strength function is about 5%.

In particular, small negative values of these strength functions are excluded.

3. Simplest photonuclear reactions (Tulupov, U. PRC'14).

3.1. The isovector giant dipole and quadruple resonances (IVGDR and IVGQR) are systematically studied by means of photonuclear reactions. The simplest reactions are photoabsorption and DSD photoneutron and inverse reactions.

To describe these reactions within the PHDOM we use the corresponding external fields as follows ($Q_{LM} = r^L Y_{LM}$):

IVGDR$\rightarrow$ $V_0(x) = -\frac{1}{2}\tau^{(3)}Q_{1M}$; ISGQR+IVGQR $\rightarrow$ $\frac{1}{2}\left(1-\tau^{(3)}\right)Q_{2M}$.

Within the accuracy $1 \ll (N-Z) \ll A$, the equations for isovector $(T=1)$ and isoscalar $(T=0)$ effective fields are decoupled. These fields $V_{LM}^{T=1}\left(V_{0,LM}^{T=1} = Q_{LM}\right)$ and $V_{2M}^{T=0}\left(V_{0,2M}^{T=0} = Q_{2M}\right)$ determine, in particular, the neutron effective fields:

$$V_{1M}^{(n)} = -\frac{1}{2}V_{1M}^{T=1};\quad V_{2M}^{(n)} = \frac{1}{2}\left(V_{2M}^{T=0} - V_{2M}^{T=1}\right).$$

The latters determine the amplitudes of the DSD photoneutron and inverse reactions. The excitation of the IVGQR (and ISGQR) in these reactions is possible only due to a p-h interaction.

3.2. Photoabsorption cross section

$$\sigma_{a,E1}(\omega) \quad (\sigma_{a,E1} + \sigma_{a,E2})$$

The adjustable parameters obtained to describe $\sigma_a^{exp}(\omega)$

Nucleus	^{89}Y	^{140}Ce	^{208}Pb
α, MeV^{-1}	0.125	0.10	0.08
k'	0.15	0.13	0.17

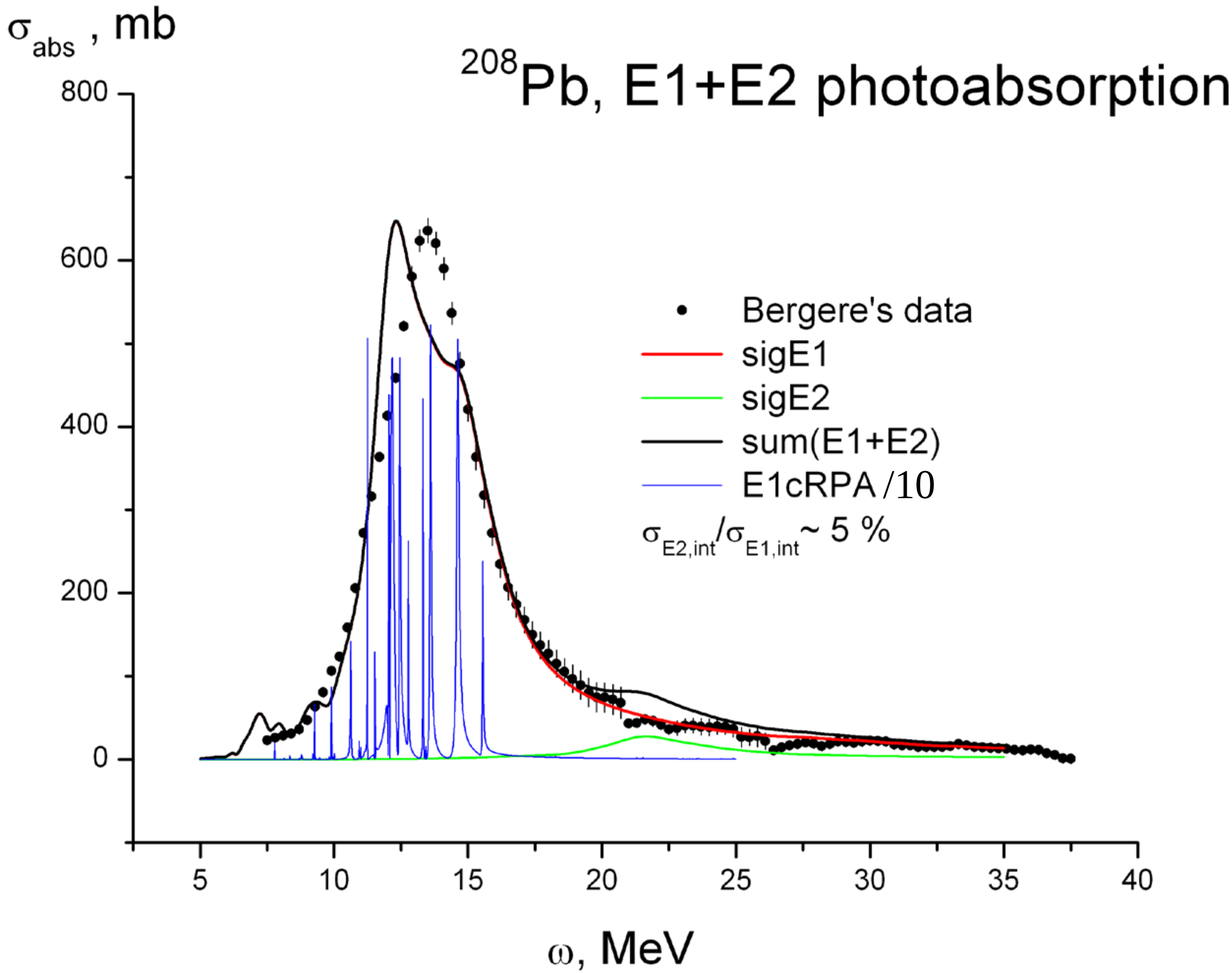
^{208}Pb

$$\delta = 7.5 - 37.5 \text{ MeV}$$

$$\sigma_{int}^{calc} = 3633 \text{ mbMeV}$$

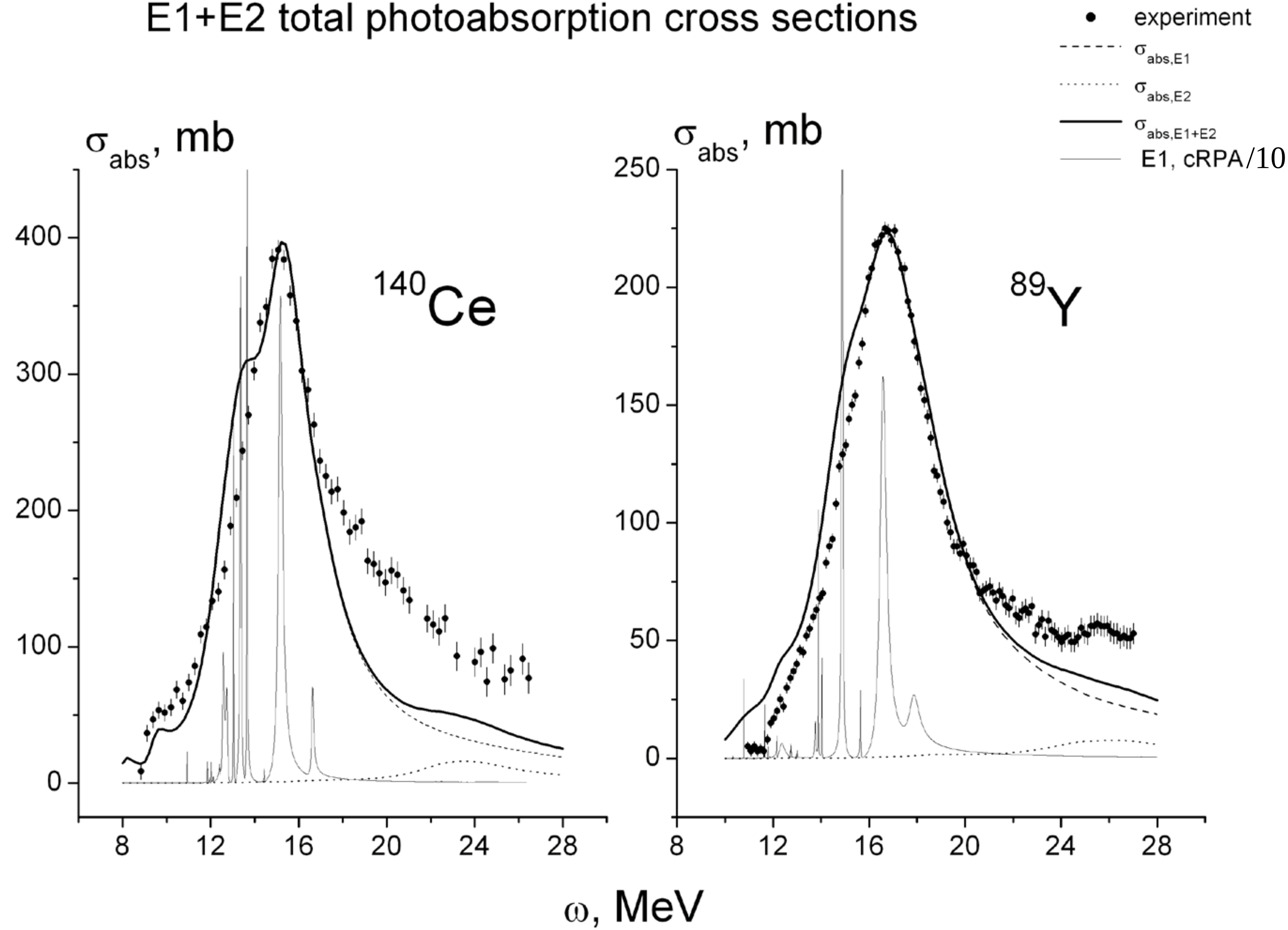
$$\sigma_{int}^{exp} = 3583 \text{ mbMeV}$$

