POPULATION OF GROUND-STATE ROTATIONAL BANDS OF SUPERHEAVY NUCLEI PRODUCED IN COMPLETE FUSION REACTIONS

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Introduction/Motivation

- The existence of heaviest nuclei with $Z > 100$ is mainly determined by their shell structure.
- Although different models provide close binding energies for nuclei with $Z < 114$, their predictions for the spins of the ground states, quasiparticle spectra, and moments of inertia are rather different.
- The limits of stability of SH nuclei in spin and excitation energy are governed by the fission barrier, which is mostly defined by the shell component.

Introduction/Motivation

- We extend the statistical approach to describe the population of yrast rotational states in the residual superheavy nuclei and the production of evaporation residues in the complete fusion reactions $^{206,208}Pb(^{48}Ca,2n)^{252,254}No$ and $^{204}Hg(^{48}Ca,2n)^{250}Fm$, taking into consideration the survival of compound nuclei against fission at different angular momenta.

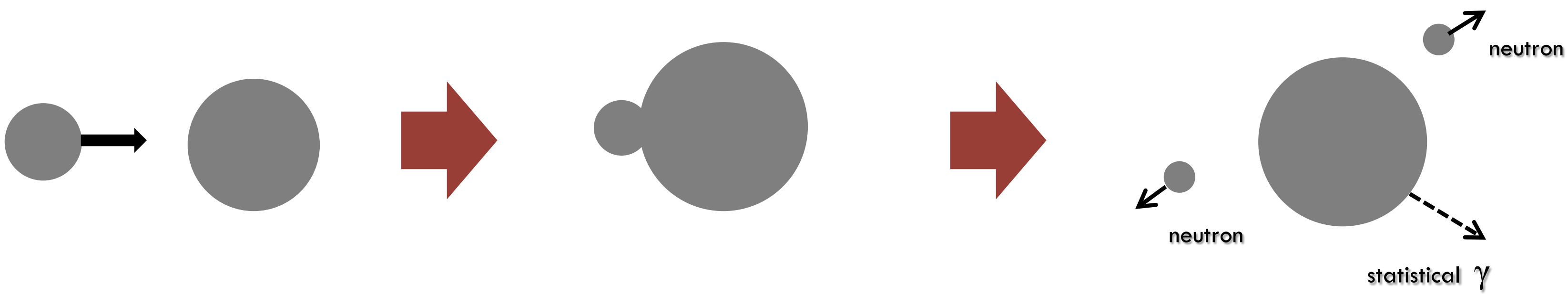
Our aim is the study of damping of the shell effects with the angular momentum for shell-stabilized nuclei.

- We try to answer the question of how the angular momentum dependence of the survival of the compound nucleus against fission is important for the production of new superheavy elements.

Model

Our model treats the formation of the yrast residual nucleus as a three-step process.

Capture | **Fusion** | **Yrast residual nucleus**



rotating dinuclear system is formed in the entrance channel by the transition of the colliding nuclei over the entrance (Coulomb) barrier.

dinuclear system evolves into the compound nucleus in the mass (charge) asymmetry coordinate

excited rotating compound nucleus transforms to the yrast residual nucleus by the emission of the neutrons and statistical γ quanta

Model

- The population of yrast rotational state L^+ with spin L and positive parity depends on the survival of the compound nucleus against fission in the *xn* evaporation channel.

- The residual excitation energy E^* after the emission of neutrons should be less than the fission barrier and quite small to be taken away by k statistical dipole γ quanta.

Model

The population cross section of the state L^{+}

$$\sigma_{\text{pop}}\left(E_{\text{c.m.}},L\right)=\sum_{k}\sigma_{\text{cap}}\left(E_{\text{c.m.}},J\right)P_{\text{cap}}\left(E_{\text{c.m.}},J\right)W_{\text{sur}}^{\text{pop}}\left(E_{CN}^{*},J,\Delta E^{*}\right)$$

$$J=L+k\,l_{g}+x\,l_{n}$$

---- the initial angular momentum of the compound nucleus

The average angular momentum carried out by neutrons and $E1$ statistical γ quanta was taken as $l_n = 0.5$ and $l_g = 1$, respectively.

$$E_{CN}^{*}\left(J\right)=E_{\text{c.m.}}-Q-\hbar^{2}J\left(J+1\right)/\left(2\Im\right)$$

---- thermal excitation energy of the produced nucleus

Model

The neutron carries out from a nucleus the average energy *2T* (by assuming the Maxwellian form of neutron spectrum) and average angular momentum 0.5.

1. step $E_1^* = E_{CN}^*(J), \quad J_1 = J$
2. step $E_2^* = E_1^* - 2T, \quad J_2 = J - 0.5$

….

