

Relativistic Energy Density Functional for Astrophysical Applications



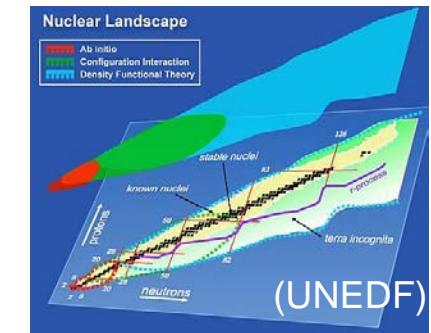
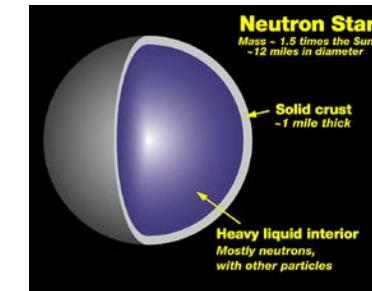
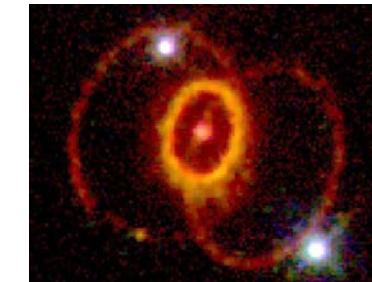
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INTRODUCTION

- Nuclei play an important role in various astrophysical scenarios and processes; e.g. supernova evolution, nucleosynthesis, etc.
- Nuclear weak-interaction processes in presupernova stellar collapse and at later stages of supernova evolution (e.g. electron capture, beta decay, neutrino-nucleus reactions,...)
→ link to charge-exchange excitations
- Neutron stars – link to nuclei: The same pressure that pushes the neutrons against the surface tension in nuclei, and determines the neutron skin thickness also supports a neutron star against gravity
- Energy density functionals (EDF) allow consistent approach to nuclear matter, finite nuclei and nuclear weak interaction processes

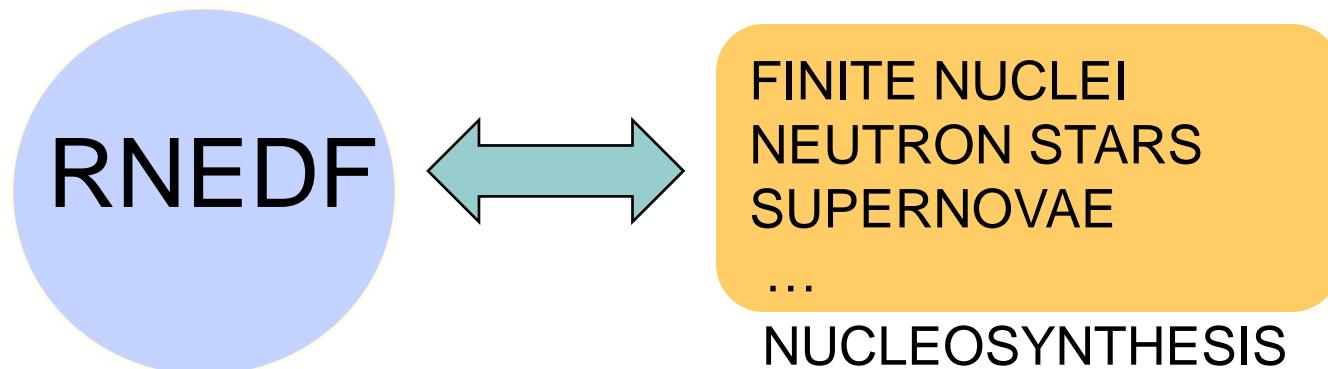


NUCLEAR PROCESSES IN STELLAR SYSTEMS

Final understanding of how supernova explosions and nucleosynthesis work, with self-consistent microscopic description of all relevant nuclear physics included, has not been achieved yet.