A consistency of the model



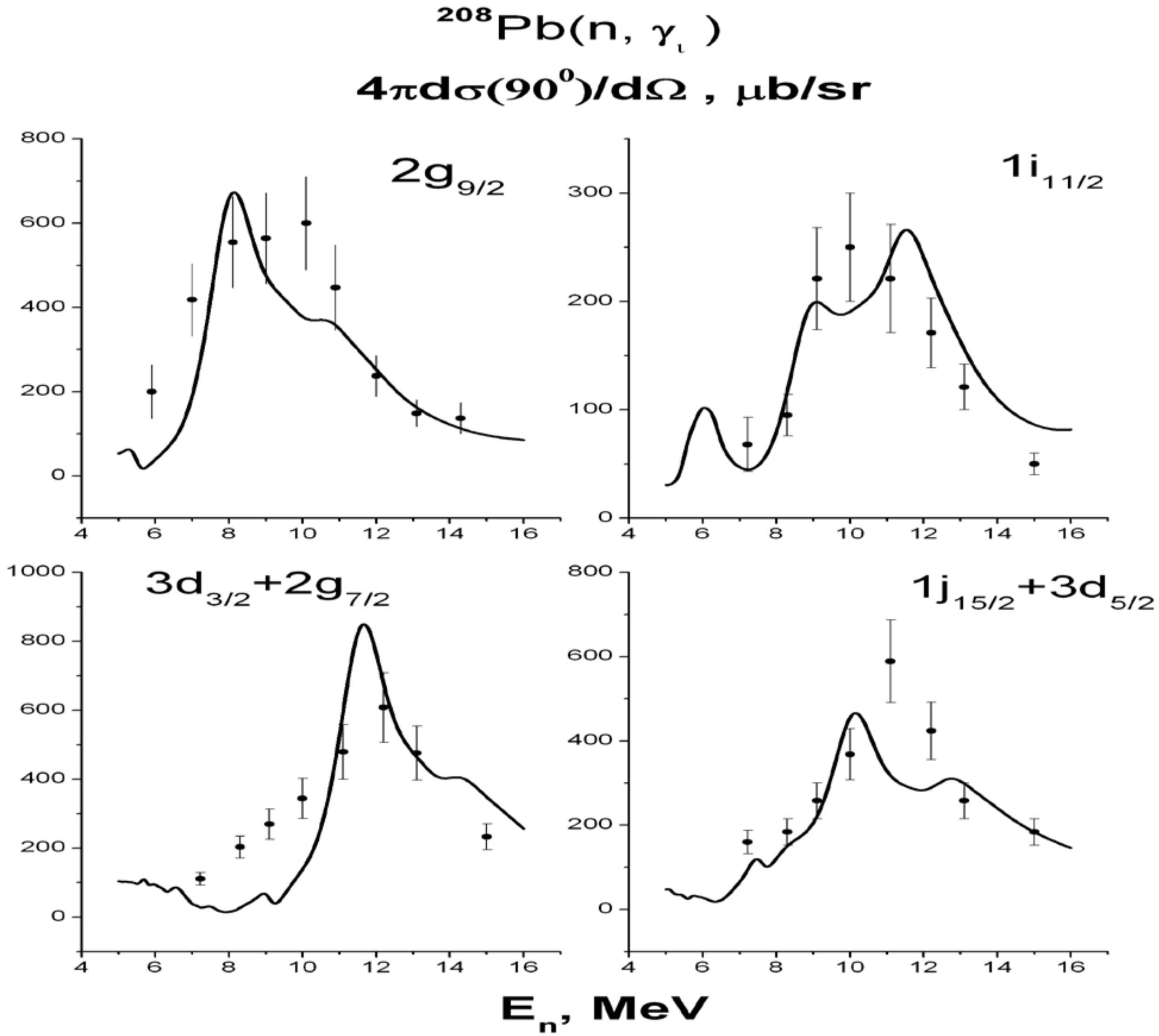
σ_{abs} , mb
^{208}Pb, E1+E2 photoabsorption
800
600
400
200
0
Bergere's data
sigE1
sigE2
sum(E1+E2)
E1cRPA /10
$\sigma_{E2,int}/\sigma_{E1,int}$ ~ 5 %
5
10
15
20
25
30
35
40
ω, MeV

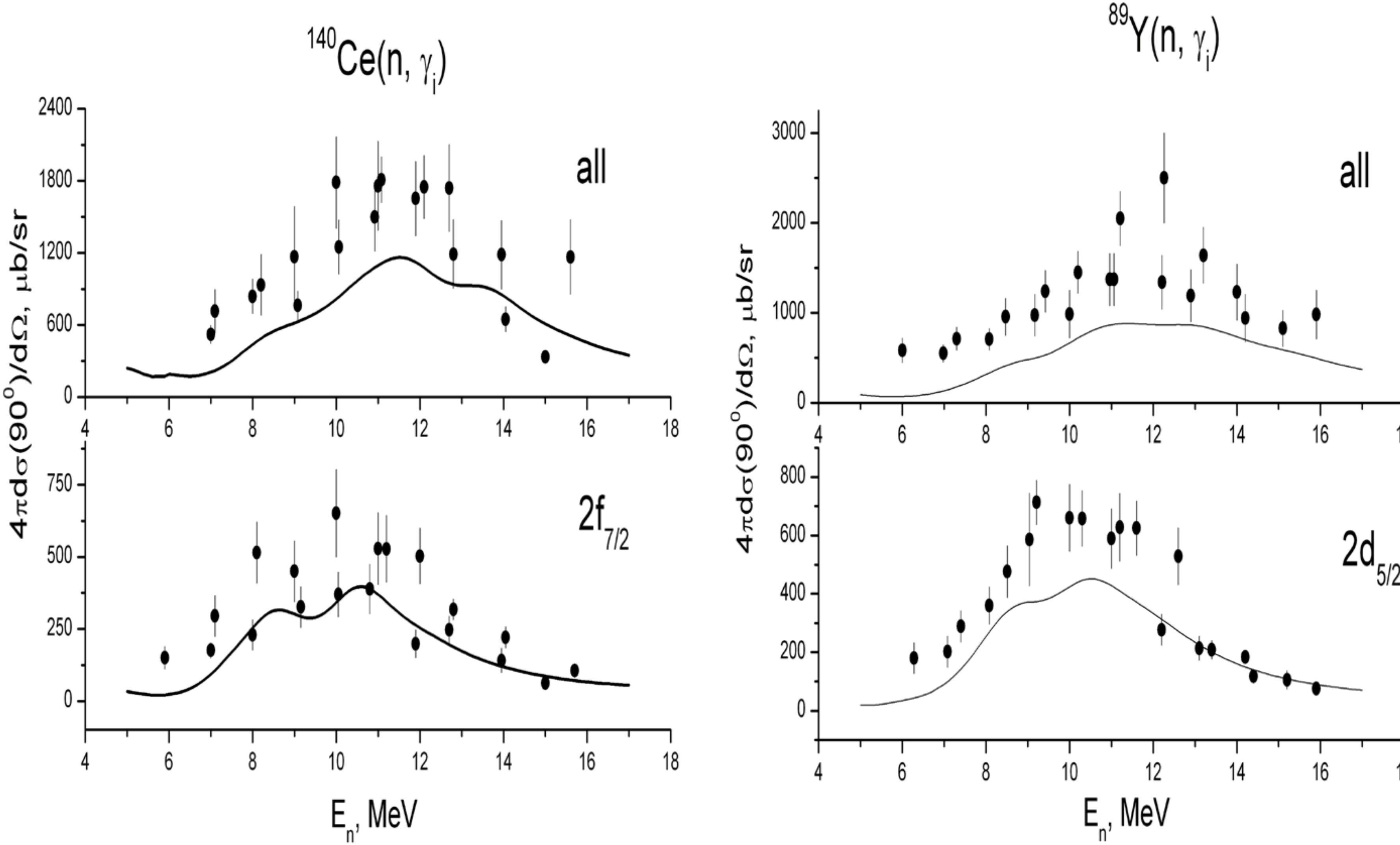
E1+E2 total photoabsorption cross sections



3.3. DSD neutron radiative capture

No free parameters!