$T = \sqrt{E_{CN}^*(J)/a}$ -- nuclear temperature

Model

The survival probability in the case of the evaporation of x neutrons and k γ quanta

$$W_{\mathrm{sur}}^{\mathrm{pop}}\left(E_{CN}^{*}, J, \Delta E^{*}\right) = P_{xnk\gamma}\left(E_{CN}^{*}, J, \Delta E^{*}\right)\prod_{i=1}^{x}\frac{\Gamma_n\left(E_i^{*}, J_i\right)}{\Gamma_f\left(E_i^{*}, J_i\right)}$$

$P_{xnk\gamma}$ ---- the probability of the realization of an *xnkγ* channel

The relative intensity of $E2$ multipolarity transitions between the yrast rotational states L^+ and $(L-2)^+$

$$I(L \to L-2) = \frac{\sum_{x=L/2}^{L_{\max}/2}\sigma_{\mathrm{pop}}\left(E_{\mathrm{c.m.}}, 2x\right)}{\sum_{x=0}^{L_{\max}/2}\sigma_{\mathrm{pop}}\left(E_{\mathrm{c.m.}}, 2x\right)}$$

Capture

Quantum diffusion approach

Sargsyan et al. EPJ A**45,** 125 (2010)
Sargsyan et al. EPJ A**47,** 38 (2011)

The quantum diffusion approach based on the following assumptions:

1. **The capture can be treated in term of a single collective variable: the relative distance *R* between the colliding nuclei.**
2. **The internal excitations (for example low-lying collective modes such as dynamical quadropole and octupole excitations of the target and projectile, one particle excitations etc.) can be presented as an environment.**
3. **Collective motion is effectively coupled with internal excitations through the environment.**

Capture

The partial capture cross-section

$$\sigma_{\text{cap}}(E_{\text{c.m.}}, J) = \frac{\pi \hbar^2}{2\mu E_{\text{c.m.}}}(2J+1)P_{\text{cap}}(E_{\text{c.m.}}, J)$$

$P_{cap}(E_{\text{c.m.}}, J)$ --- the partial capture probability at fixed energy and angular momenta

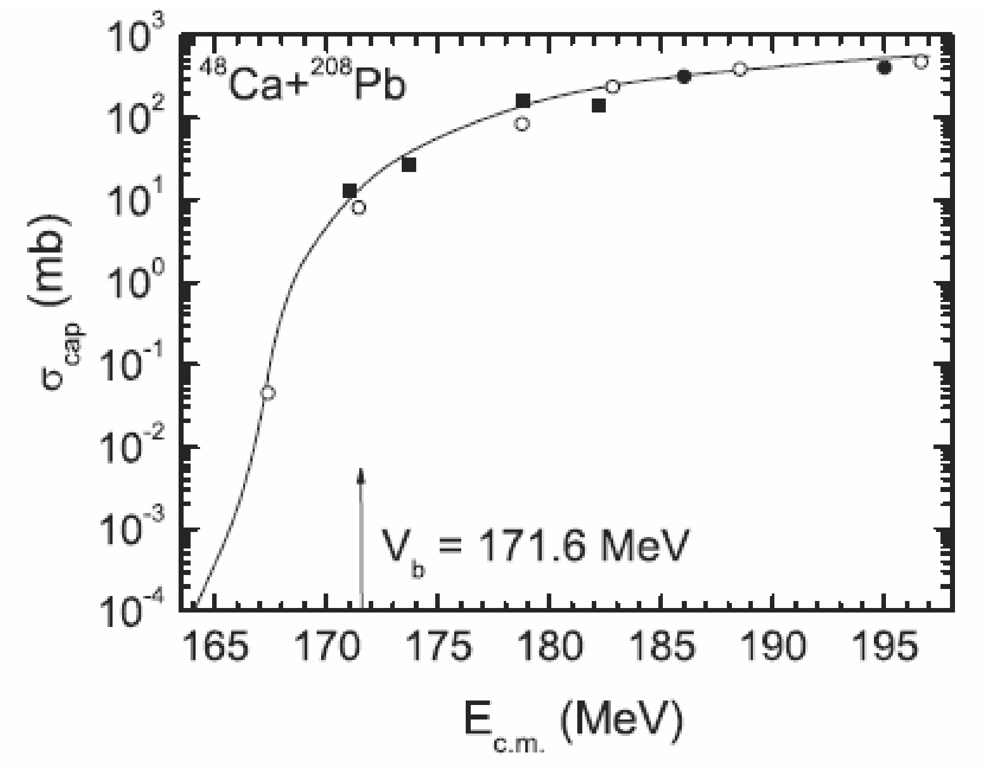
The partial capture probability obtained by integrating the propagator G from the initial state (R_0,P_0) at time t=0 to the finale state (R, P) at time t:

$$P_{\text{cap}} = \lim_{t\to\infty} \int_{-\infty}^{R_m} dR \int_{-\infty}^{\infty} dP\, G(R,P,t|R_0,P_0,0) = \lim_{t\to\infty} \frac{1}{2}\operatorname{erfc}\left[\frac{-R_m + \overline{R(t)}}{\sqrt{\Sigma_{RR}(t)}}\right]$$

Dodonov and Man'ko, Trudy Fiz. Inst. Akad. Nauk **167**, 7 (1986).

The quantum-mechanical dissipative effects and non-Markovian effects accompanying the passage through the potential barrier are taken into consideration in our formalism.