OUR GOAL: Universal relativistic nuclear energy density functional (RNEDF) for

- properties of finite nuclei (masses, radii, excitations)
- nuclear equation of state (EOS)
- neutron star properties (mass/radius, ...)
- electron capture in presupernova collapse
- neutrino-nucleus reactions and beta decays of relevance for the nucleosynthesis
- other astrophysically related phenomena...



THEORY FRAMEWORK

T. Niksic, et al., Comp. Phys. Comm. 185, 1808 (2014).

- The implementation of density functional theory in the relativistic framework in terms of self-consistent relativistic mean-field model → talks J. Meng, E. Khan, H. Liang
- The basis is an effective Lagrangian with four-fermion (contact) interaction terms includes the isoscalar-scalar, isoscalar-vector, isovector-vector interactions

$$\begin{aligned}\mathcal{L} = & \bar{\psi}(i\gamma \cdot \partial - m)\psi \\ & - \frac{1}{2}\alpha_S(\hat{\rho})(\bar{\psi}\psi)(\bar{\psi}\psi) - \frac{1}{2}\alpha_V(\hat{\rho})(\bar{\psi}\gamma^\mu\psi)(\bar{\psi}\gamma_\mu\psi) \\ & - \frac{1}{2}\alpha_{TV}(\hat{\rho})(\bar{\psi}\vec{\tau}\gamma^\mu\psi)(\bar{\psi}\vec{\tau}\gamma_\mu\psi) \\ & - \frac{1}{2}\delta_S(\partial_\nu\bar{\psi}\psi)(\partial^\nu\bar{\psi}\psi) - e\bar{\psi}\gamma \cdot A \frac{(1-\tau_3)}{2}\psi\end{aligned}$$

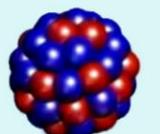
- many-body correlations encoded in density-dependent coupling functions that are motivated by microscopic calculations but parametrized in a phenomenological way
- In addition: pairing correlations in finite nuclei
 - Relativistic Hartree-Bogoliubov model (with separable form of the pairing force)
- Relativistic Q(RPA)

CONSTRAINING THE FUNCTIONAL

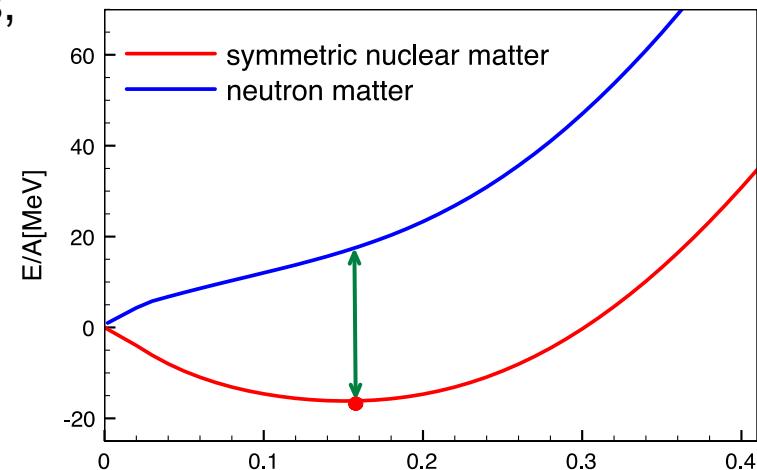
- The model parameters $\mathbf{p} = (p_1, \dots, p_n)$ are constrained directly by many-body observables using χ^2 minimization

$$\chi^2(\mathbf{p}) = \sum_{i=1}^m \left(\frac{\mathcal{O}_i^{\text{theo.}}(\mathbf{p}) - \mathcal{O}_i^{\text{ref.}}}{\Delta \mathcal{O}_i^{\text{ref.}}} \right)^2$$

- Calculated values are compared to experimental, observational, and pseudo-data, e.g.