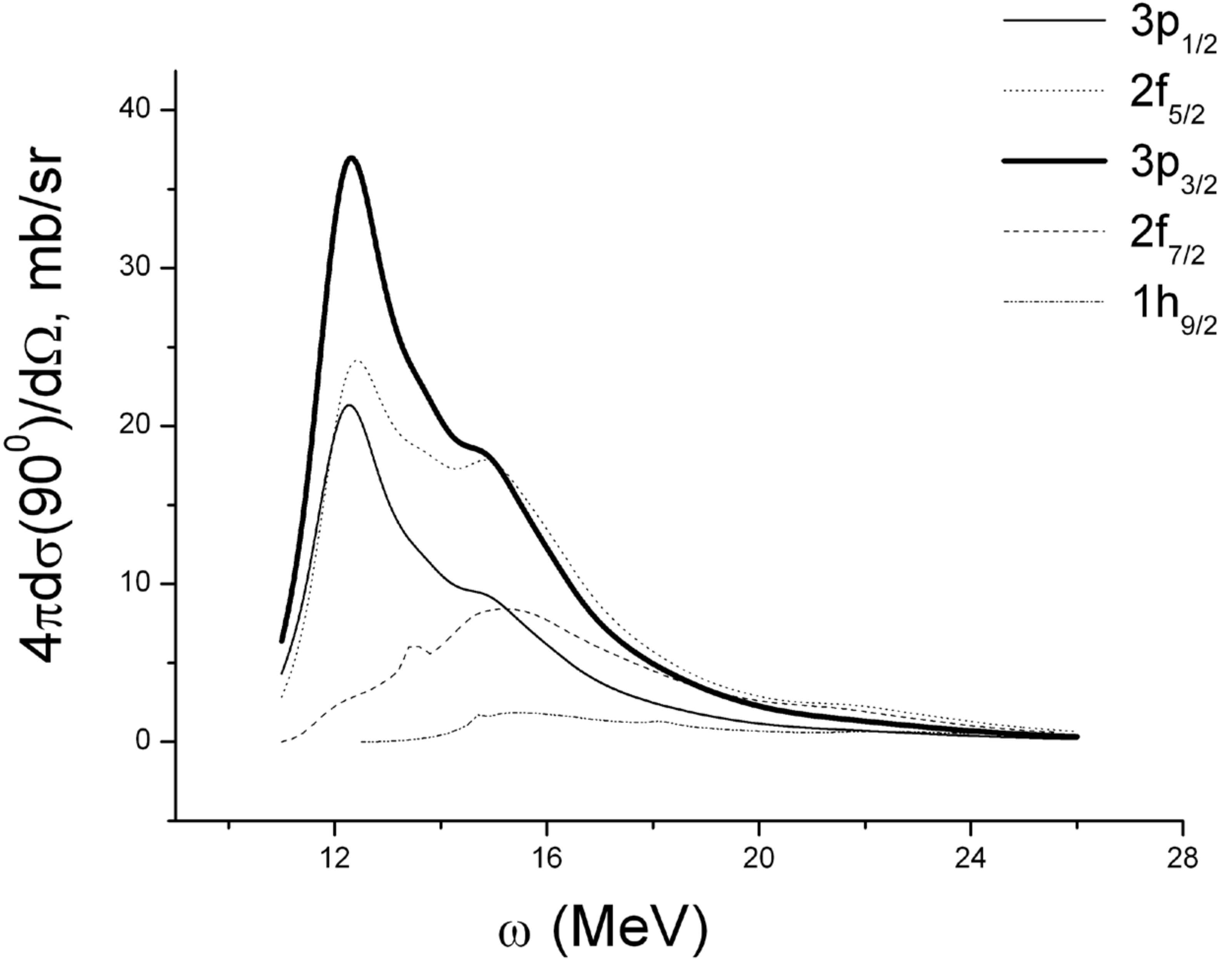
^{140}Ce(n, γ_i)
^{89}Y(n, γ_i)
all
all
$2f_{7/2}$
$2d_{5/2}$
$4\pi d\sigma(90^o)/d\Omega$, μb/sr
$4\pi d\sigma(90^o)/d\Omega$, μb/sr
E_n, MeV
E_n, MeV

3.4. Partial DSD ^{208}Pb(γ,n) reaction cross sections (predictions).

Partial branching ratios for IVGDR direct neutron decay

μ	$3p_{1/2}$	$2f_{5/2}$	$3p_{3/2}$	$1i_{13/2}$	$1h_{9/2}$	$1f_{7/2}$
b_μ	1.79	3.61	3.10	1.37	2.15	0.53

$$b_{\mathrm{tot}} = 12.55\%$$



3p1/2
2f5/2
3p3/2
2f7/2
1h9/2
4πdσ(90⁰)/dΩ, mb/sr
ω (MeV)
0
10
20
30
40
12
16
20
24
28

3.5. IVGDR+IVGQR

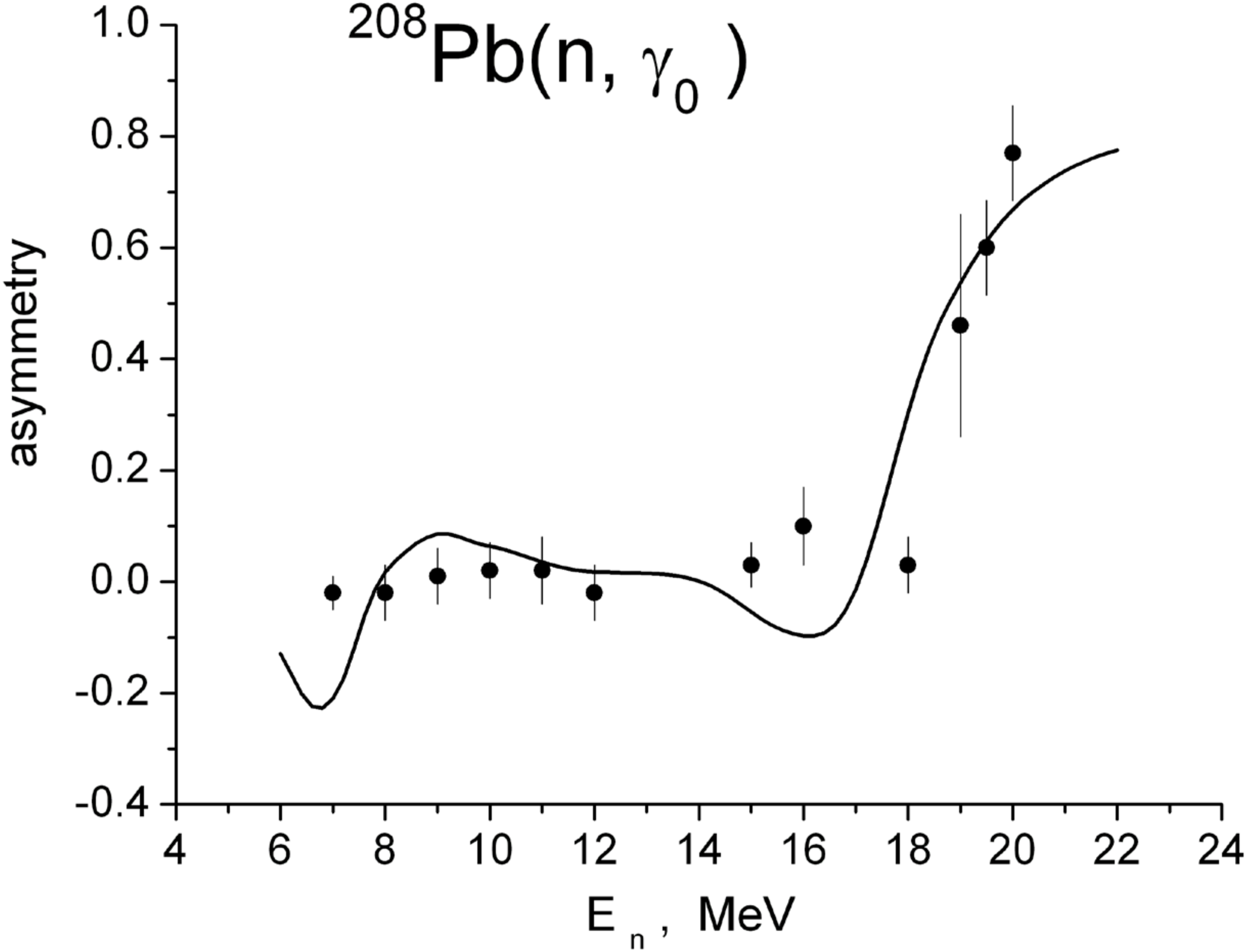
The asymmetry of the DSD partial differential (γ,n) and inverse reaction cross sections is linear on the E2-reaction amplitude and, therefore, is the appropriate subject for study of the IVGQR in photonuclear reactions.

