Strength functions from large scale nuclear shell model and their applications in nuclear astrophysics

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UNDER EXTREME CONDITIONS"

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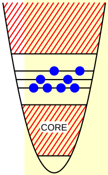
Shell model approach

Calculations Ab Initio

- Realistic NN interactions
- Diagonalization in $N\hbar\omega$ h.o.space



- define valence space
- $H_{\text{eff}}\Psi_{\text{eff}} = E\Psi_{\text{eff}}$

⇝ INTERACTIONS

- build and diagonalize Hamiltonian matrix

⇝ CODES

Weak processes:

- β decays
- $\beta\beta$ decays

$[T^{0\nu}_{1/2}(0^+ \to 0^+)]^{-1} = G_{0\nu}|M^{0\nu}|^2\langle m_\nu\rangle^2$

☞ASTROPHYSICS

☞PARTICLE PHYSICS

Collective excitations:

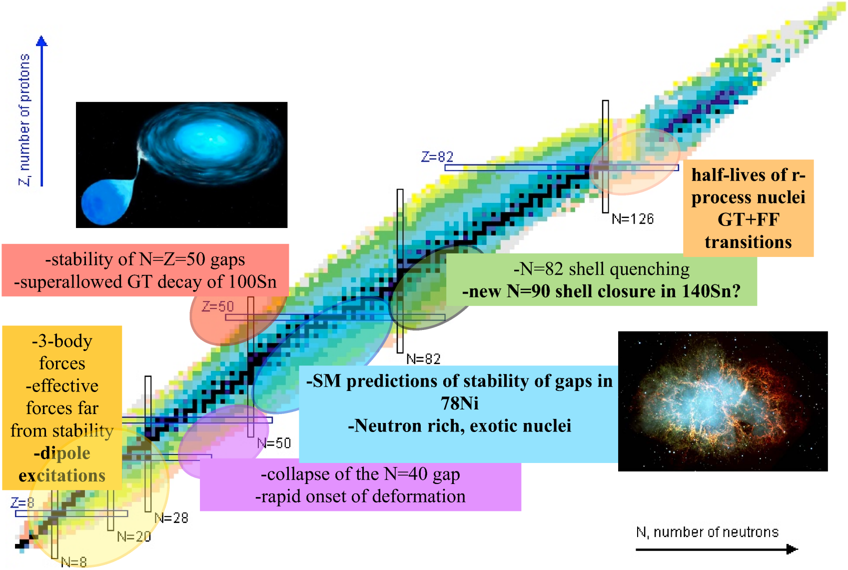
- deformation, superdeformation
- superfluidity
- symmetries

Shell evolution far from stability:

- Shell quenching
- New magic numbers

☞ASTROPHYSICS

SM with empirical interactions: regions of activity



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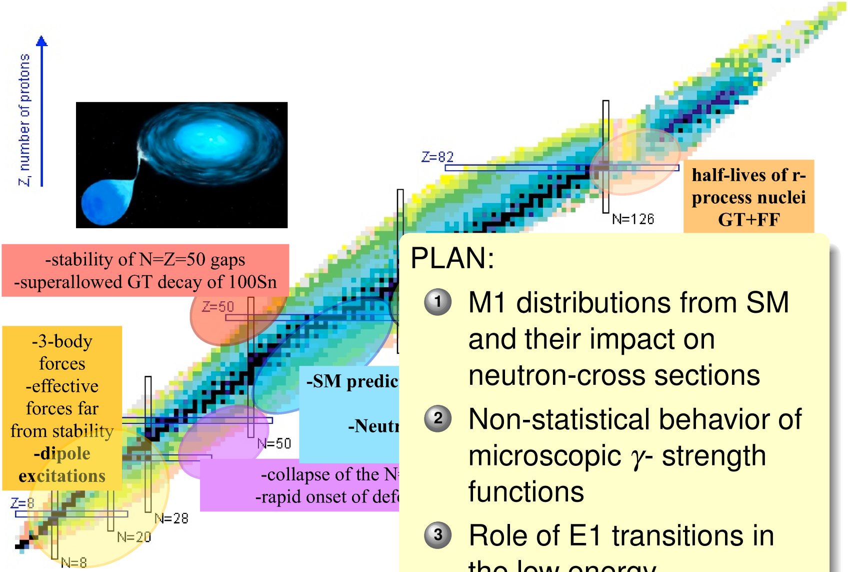