- properties of finite nuclei** – e.g., binding energies, charge radii, diffraction radii, surface thicknesses, pairing gaps, etc.,...
- nuclear matter properties** – equation of state, binding energy and density at the saturation, symmetry energy J & L, incompressibility...



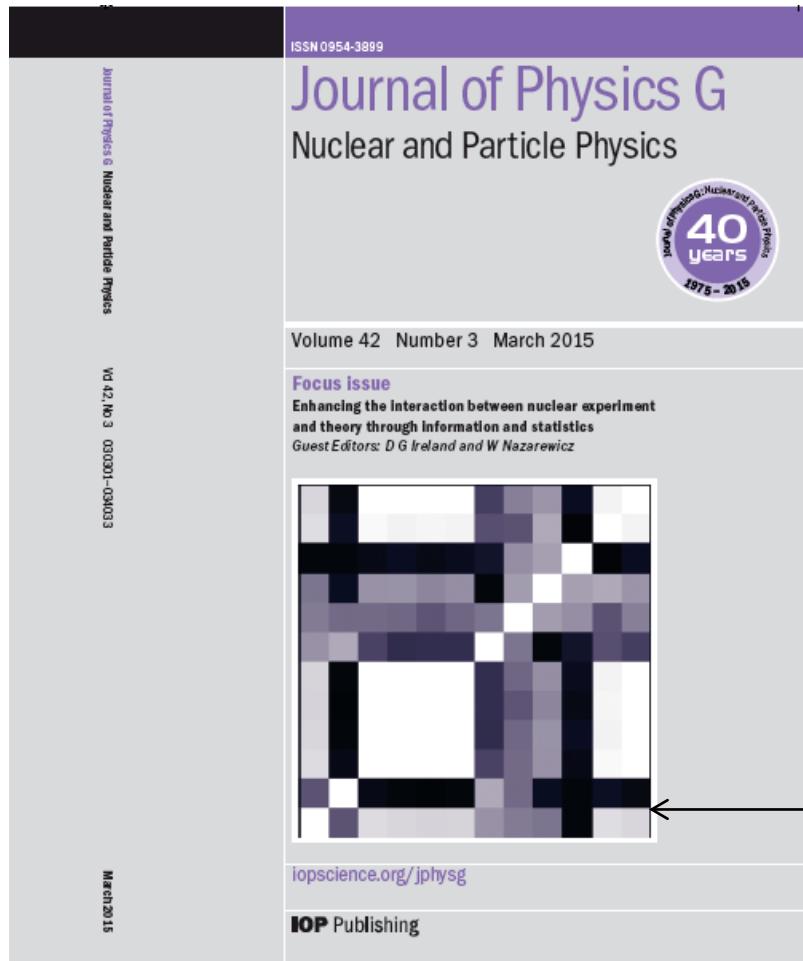
- Isovector channel of the EDF is weakly constrained by exp. data such as binding energies and charge radii. Possible observables for the isovector properties:
neutron radii, neutron skins, dipole polarizability, pygmy dipole strength, neutron star radii

COVARIANCE ANALYSIS IN THE FRAMEWORK OF EDFs

The quality of χ^2 minimization to exp. data is an indicator of the statistical uncertainty

- Curvature matrix:

$$\mathcal{M}_{ij} = \frac{1}{2} \partial_{p_i} \partial_{p_j} \chi^2|_{\mathbf{p}_0}$$



Covariance between two quantities A & B:

$$\overline{\Delta A \Delta B} = \sum_{ij} \partial_{p_i} A (\hat{\mathcal{M}}^{-1})_{ij} \partial_{p_j} B$$

1) variance $\overline{\Delta^2 A}$ defines statistical uncertainty of calculated quantity