$$\alpha_\mu = \frac{d\sigma_\mu^{(-)}(\omega,\theta_1)}{d\sigma_\mu^{(+)}(\omega,\theta_1)};$$

$$\frac{d\sigma_\mu^{(\mp)}(\omega,\theta_1)}{d\Omega} = \frac{d\sigma_\mu(\omega,\theta_1)}{d\Omega} \mp \frac{d\sigma_\mu(\omega,\pi-\theta_1)}{d\Omega};$$

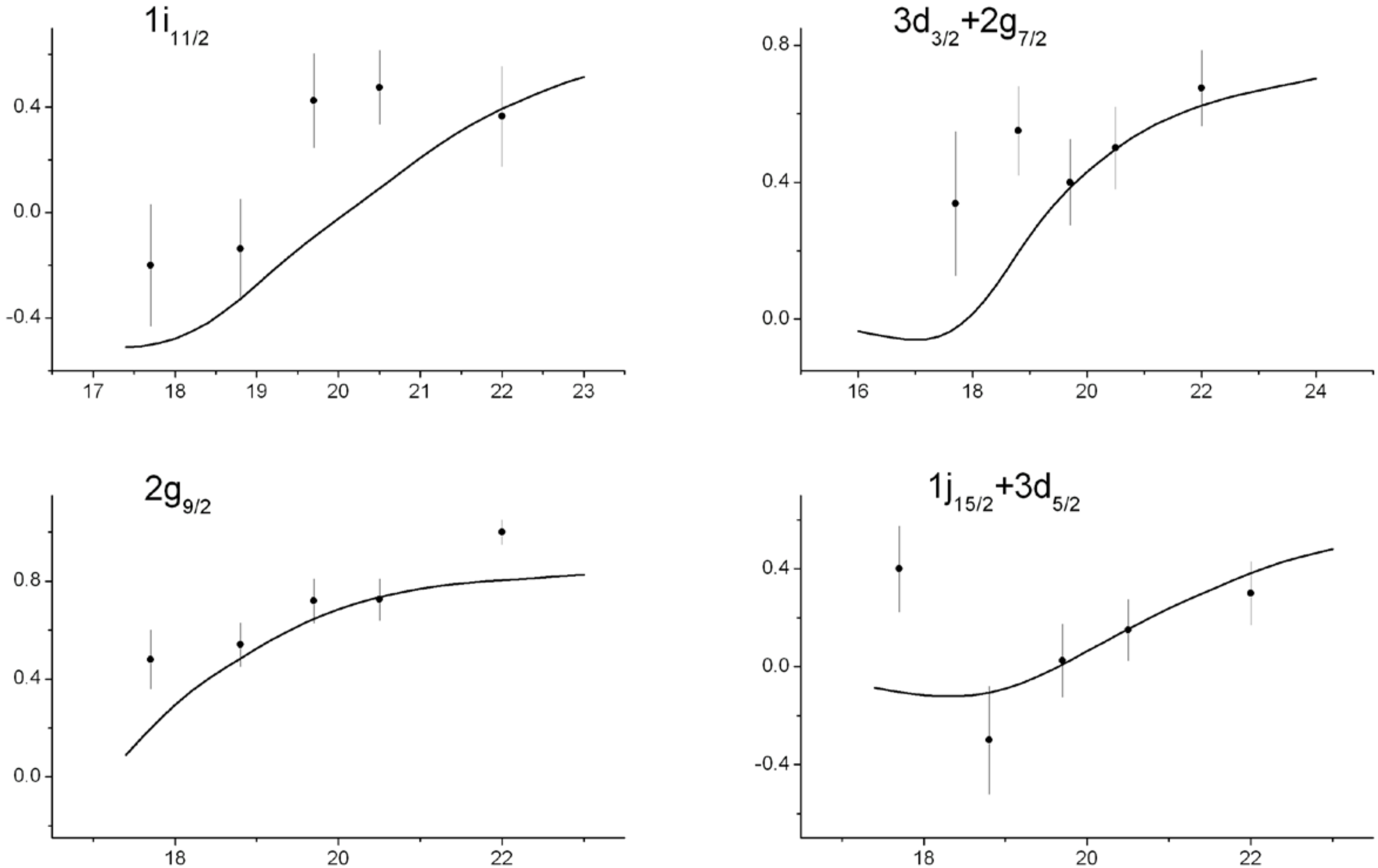
$$\theta_1 = 55^\circ$$

The adjustable parameter $k_2' = 0.1$

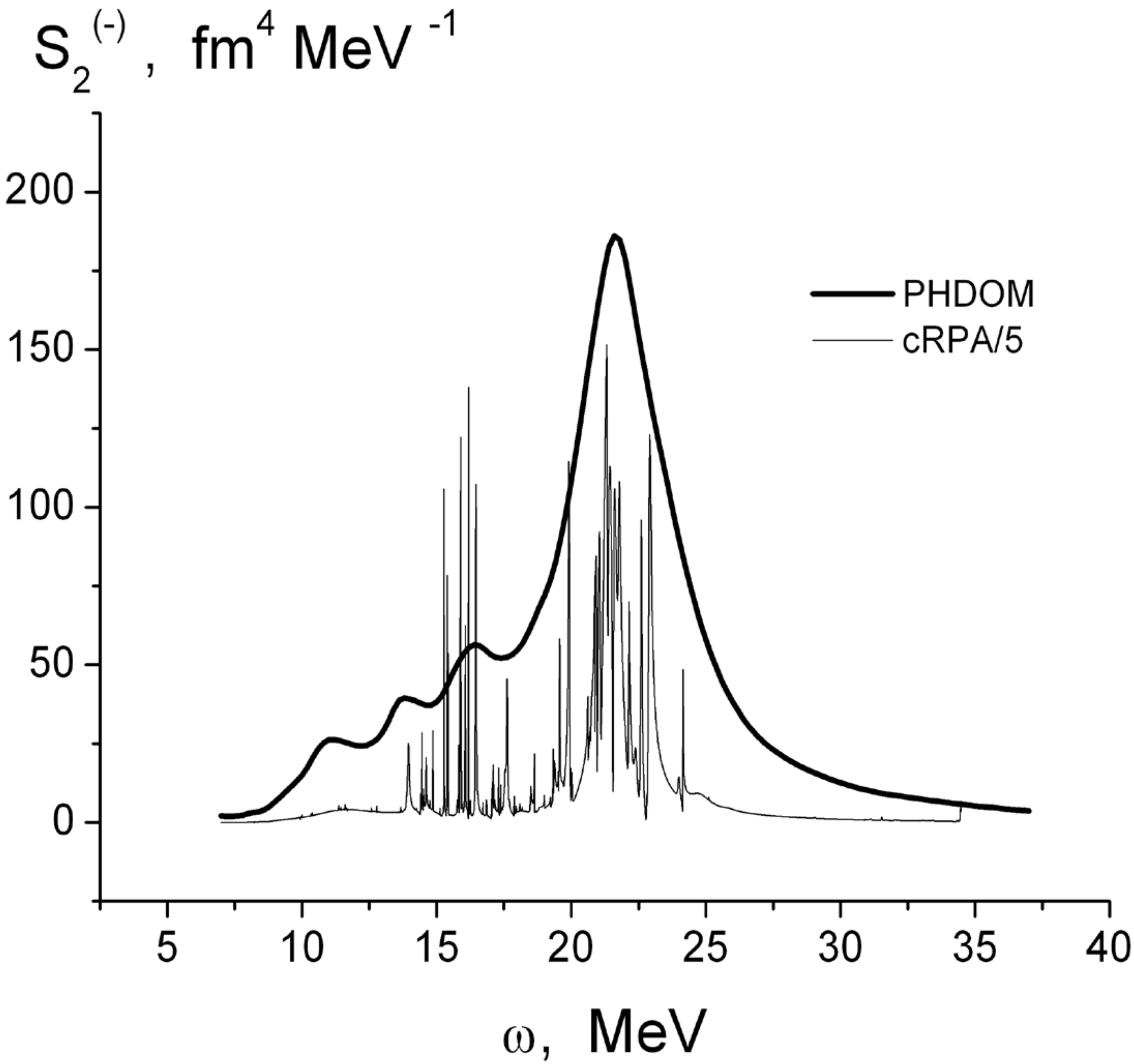


$^{208}Pb(n, \gamma_0)$
asymmetry
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
4
6
8
10
12
14
16
18
20
22
24
E_n, MeV