Capture

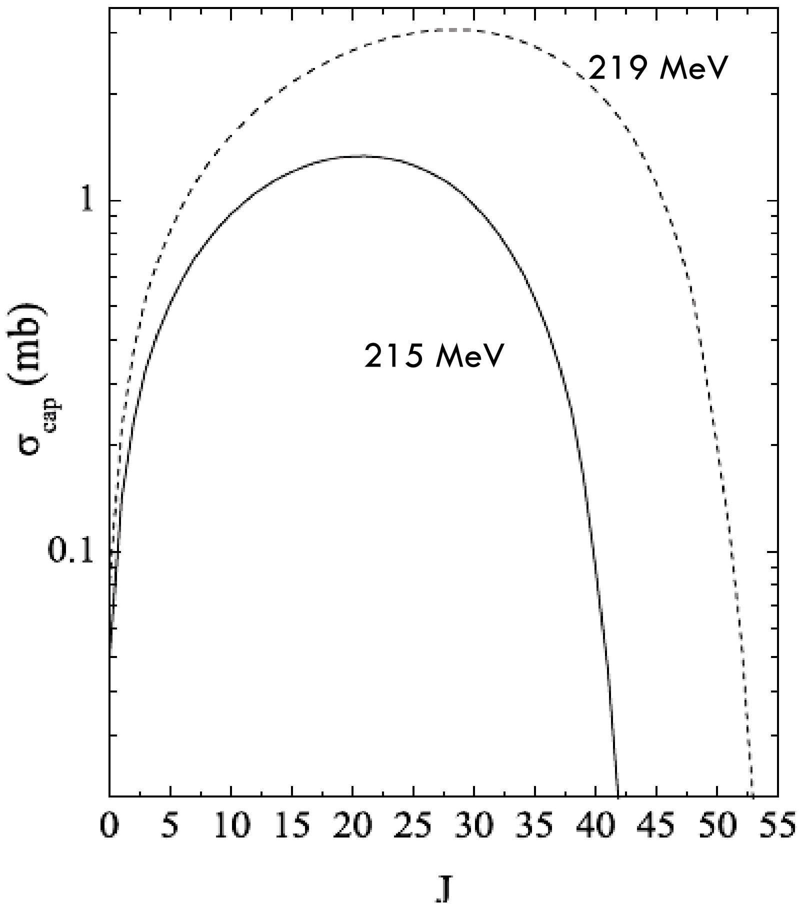


Experimental data:

*R. Bock et al., NPA **388, 334 (1982).***
*A. J. Pacheco et al., PRC **45, 2861 (1992).***
*E. Prokhorova et al., NPA **802, 45 (2008).***

- ✓ The calculated capture cross section is in good agreement with the available experimental data.
- ✓ With this approach, many heavy-ion capture reactions at energies above and well below the Coulomb barrier have been successfully described.