PLAN:

1. M1 distributions from SM and their impact on neutron-cross sections
2. Non-statistical behavior of microscopic γ- strength functions
3. Role of E1 transitions in the low energy enhancement

Neutron capture cross sections

$$\sigma^{\mu\nu}_{(n,\gamma)}(E_i,n) = \frac{\pi\hbar^2}{2M_{i,n}E_{i,n}} \frac{1}{(2J_i^{\mu}+1)(2J_n+1)} \sum_{J,\pi}(2J+1)\frac{T_n^{\mu}T_\gamma^{\nu}}{T_{tot}},$$

where:

$E_{i,n}, M_{i,n}$- center-of-mass energy, reduced mass of the system
$J_n = 1/2$-neutron spin
transmission coefficients:
$T_n^{\mu} = T_n(E,J,\pi;E_i^{\mu},J_i^{\mu},\pi_i^{\mu})$ $T_\gamma^{\nu} = T_\gamma(E,J,\pi;E_m^{\nu},J_m^{\nu},\pi_m^{\nu})$

For a given multipolarity

$$T_{XL}(E,J,\pi,E^{\nu},J^{\nu},\pi^{\nu}) = 2\pi E_\gamma^{2L+1} f_{XL}(E,E_\gamma)$$

Key ingredients in Hauser-Feschbach calculations:

- description of gamma emission spectra of a compound nucleus
- Brink-Axel hypothesis

Lanczos strength function method

$$S = |\hat{O}|\psi_i\rangle| = \sqrt{\langle\psi_i|\hat{O}^2|\psi_i\rangle}$$

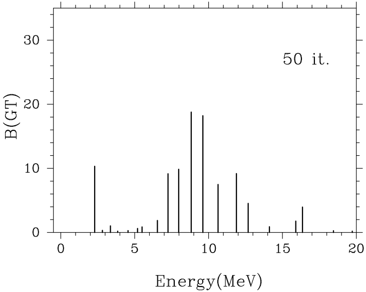
The operator $\hat{O}$ does not commute with H and $\hat{O}|\psi_i\rangle$ is not necessarily the eigenstate of the Hamiltonian. But it can be developed in the basis of energy eigenstates:

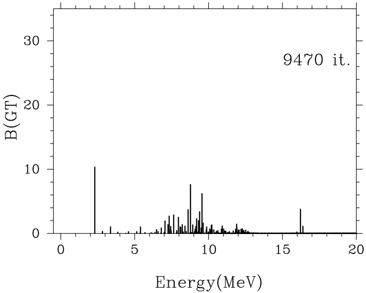
$$\hat{O}|\psi_i\rangle = \sum_f S(E_f)|E_f\rangle,$$

where $S(E_f) = \langle E_f|\hat{O}|\psi_i\rangle$ is called **strength function**.

If we carry Lanczos procedure using $|O\rangle = \hat{O}|\psi_i\rangle$ as initial vector then H is diagonalized to obtain eigenvalues $|E_f\rangle$ and after N iterations we have the also the strength function:
$\tilde{S}(E_f) = \langle E_f|O\rangle = \langle E_f|\hat{O}|\psi_i\rangle$.

How good is the strength function $\tilde{S}$ after N iterations compared to the exact one S?