^{209}Bi(n, γ_i) asymmetry



E_n, MeV



S2(-), fm4 MeV -1
200
150
100
50
0
5
10
15
20
25
30
35
40
ω, MeV
PHDOM
cRPA/5

4. Isoscalar monopole (ISM) excitations in medium-heavy mass spherical nuclei (Gorelik, Shlomo, Tulupov, U., PAN ,`15)

Investigations of ISM excitations allow to get info about nuclear matter incompressibility coefficient.

4.1. First, we study within the PHDOM the <u>ISM relative energy-weighted strength functions</u>

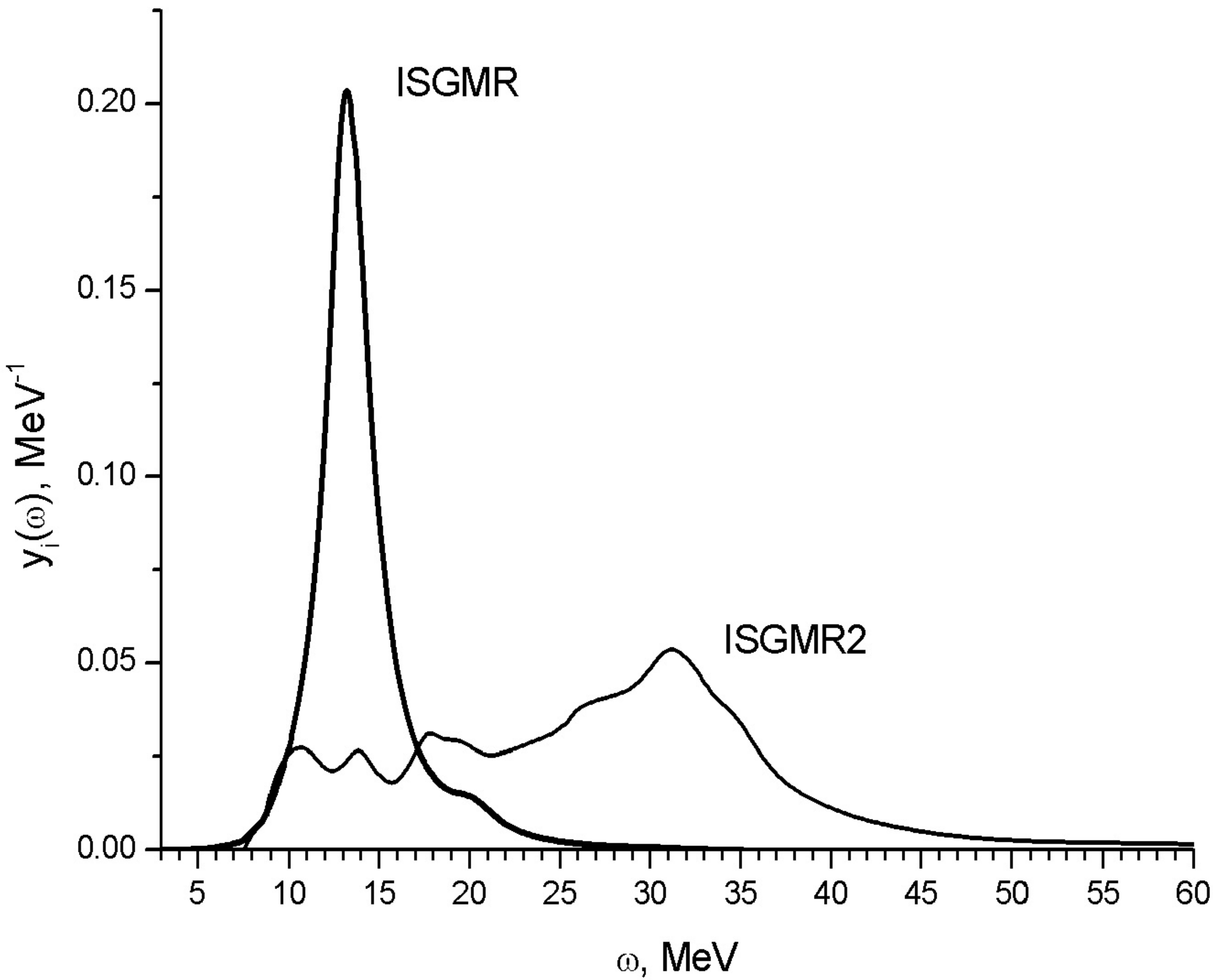
$$y_i(\omega) = \omega S_{V_{0,i}}(\omega) / EWSR_{V_{0,i}} ,$$

corresponding to the ISM external fields $V_{0,i}(x)$

$V_{0,1} = r^2 Y_{00}$ and $V_{0,2} = r^2(r^2 - \eta) Y_{00}$

(η is an adjustable parameter) which lead to excitation of the isoscalar monopole giant resonance (ISGMR) and its overtone (ISGMR2).

The strength functions calculated within the PHDOM for ^{208}Pb are shown in the following figure.



ISGMR
ISGMR2
0.20
0.15
0.10
0.05
0.00
$y_i(\omega)$, MeV^{-1}
5
10
15
20
25
30
35
40
45
50
55
60
ω, MeV

4.2. To deduce the ISM strength distribution from the inelastic (α, α')-scattering cross sections at small angles actually it is necessary to know the ISM energy-averaged double transition density:

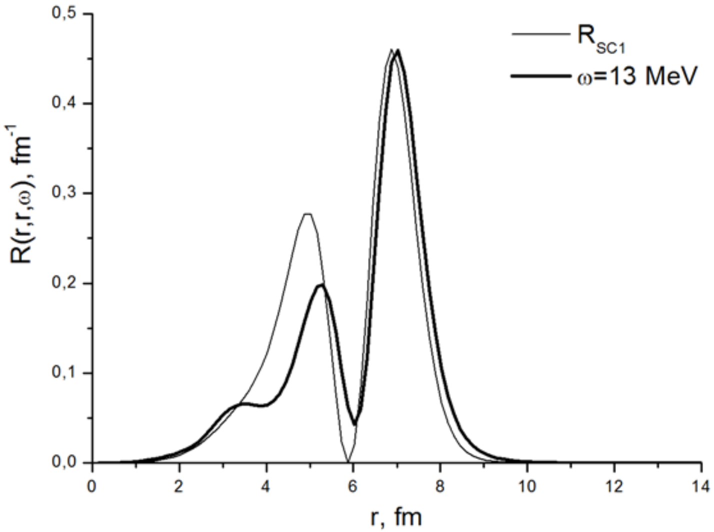
$$\rho(r, r', \omega) = \langle \rho(r, \omega) \rho(r', \omega) \rangle$$

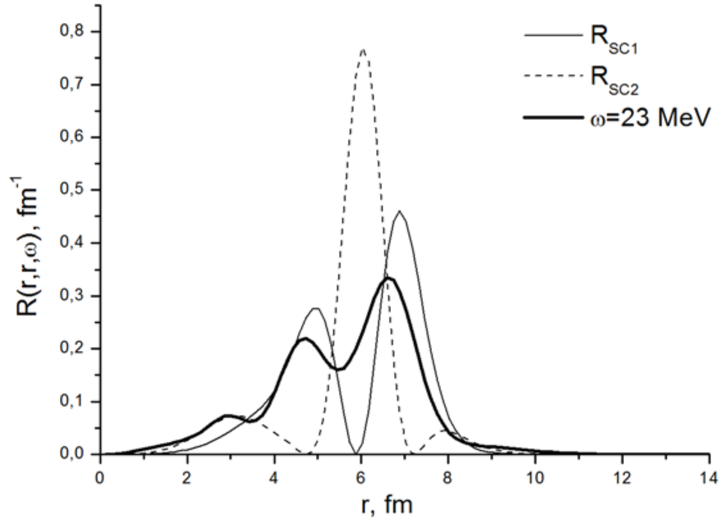
at arbitrary energies. For ^{208}Pb we evaluate within the PHDOM the corresponding reduced quantity