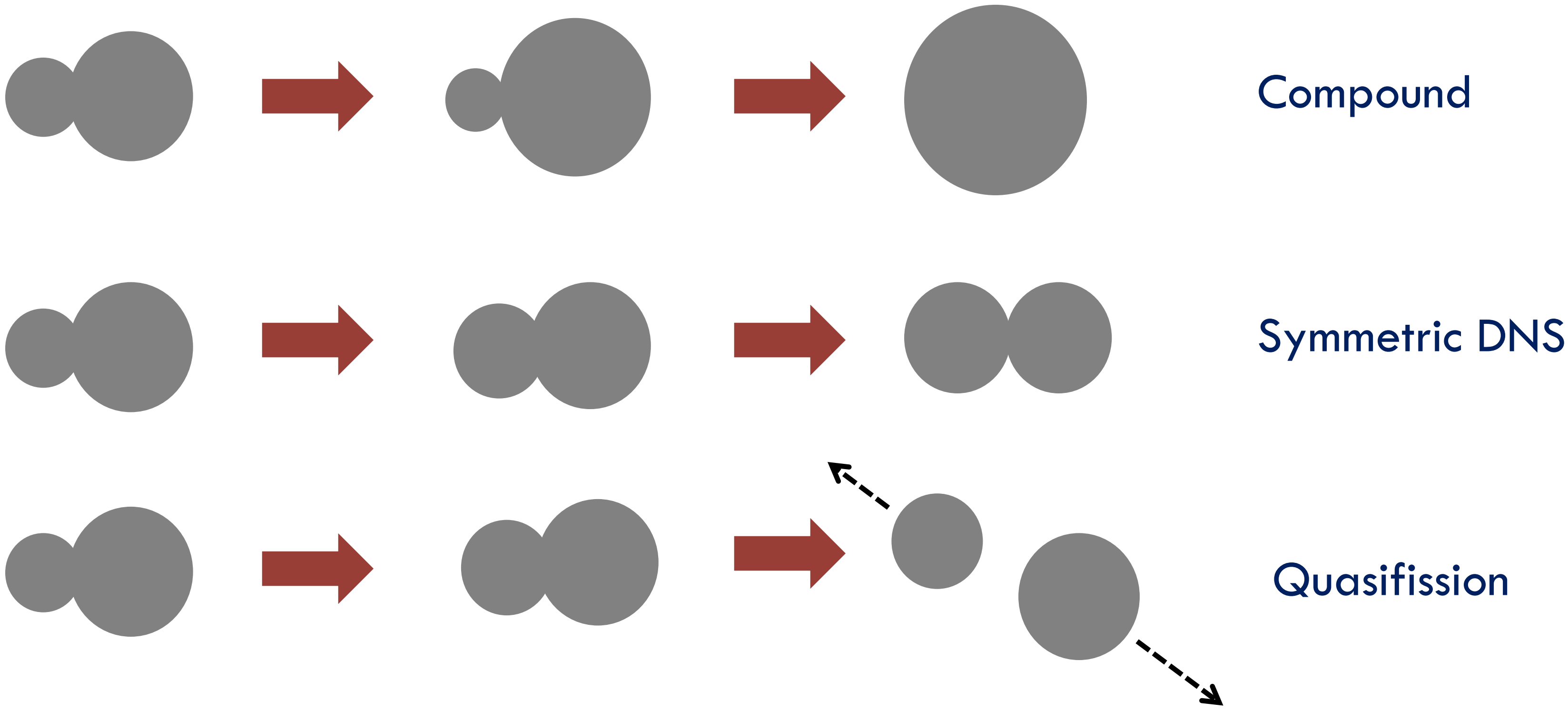
Capture



- At small angular momenta, the function $\sigma_{cap}(E_{c.m.}, J)$ growth is due to the factor $2J + 1$
- At larger J, the system of colliding nuclei turns out in the sub-barrier region and the capture cross section falls down, since in this region the decrease of P_{cap} with J is not compensated by the factor $2J + 1$.

Complete fusion

In DNS model the compound nucleus is reached by a series of transfers of nucleons or small clusters from the light nucleus to the heavier one in a touching configuration.



Complete fusion

The dynamics of DNC considered as an evolution of two coordinates:

$$\eta = \frac{A_1 - A_2}{A_1 + A_2}$$ -- **the diffusion in mass asymmetry leads to fusion or symmetrization**

R -- **the diffusion in the internuclear distance leads to quasifission**

$\boxed{\text{The probability of complete fusion}}$

$$P_{CN} = \frac{\lambda_\eta}{\lambda_\eta + \lambda_\eta^{\mathrm{sym}} + \lambda_R}$$

We use two-dimensional Kramers-type expressions for the rates

Adamian at al., NPA ***678, 24 (2000)***

Adamian at al., NPA ***633, 409 (1998)***

In the reactions considered, the internal fusion barrier and quasifission barrier are almost equal to each other. Therefore, $\lambda_\eta \approx \lambda_\eta^{\mathrm{sym}} + \lambda_R$ and $P_{CN} \approx 0.5$

At treated narrow intervals of beam energies (about 28 MeV) and angular momenta (about 20), the dependence of $\boldsymbol{P_{CN}}$ on $\boldsymbol{E_{c.m.}}$ and $\boldsymbol{J}$ can be neglected.

Channel widths and level density

The decay width of channel *i* is given in terms of the probability of this process

$$\Gamma_i = \frac{R_{CN_i}}{2\pi\,\rho\left(E^*_{CN}, J\right)}$$

The probability of neutron emission

$$R_{CN_n}\left(E^*_{CN}, J\right) = \sum_{j=J-1/2}^{j=J+1/2} \left(\int_0^{E^*_{CN}-B_n} d\varepsilon\, \rho_d\left(E^*_{CN} - B_n - \varepsilon, j\right) T_J\left(A-1, \varepsilon\right) \right)$$

experimental values of neutron binding energies are used: *Audi, NPA **729, 337 (2003)***

level density of the daughter nucleus

transition coefficients: *Mashnik, arXiv:nucl-th/0208048.*

The fission probability in the case of a one-hump barrier

$$R_{CN_f}\left(E^*_{CN}, J\right) = \int_0^{E^*_{CN}-B_f\left(E^*_{CN}, J\right)} \frac{\rho_f\left(E^*_{CN} - B_f\left(E^*_{CN}, J\right) - \varepsilon, J\right)}{1+\exp\left[2\pi\left(\varepsilon + B_f\left(E^*_{CN}, J\right) - E^*_{CN}\right)/(\hbar\omega)\right]} d\varepsilon$$

The level density is calculated with the Fermi-gas model

*Ignatyuk, Sov. J. Nucl. Phys. **29, 875 (1979).***

Fission barrier

The fission barrier has the liquid-drop and microscopical parts

$$B_f\left(E^*_{CN}, J\right) = B_f^{LD}\left(J\right) + B_f^M$$

$B_f^{LD}\left(J\right)$ **-- liquid-drop part:** *Sierk, PRC* **33, 2039 (1986)**

$B_f^M = \delta W_{sd}^A - \delta W_{gr}^A$ **-- microscopical part:** is related to the shell correction at the ground state and the shell at the saddle point.

Usually, one neglects the shell correction at the saddle point

$$B_f^M = B_f^M\left(E^*_{CN} = 0\right) \approx \left|\delta W_{gr}^A\left(E^*_{CN} = 0\right)\right|$$

The predicted values of shell corrections:
Acta Phys. Pol. B **34, 2153 (2003)**, ibid. **34, 2073 (2003)**, ibid. **34, 2141 (2003)**; ibid. **32, 691 (2001)**