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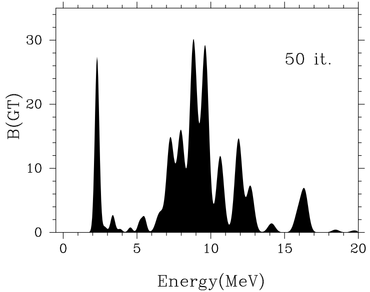
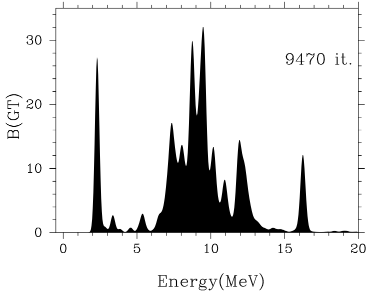
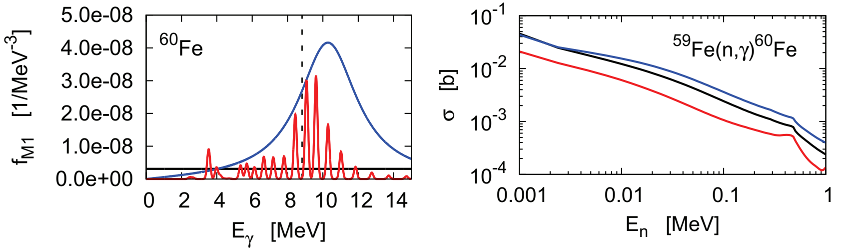




Impact of the realistic M1 fragmentation on the neutron capture cross sections

M1 microscopic strength functions in iron chain (^{53}Fe-^{70}Fe), impact on (n,γ) cross sections, tests of Brink-Axel hypothesis

H.-P. Loens K. Langanke, G. Martinez-Pinedo and K. Sieja, EPJ A48 (2012) 48



- State-by-state cross section 2 times larger than using Brink hypothesis
- Using SF of 2^+ state instead of 0^+ leads to larger cross sections

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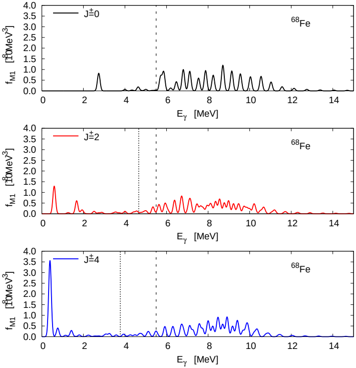

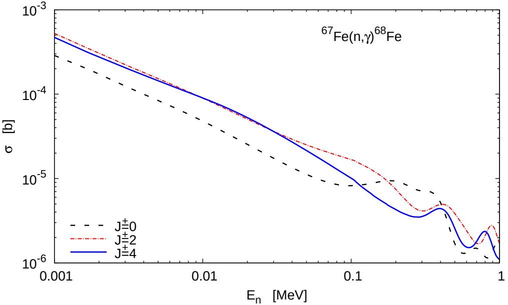

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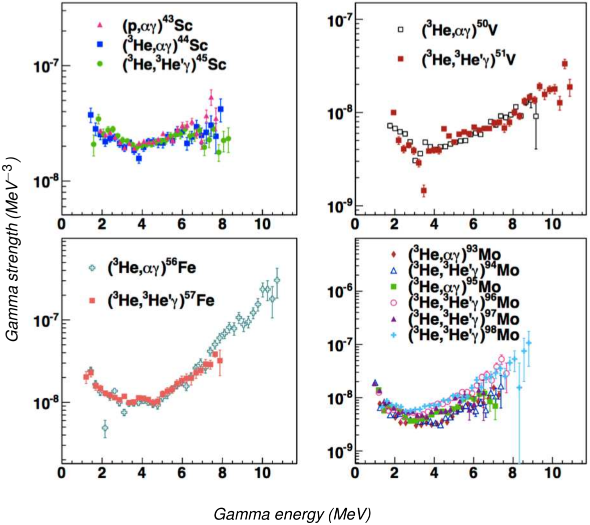
Low energy enhancement of the γ-strength function