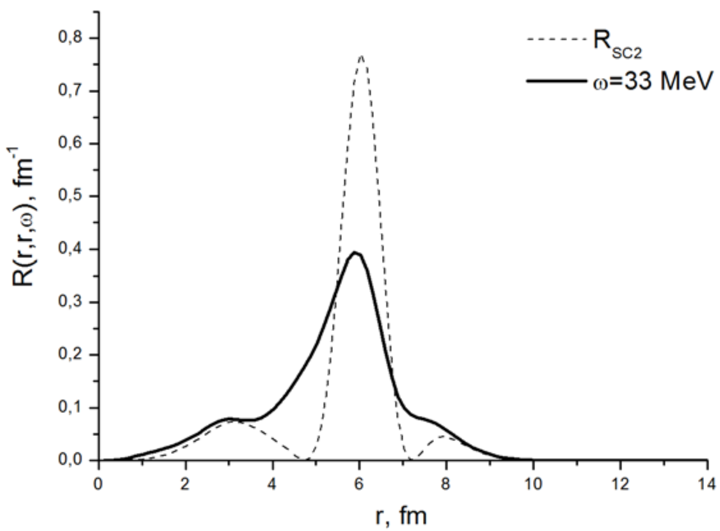
$$R(r, r', \omega) = \rho(r, r', \omega) / \int \rho(r = r', \omega) dr$$

and compare the results for $R(r = r', \omega)$ with those obtained with the use of the semi-classical collective model transition densities $\rho_{sc,i}(r)$ (independent of ω):

$$R_{sc,i}(r = r') = \rho_{sc,i}^2(r) / \int \rho_{sc,i}^2(r) dr\,.$$


R(r,r,ω), fm-1
r, fm
RSC1
ω=13 MeV
0,0
0,1
0,2
0,3
0,4
0,5
0
2
4
6
8
10
12
14


R(r,r,ω), fm-1
r, fm
RSC1
RSC2
ω=23 MeV
0,0
0,1
0,2
0,3
0,4
0,5
0,6
0,7
0,8
0
2
4
6
8
10
12
14


R(r,r,ω), fm-1
r, fm
RSC2
ω=33 MeV
0,0
0,1
0,2
0,3
0,4
0,5
0,6
0,7
0,8
0
2
4
6
8
10
12
14

III. SQDOM

- Being the oldest nuclear model, the OM was originally formulated in terms of the *S*-matrix for nucleon scattering by the potential having an imaginary part. The dispersive version of the model has been formulated in a rather formal way (Mahaux, Sartor, Adv. Nucl. Phys. 1991).
- Being interested in description of damping of deep-hole states in medium-heavy spherical nuclei, we start formulation of the SQDOM from the energy-averaged Dyson equation for the s-p Green's function. As a result, we get the following relationships presented below in a simplified form (Kolomiytsev, Igashov, U., PAN '14, NSRT-2015).

3.1 Basic relationships

- Equation for the radial Green's function of an OM Shrödinger equation:

$$\{h_{0,jl}(r) + \Delta(r,\varepsilon) - iW(r,\varepsilon)\mathrm{sgn}(\varepsilon-\mu) - \varepsilon\}g_{jl}(r,r',\varepsilon) = -\delta(r-r').$$

- $h_{0,jl}(r)$ is the radial part of a s-p Hamiltonian adopted to describe the single-quasiparticle spectra near the Fermi energy (chemical potential μ) for the doubly-closed-shell nuclei;
- $W(r,\varepsilon) = W(\varepsilon)f_{WS}(r)$ is the OM potential imaginary part taken as the even function of excitation energy $E = |\varepsilon-\mu|$ for particles and holes: $W(\varepsilon) = W(E)$ ($f_{WS}(r)$ is the Woods-Saxon function);
- $\Delta(r,\varepsilon) = \Delta(\varepsilon)f_{WS}(r)$, $\Delta = \Delta^p + \Delta^d$, where $\Delta^p(\varepsilon)$ simulates the mean-field energy dependence, while $\Delta^d(\varepsilon)$ is due to the spreading effect and satisfies the dispersive relationship:

$$\Delta^d(\varepsilon) = \frac{2E}{\pi}\ P.V.\int_0^\infty \frac{W(E')}{E^2 - E'^2}\,dE'$$

In fact, this equation follows from the spectral expansion for the 3-particle Green function which determines the s-p self-energy operator.

3.2 From the spectral expansion of the s-p Green function follows the number of particles conversation (the model unitarity):

$$N = \sum_{jl} (2j+1) n_{jl}\, ; n_{jl} = \int S_{jl}^{(-)}(\varepsilon) d\varepsilon\, ,$$

Where the s-h strength function is defined as follows:

$$S_{jl}^{(-)}(\varepsilon) = \frac{1}{\pi} \mathrm{Im} \int g_{jl}(r = r', \varepsilon < \mu) dr\, .$$

3.3 The OM potential energy-dependent part, $\Delta(\varepsilon)$, is the main source of model unitarity violation. In the limit $W \to 0$, the OM Green function's poles ε_λ ($\lambda = \{n_r, j, l\}$), corresponding to s-h excitation energies $|E_\lambda| = |\varepsilon_\lambda - \mu|$, are determined by the equation

$$\{h_{0,jl}(r) + \Delta(r, \varepsilon_\lambda) - \varepsilon_\lambda\} \chi_\lambda(r) = 0$$

(the s-h wave functions are supposed to be normalized to unity).

From the equation for $g_{jl}(r,r',\varepsilon)$ one gets:

$$g_{jl}(r,r',\varepsilon \to \varepsilon_\lambda) \to \frac{\chi_\lambda(r)\chi_\lambda(r')}{\varepsilon - \varepsilon_\lambda - \frac{i\Gamma_\lambda}{2}} R_\lambda,$$

$$R_\lambda = \frac{1}{1 - \Delta'(\varepsilon_\lambda) f_\lambda}; \Gamma_\lambda = 2W(\varepsilon_\lambda) f_\lambda R_\lambda$$

(the realistic supposition $\left(W'(\varepsilon_\lambda)\right)^2 \ll \Delta'(\varepsilon_\lambda)$ was used).

The s-h strengths $S_\lambda^{(-)} = \int S_\lambda^{(-)}(\varepsilon) d\varepsilon = R_\lambda$ are found to be strongly overestimated (realistic values of R_λ are about factor of 2).

3.4 To restore the model unitarity, one can use renormalized basic equation:

$$\left\{h_{0,jl}(r) + \Delta(r,\varepsilon) - \frac{iW(r,\varepsilon)}{R(\varepsilon)}\operatorname{sgn}(\varepsilon-\mu) - \varepsilon\right\} g_{jl}^{R}(r,r',\varepsilon) = -\frac{1}{R(\varepsilon)}\delta(r-r').$$