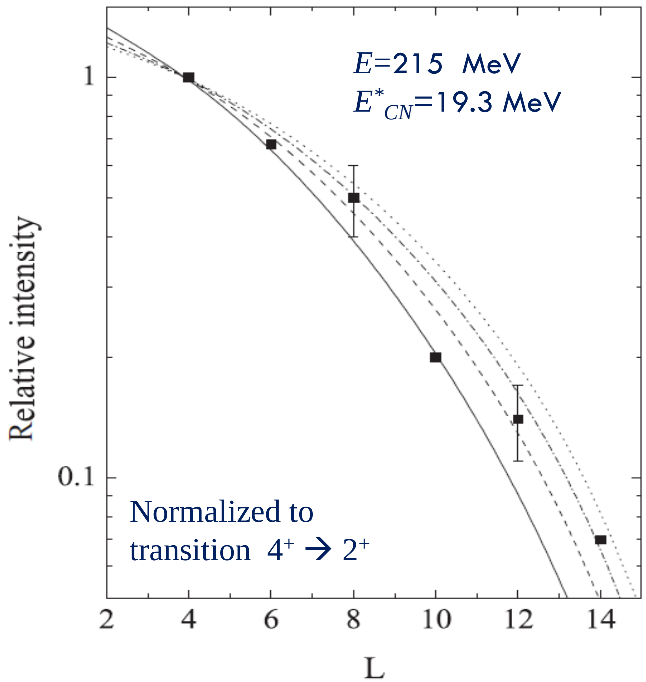
The damping of the shell effects with the growth of the excitation energy and angular momentum are approximated by the exponential damping factors

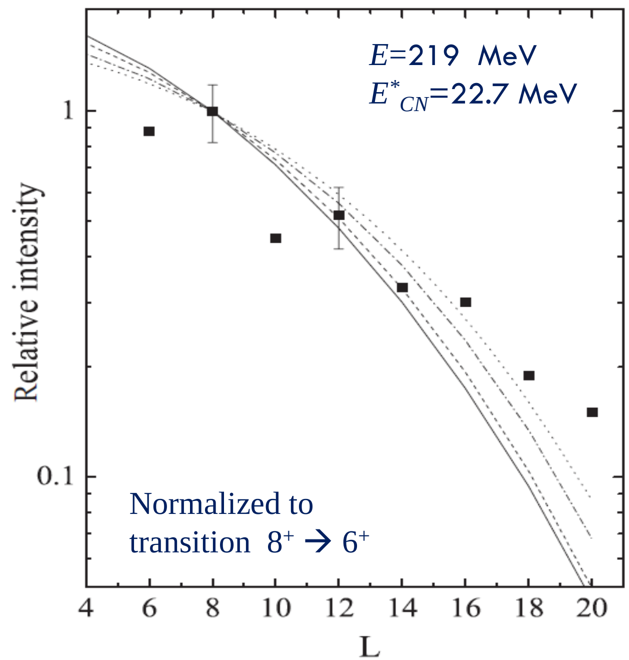
$$B_f\left(E^*_{CN}, J\right) = B_f^{LD}\left(J\right) + B_f^M\left(E^*_{CN} = 0\right) \times \exp\left(-E^*_{CN}\left(J\right)/E_D\right) \times \exp\left(-J\left(J+1\right)/D\right)$$

Population of yrast rotational states of ^{254}No

The relative transition intensities in the rotational band of ^{254}No

produced in the reaction ^{208}Pb(^{48}Ca,2n)^{254}No





Experimental data:

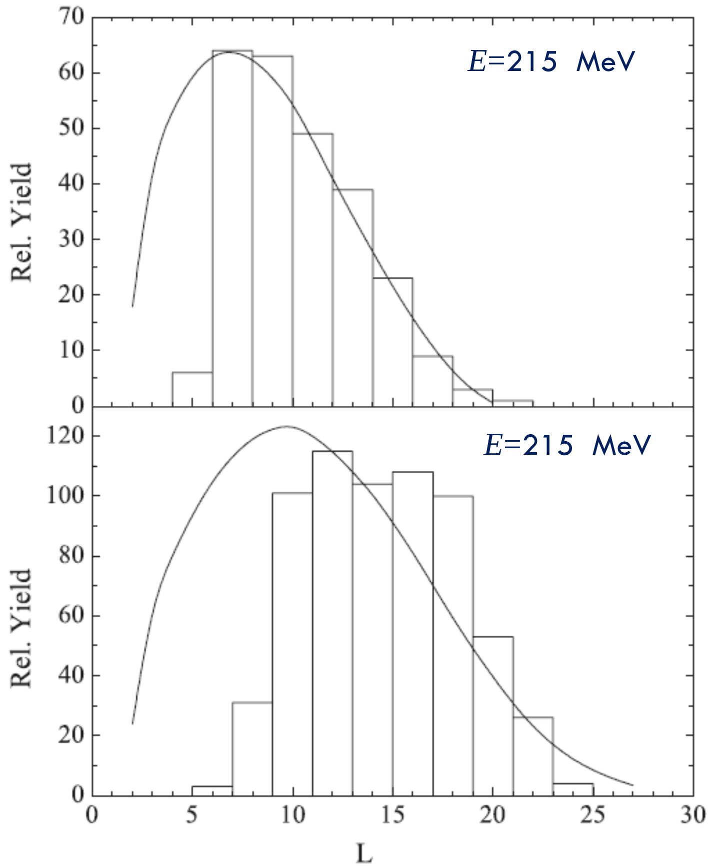
*P. Reiter et al., PRL **82, 509 (1999)***

*P. Reiter et al., PRL **84, 3542 (2000).***

- Solid, dashed, dash-dotted, and dotted curves correspond to D = 600, 800, 1000, and 1200, respectively.

- ✓ Higher beam energy leads to the large population of high-spin states. This difference is because of the capture process: the partial capture cross section is larger at beam energy 219 MeV.
- ✓ The variation of the damping parameter D from 600 to 1200 does not strongly affect the results of the calculation.