Data from Oslo group

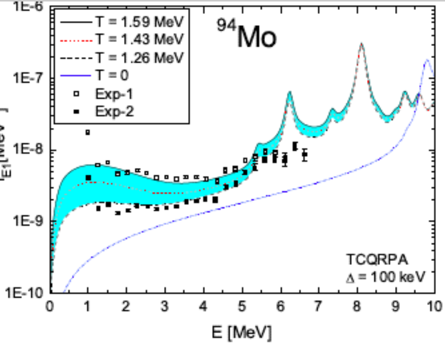


- Microscopic strength functions are different from global parametrizations
- Low energy enhancement of γ-strength observed in different regions of nuclei
- It can influence the (n, γ) rates of the r-process by a factor of 10!

  A.C. Larsen and S. Goriely, Phys. Rev. C82 (2010) 014318
- Evidence for the dipole nature of low energy enhancement in ^{56}Fe

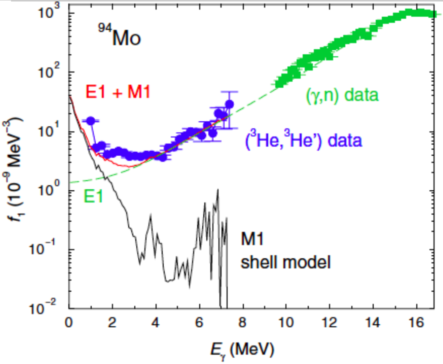
  A. C. Larsen et al., Phys. Rev. Lett. 111 (2013) 242504

What theory says about it?



E. Litvinova and N. Belov, Phys. Rev. C88 (2013) 031302R

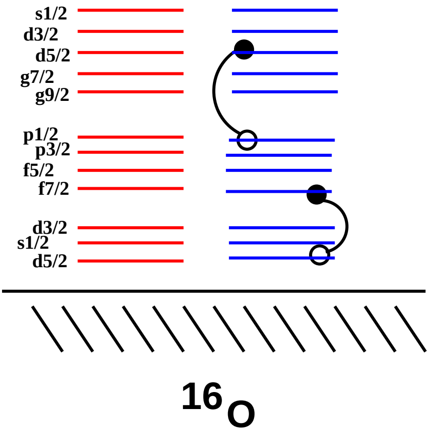
- Thermal continuum QRPA calculations
- Enhancement due to transitions between thermally unblocked s.p. states and the continuum
- Note the difference between $T = 0$ (ground state) and $T > 0$ (excited state) E1 strength distribution



R. Schwengner et al., PRL111 (2013) 232504

- Shell model transitions between a large amount of states
- Enhancement due to the M1 transitions between states in the region near the quasicontinuum
- A general mechanism to be found throughout the nuclear chart *B.A. Brown and A.C. Larsen., PRL113 (2014) 252502*

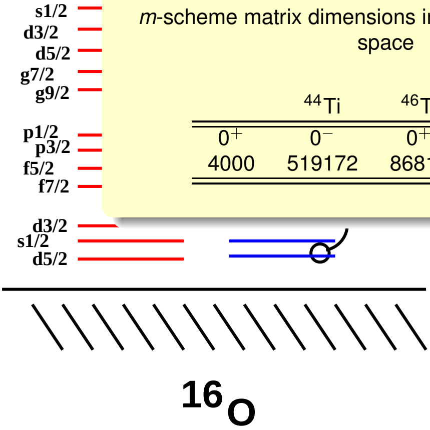
SM calculations in ^{44,46}Ti nuclei



PURPOSE:
Obtain M1/E1/(E2) SF within the same framework

- Full *fp*-calculations for positive parity states
- Full $1\hbar\omega$ calculations for negative parity states- all 1p-1h excitations from *sd* and to *gds* shells
  ☞Exact removal of spurious COM states
- $H_{SM} =$
  $\sum_i \varepsilon_i c_i^\dagger c_i + \sum_{i,j,k,l} V_{ijkl} c_i^\dagger c_j^\dagger c_l c_k + \beta_{c.m.} H_{c.m.}$
- All positive parity states below 16MeV (spins 0-11)
- in ^{44}Ti, 100-350 negative parity states per spin (from 0 to 11)