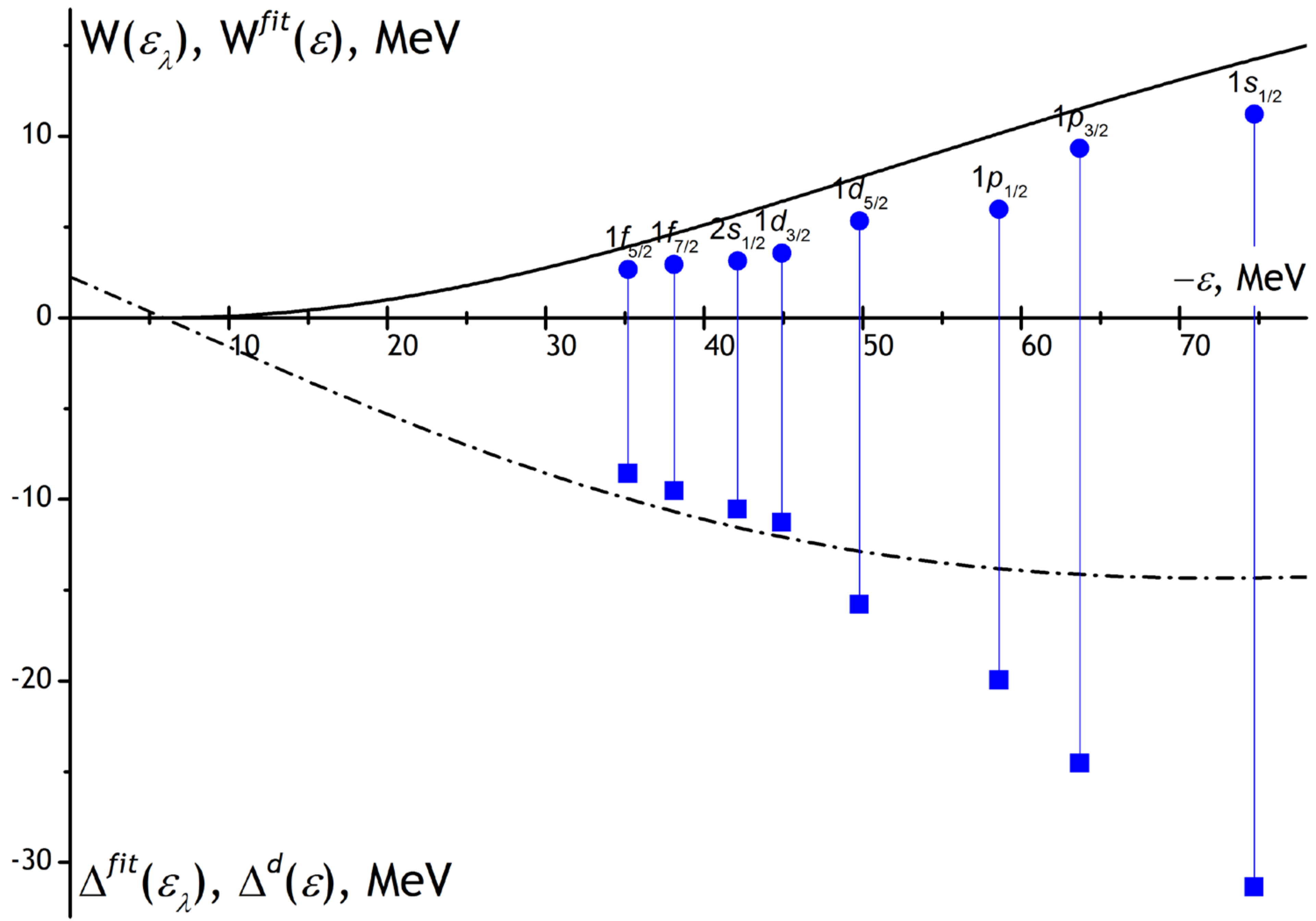
In a vicinity of the s-h energy ε_λ one gets:

$$g_{jl}^{R}(r,r',\varepsilon\to\varepsilon_\lambda) \to \frac{\chi_\lambda(r)\chi_\lambda(r')}{\varepsilon-\varepsilon_\lambda-\frac{i\Gamma_\lambda^R}{2}}$$

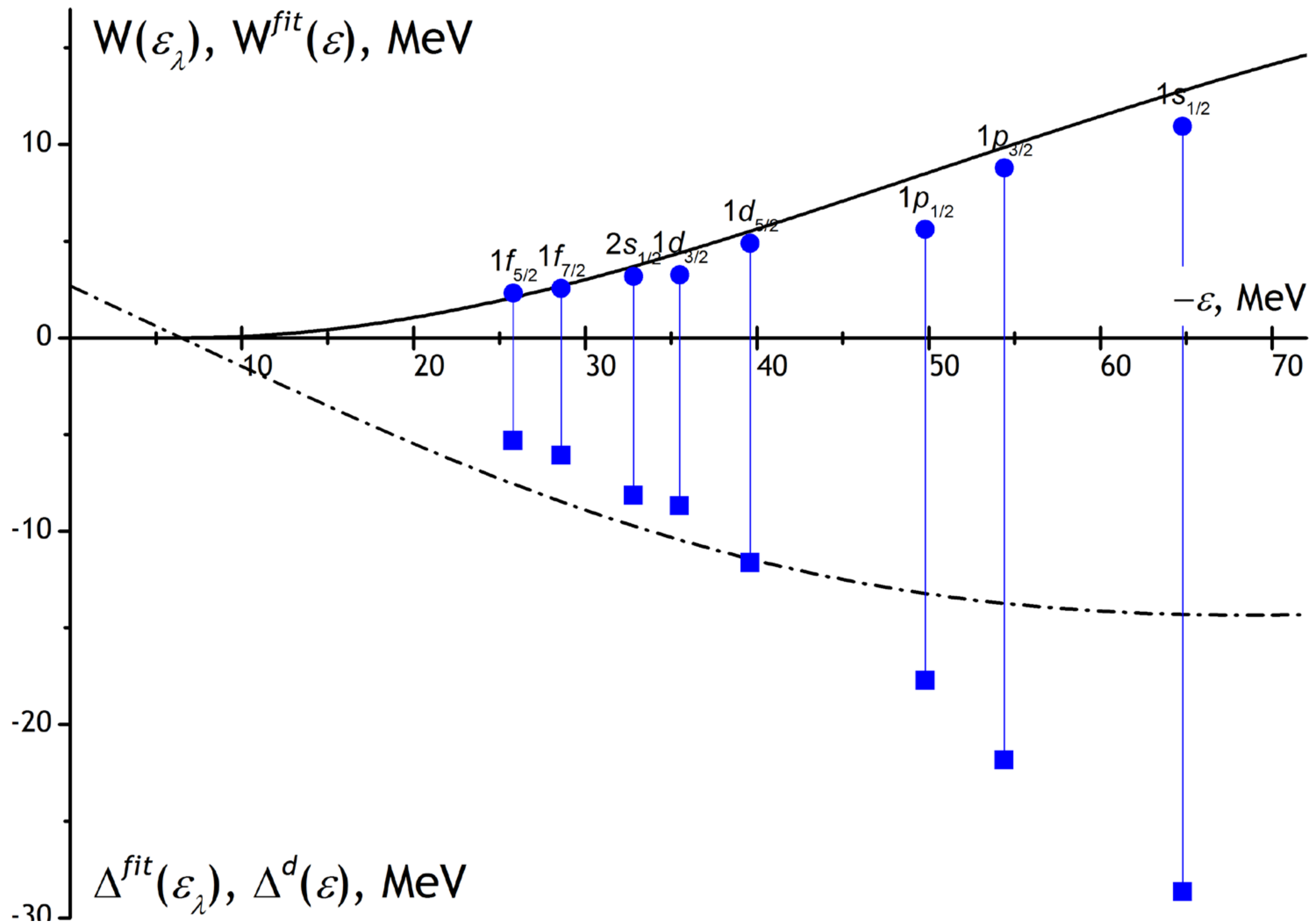
with $\Gamma_\lambda^R = 2W(\varepsilon_\lambda)f_\lambda$ and with the s-h strength $S_\lambda^{(-)R} \approx 1$.

3.5 Implementation of the SQDOM to deep-hole states

- Contribution of the spreading effect to the depp-hole state excitation energy for ^{90}Zr and ^{208}Pb is estimated with the use of experimental deep-hole energies ε_λ and total widths Γ_λ deduced from the $(p,2p)$- and (p,pn)-reactions (Vorobyov et al., PAN'95). Being based on the above-given eqs. for $\chi_\lambda(r)$ and Γ_λ^R one can find the quantities $W(\varepsilon_\lambda), \Delta(\varepsilon_\lambda)$. The set of values $W(\varepsilon_\lambda)$ is then adopted by a proper function $W^{fit}(|E|)$, which is further used to estimate the OM potential dispersive part $\Delta^d(\varepsilon)$. The results shown in Figs. for neutron and proton deep-hole states in ^{208}Pb illustrate significant contribution of the spreading effect to the deep-hole states excitation energy.