Entry spin distribution of ^{254}No



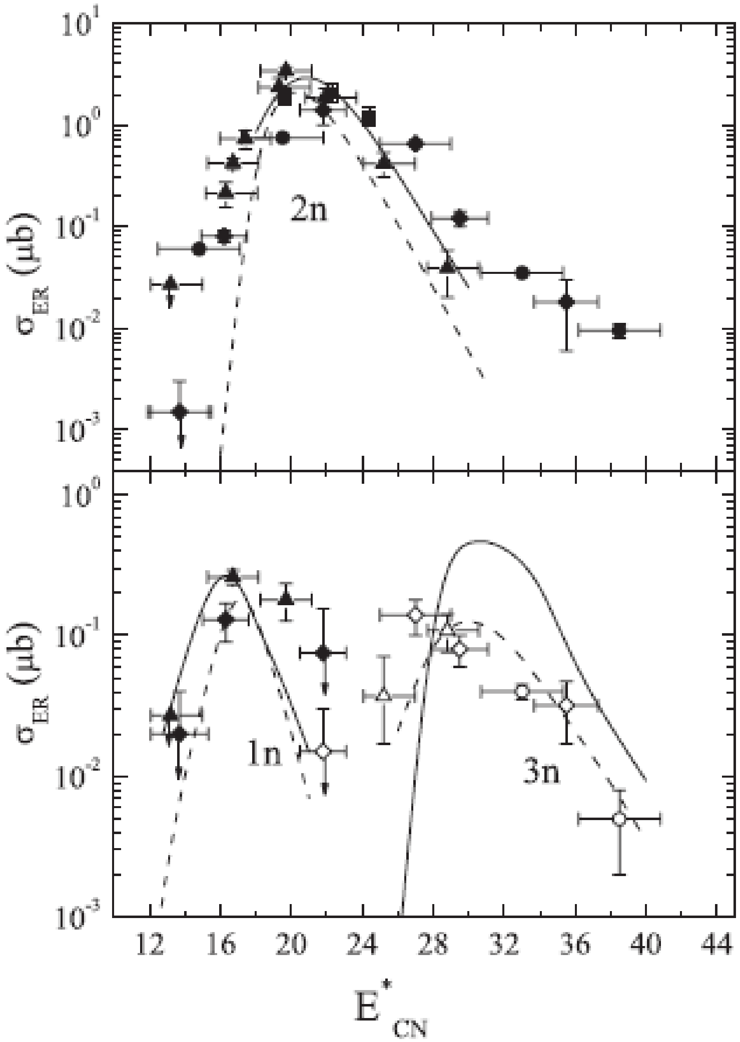
Experimental data:
P. Reiter et al., PRL **84, 3542 (2000).**

Produced in the reaction $^{208}Pb(^{48}Ca,2n)^{254}No$

Calculations (lines) are done by using the damping parameter D = *1000.*

- ✓ More high-spin states survive against fission with increasing E. This could be explained by a larger capture cross section for large values of the initial angular momentum at beam energy 219 MeV!
- ✓ Our results are close to the experimental values, especially near the maxima of spin distributions.

Evaporation-residue cross sections and excitation functions of $^{48}Ca + ^{208}Pb$



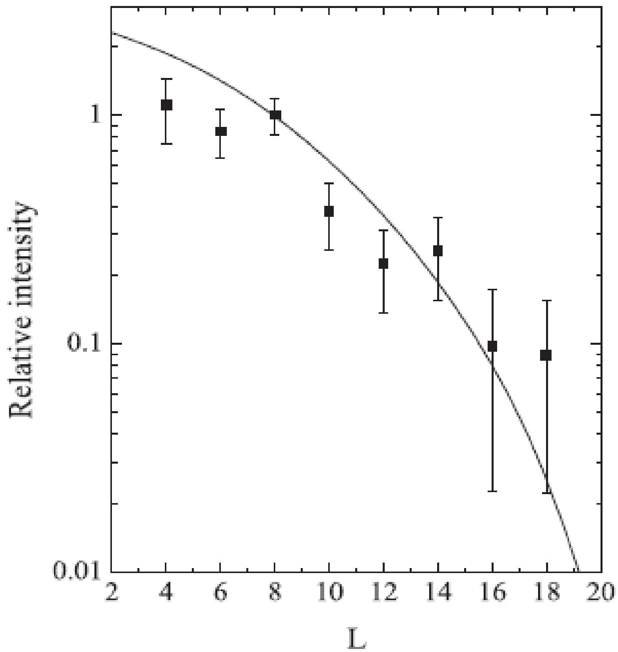
Calculations (lines) are done by using the damping parameter *D = 1000*.

Experimental data:
Belozerov et al., EPJ. A **16, 447 (2003)**
Oganessian et al., PRC **64, 054606** *(2001)*
Gaggeler et al., NPA **502, 561 (1989)**
Yeremin et al., JINR Rapid Commun. **6[92–98], 21 (1998).**

- ✓ The systematic uncertainty in the definition of σ_{ER} is up to a factor of 3.
- ✓ The dependence of the calculated results on the parameter *D* is rather weak. (in the maximum of a 2*n* evaporation channel, the variation of *D* from 600 to 1200 leads to the variation of σ_{ER} from 2.5 to 3.6 μb.