SM calculations in ^{44,46}Ti nuclei



...it's quite a challenging problem...

m-scheme matrix dimensions in *sd* – *pf* – *gds* model space

	^{44}Ti	^{46}Ti	
0^+	0^-	0^+	0^-
4000	519172	86810	9448410

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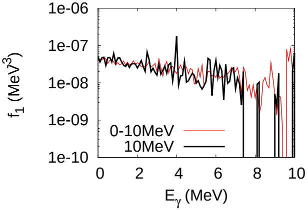
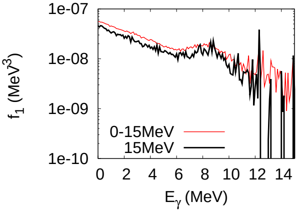
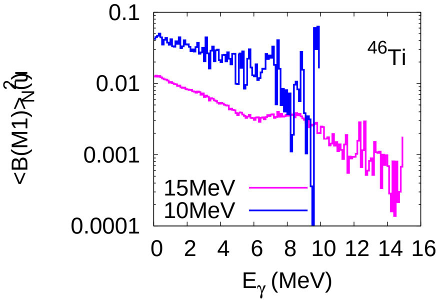
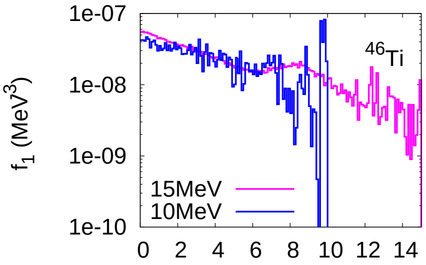
M1 calculations in ^{46}Ti





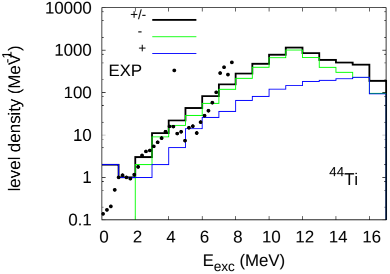




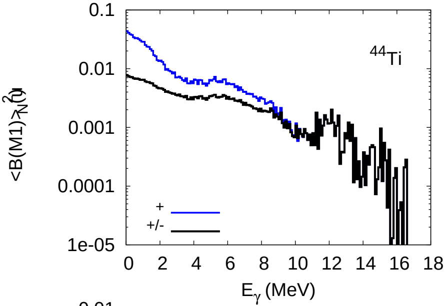
- SF is dominated by the SF of the highest energy state

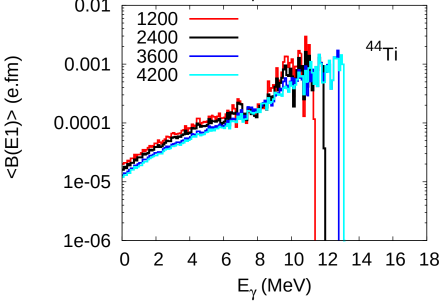
M1/E1 calculations in ^{44}Ti

EXP: *A. C. Larsen et al., Phys. Rev. C85 (2012) 014320*



- Different behavior of $E1$ and $M1$ strength at low energy





Summary

- SM Lanczos strength function is a powerful tool for obtaining low energy distributions for transition (GT, M1, E1, ...) operators
- Using microsopic strength functions instead of global parameterizations influences the calculated cross sections
- The low energy enhancement of M1 γ-strength functions is found in ^{46,44}Ti isotopes, as was found before in other regions
- E1 strength from SM in ^{44}Ti is obtained. It seems to have a different behavior, with no low energy upbend
- Further calculations necessary:
  - other nuclei
  - reason for M1 enhancement
  - ...