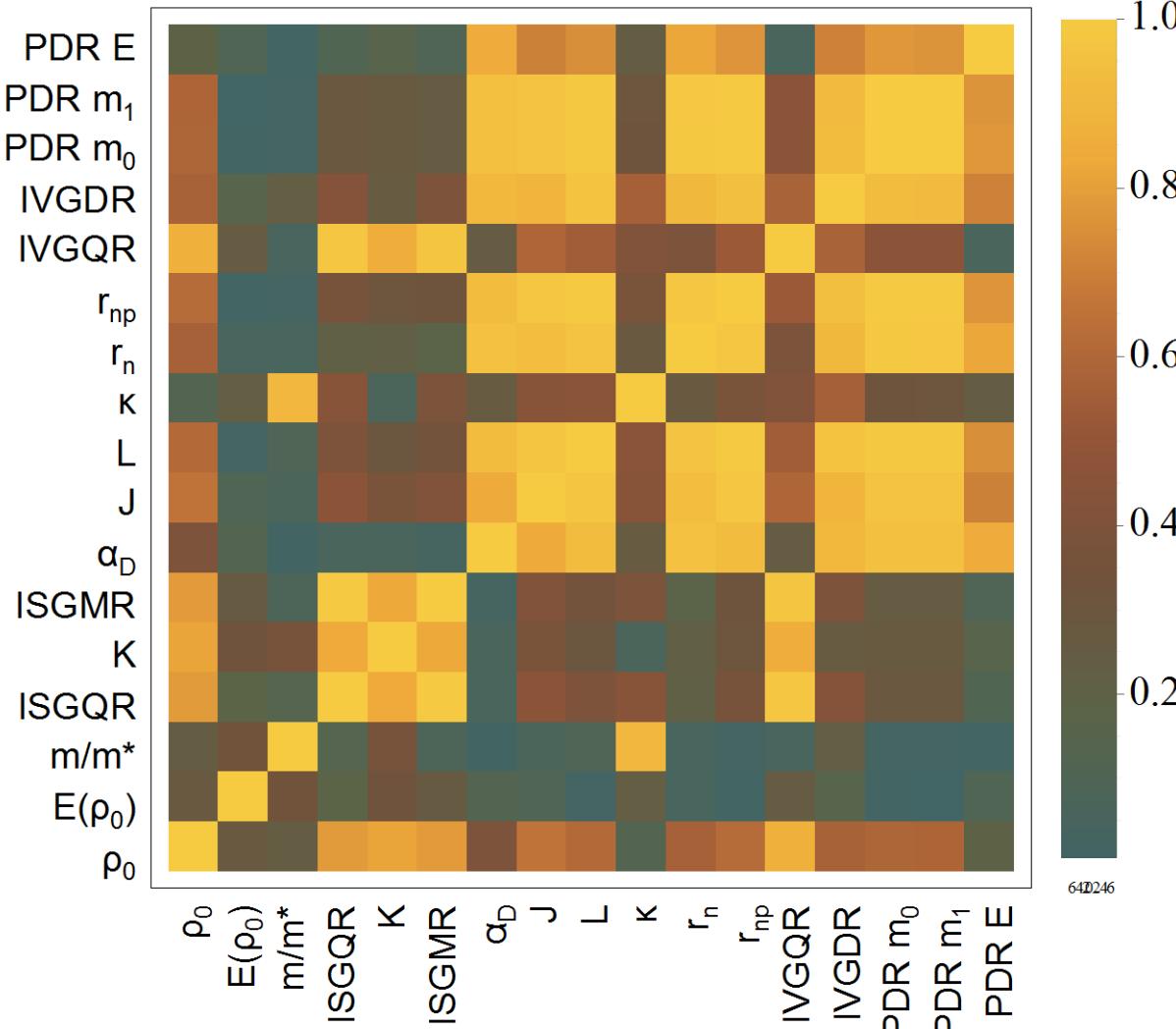
2) correlations between quantities A & B: $c_{AB} = \frac{|\overline{\Delta A \Delta B}|}{\sqrt{\overline{\Delta A^2} \overline{\Delta B^2}}}$

- see: J. Dobaczewski, W. Nazarewicz, P.-G. Reinhard, JPG 41, 074001 (2014).
- X. Roca Maza et al., JPG 42, 034033 (2015)
- T. Niksic et al., JPG 42, 034008 (2015)
- Correlation matrix indicates important correlations between various quantities.