Population of yrast rotational states of ^{252}No and ^{250}Fm

$^{208}Pb(^{48}Ca,2n)\,^{252}No$



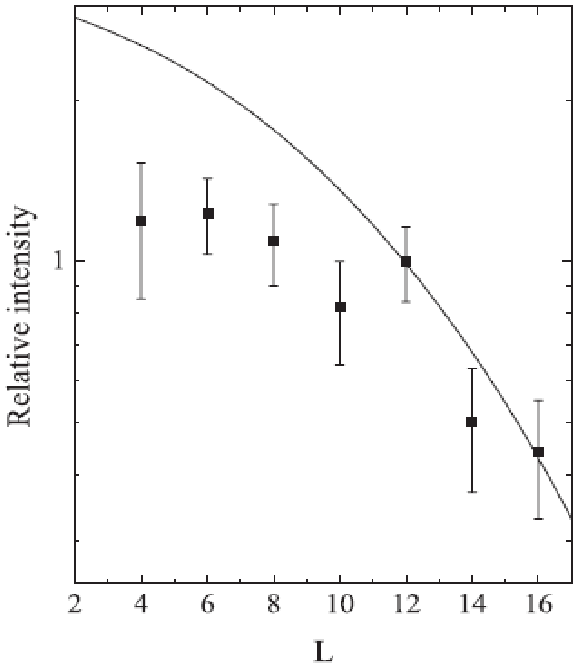
Experimental data:

Herzberg et al., PRC **65, 014303 (2001).**

Beam energy 215.5 MeV

$$\sigma_{ER}^{exp} \approx 220\text{ nb},\ \ \sigma_{ER}^{th} = 210\text{ nb}$$

$^{204}Hg(^{48}Ca,2n)\,^{250}Fm$



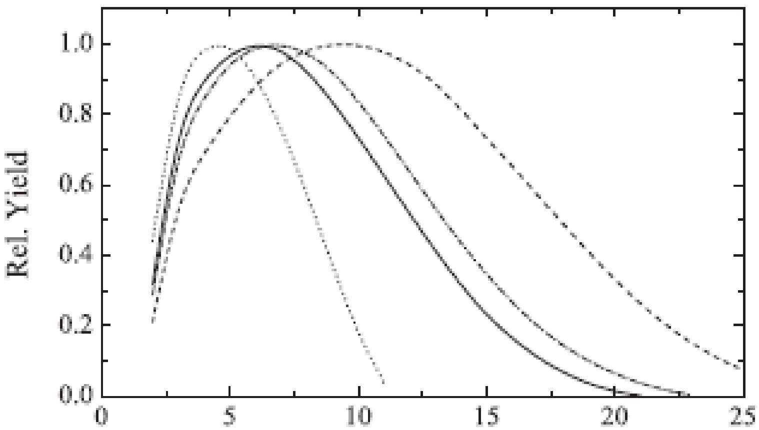
Experimental data:

Bastin et al., PRC **73, 024308 (2006).**

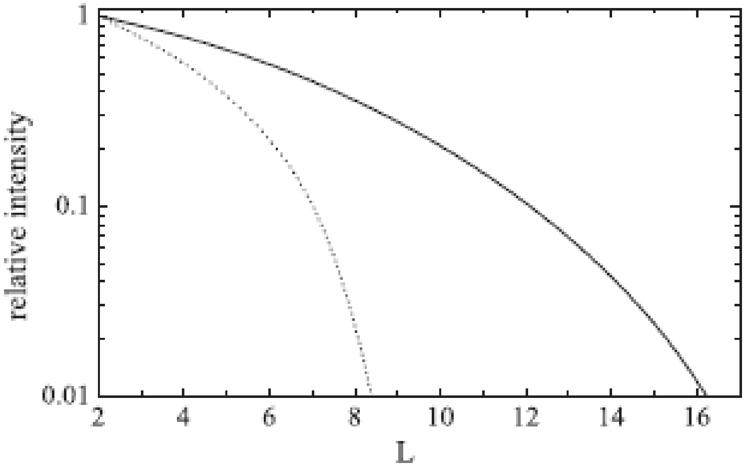
Beam energy 209 MeV

$$\sigma_{ER}^{exp} \approx 980^{+160}_{-160}\text{ nb},\ \ \sigma_{ER}^{th} = 790\text{ nb}$$

Entry spin distribution of ^{252}No and ^{250}Fm



The entry spin distributions of ^{250}Fm and ^{252}No,
^{48}Ca(205 MeV)+^{204}Hg→^{250}Fm+2*n* (solid line),
^{48}Ca(209 MeV)+^{204}Hg→^{250}Fm+2*n* (*dashed* line),
^{48}Ca(211.5 MeV)+^{206}Pb→^{252}No+2*n (dotted line)*,
^{48}Ca(215.5 MeV)+^{206}Pb→^{252}No+2*n (dash-dotted line).*