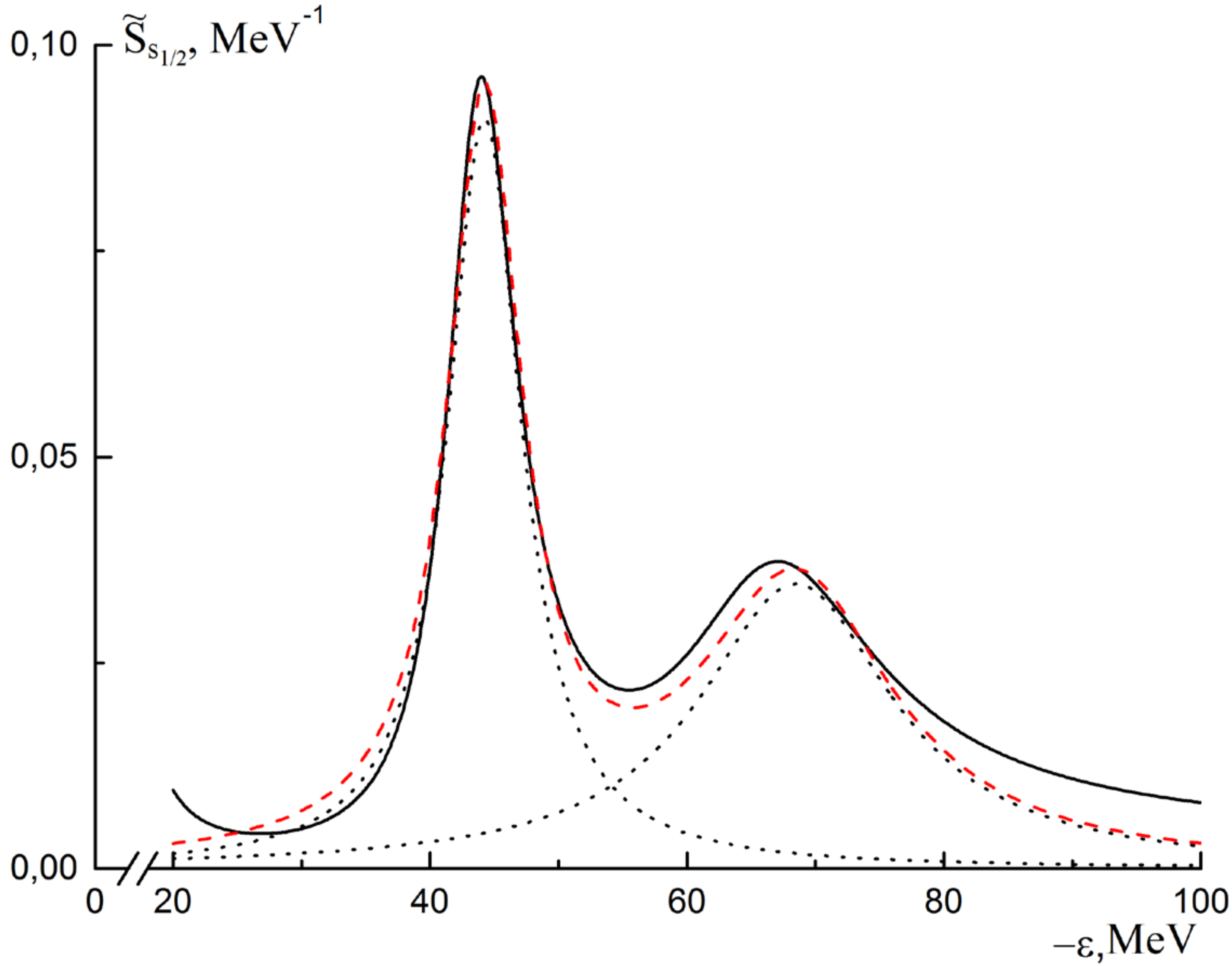


$W(\varepsilon_\lambda)$, $W^{fit}(\varepsilon)$, MeV
$\Delta^{fit}(\varepsilon_\lambda)$, $\Delta^{d}(\varepsilon)$, MeV
$-\varepsilon$, MeV
$1s_{1/2}$
$1p_{3/2}$
$1p_{1/2}$
$1d_{5/2}$
$1d_{3/2}$
$2s_{1/2}$
$1f_{7/2}$
$1f_{5/2}$
10
0
-10
-20
-30
10
20
30
40
50
60
70

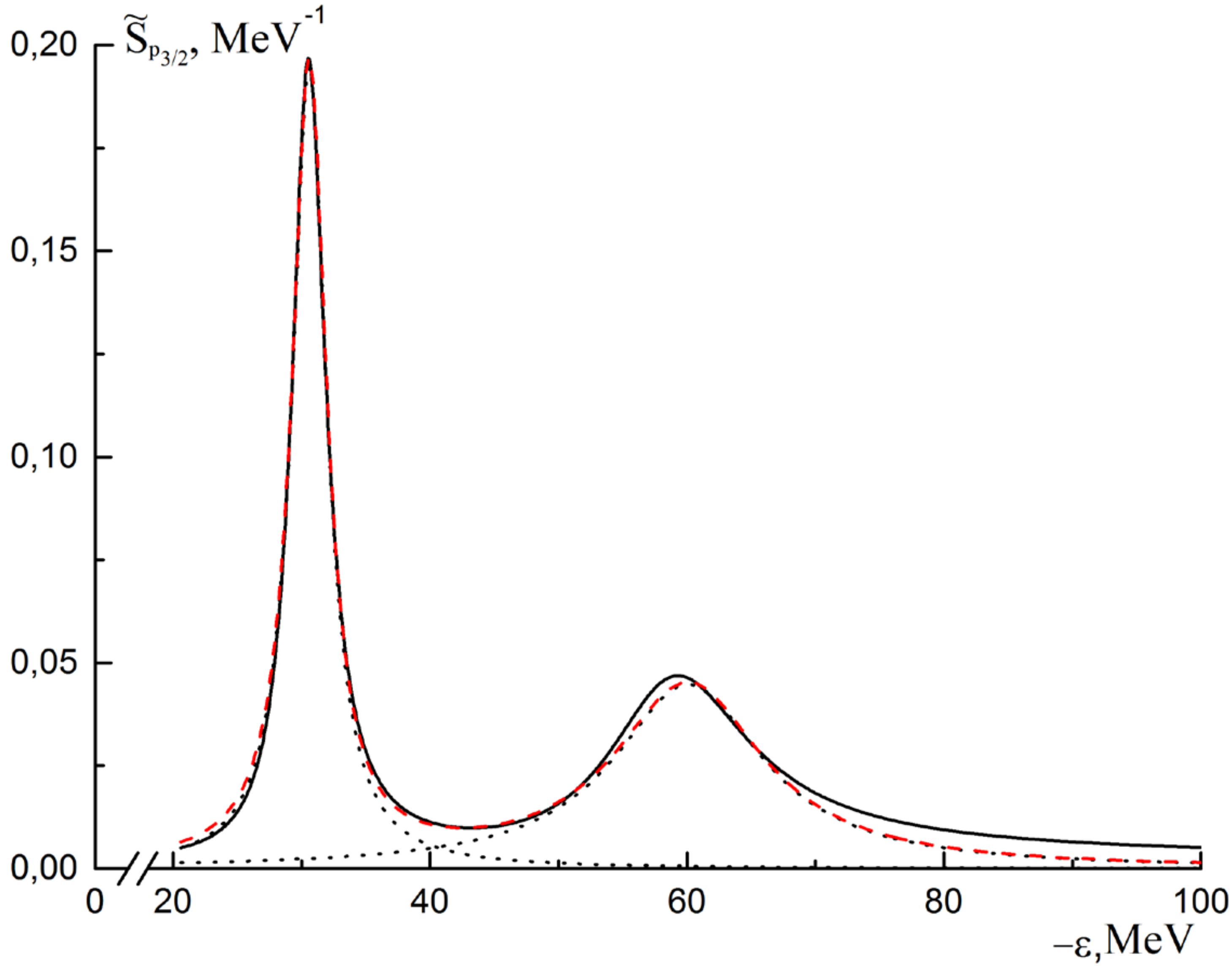


$W(\varepsilon_\lambda)$, $W^{fit}(\varepsilon)$, MeV
$\Delta^{fit}(\varepsilon_\lambda)$, $\Delta^{d}(\varepsilon)$, MeV
$-\varepsilon$, MeV
$1f_{5/2}$
$1f_{7/2}$
$2s_{1/2}$
$1d_{3/2}$
$1d_{5/2}$
$1p_{1/2}$
$1p_{3/2}$
$1s_{1/2}$
10
0
-10
-20
-30
10
20
30
40
50
60
70

- Using the renormalized OM Green function we can evaluate the s-h strength functions for the case of rather overlapping s-h resonances (e.g. $s_{1/2}$ and $p_{3/2}$ deep neutron-hole states in ^{208}Pb). In such a case representation of the strength functions as a superposition of proper lorentzians is rougthly possible (Figs.)



$\widetilde{S}_{s_{1/2}}$, MeV^{-1}
0,10
0,05
0,00
0
20
40
60
80
100
$-\varepsilon$,MeV



$\widetilde{S}_{p_{3/2}}$, MeV^{-1}
0,20
0,15
0,10
0,05
0,00
0
20
40
60
80
100
$-\varepsilon$,MeV

IV. Conclusive remarks

The presented p-h dispersive optical model allowing to take into account the main relaxation modes of high-energy nuclear excitations seems to be an “economical” model for description of main properties of these excitations in medium-heavy mass “hard” spherical nuclei. Some implementations of the model demonstrate its abilities.

The microscopically based transition to the single-quasi-particle optical model open the possibilities to further development of this model.

Many thanks for your attention!