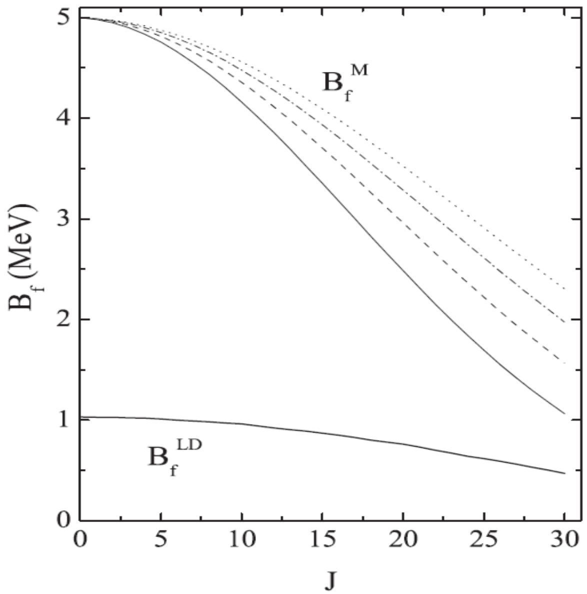


Relative transition intensities in the yrast rotational bands of ^{250}Fm (solid line) and ^{252}No (dotted line), produced in the reactions
^{48}Ca(205 MeV)+^{204}Hg→^{250}Fm+2n
^{48}Ca(211.5 MeV)+^{206}Pb→^{252}No+2n.

Summary

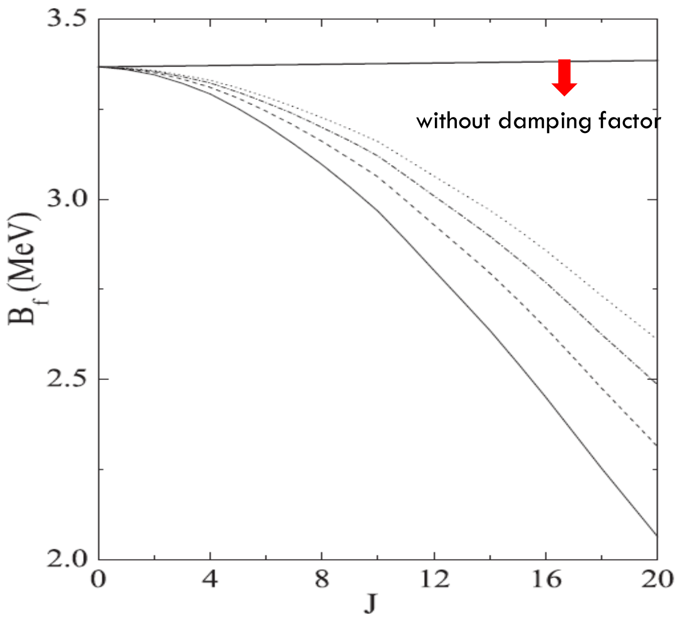
- ✓ Using the statistical and quantum diffusion approaches, we studied the population of rotational bands in superheavy nuclei produced in fusion-evaporation reactions $^{206,208}Pb(^{48}Ca,2n)^{252,254}No$ and $^{204}Hg(^{48}Ca,2n)^{250}Fm$.
- ✓ The Fermi-gas model was applied to calculate the level densities.
- ✓ The interval D = 600–1000 for the damping parameter was found for the description of the damping of the shell effects with the angular momentum and used in the calculations of the relative transition intensities in the ground-state rotational bands, the entry spin distributions of the evaporation residues and the evaporation-residue cross sections.
- ✓ Angular momentum dependence of these observables mainly comes from the partial capture and survival probabilities.
- ✓ At low and moderate angular momentum values, the centrifugal forces are not dangerous for the production of superheavy elements, especially in "the island of stability."

Fission barrier for ^{256}No



The dependence of the liquid-drop and microscopical parts of the fission barrier at E^*_{CN}=0.

- Solid, dashed, dash-dotted, and dotted curves correspond to D = 600, 800, 1000, and 1200, respectively.
- While the value B_f^{LD} slowly decreases with the growth of J, the value B_f^M rapidly falls down due to the damping of the shell effects.
- The difference between the microscopical components at the variation of the parameter D is rather small at J < 10, but becomes considerable at larger angular momenta.



The dependence of the total fission barrier at E^*_{CN}=19.3 MeV and J = 0

The dependence of the fission barrier on the angular momentum is mainly governed by the damping of the shell effects on the angular momentum!

Evaporation residue

Partial evaporation residue cross section

$$\sigma_{ER}\left(E_{\mathrm{c.m.}},J\right)=\sigma_{\mathrm{cap}}\left(E_{\mathrm{c.m.}},J\right)P_{CN}\left(E_{\mathrm{c.m.}},J\right)W_{\mathrm{sur}}\left(E_{\mathrm{c.m.}},J\right)$$

Survival probability of the excited compound nucleus

$$W_{\mathrm{sur}}\left(E_{CN}^{*},J\right)=P_{xn}\left(E_{CN}^{*}\right)\prod_{i=1}^{x}\frac{\Gamma_{n}\left(E_{i}^{*},J_{i}\right)}{\Gamma_{f}\left(E_{i}^{*},J_{i}\right)}$$

Total evaporation-residue cross section

$$\sigma_{ER}\left(E_{\mathrm{c.m.}}\right)=\sum_{J=0}^{J_{\max}}\sigma_{ER}\left(E_{\mathrm{c.m.}},J\right)$$

Maximal value of the angular momentum

$$E_{\mathrm{c.m.}}-Q-\sum_{i=1}^{x}B_{n}(i)-\hbar^{2}J_{\max}\left(J_{\max}+1\right)/\left(2\Im\right)>0$$