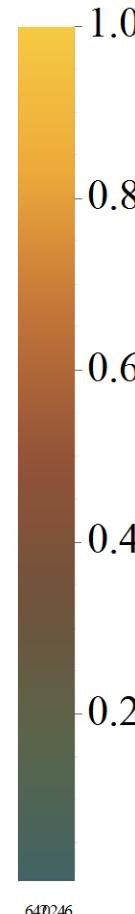
→ talk P.-G. Reinhard

CORRELATIONS: NUCLEAR MATTER AND PROPERTIES OF NUCLEI

^{208}Pb



c_{AB}



Correlation matrix between nuclear matter properties and several quantities in ^{208}Pb (DDME-min1)

- neutron skin thickness, properties of giant resonances, pygmy strength

$c_{AB}=1$: A & B strongly correlated

$c_{AB}=0$: A & B uncorrelated

CONSTRAINING THE SYMMETRY ENERGY

Nuclear matter equation of state:

$$E(\rho, x) = E_{SNM}(\rho) + E_{sym}(\rho)(1 - 2x)^2 + \dots$$

$$\rho = \rho_n + \rho_p, x = \rho_p/\rho$$

Symmetry energy term:

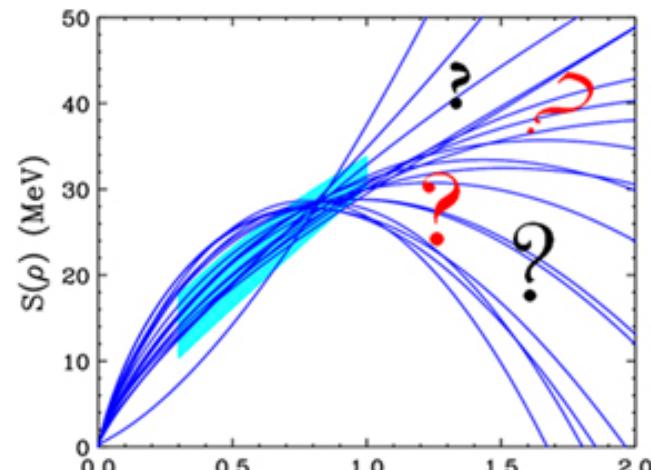
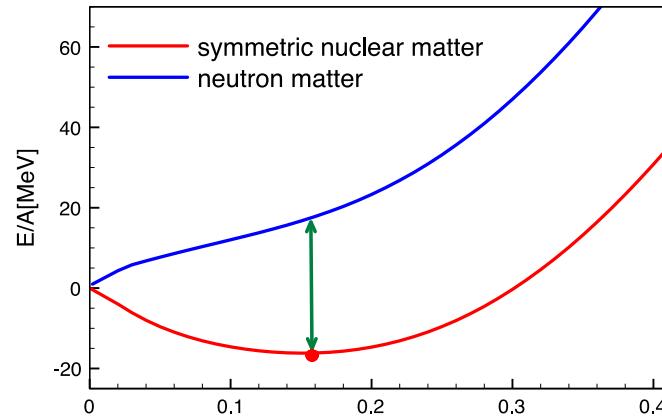
$$E_{sym}(\rho) \equiv S_2(\rho) = J - L\epsilon + \dots$$

$$\epsilon = (\rho_0 - \rho)/(3\rho_0)$$

$$L = 3\rho_0 \frac{dS_2(\rho)}{dr} \Big|_{\rho_0}$$

J – symmetry energy at saturation density

L – slope of the symmetry energy (related to the pressure of neutron matter)

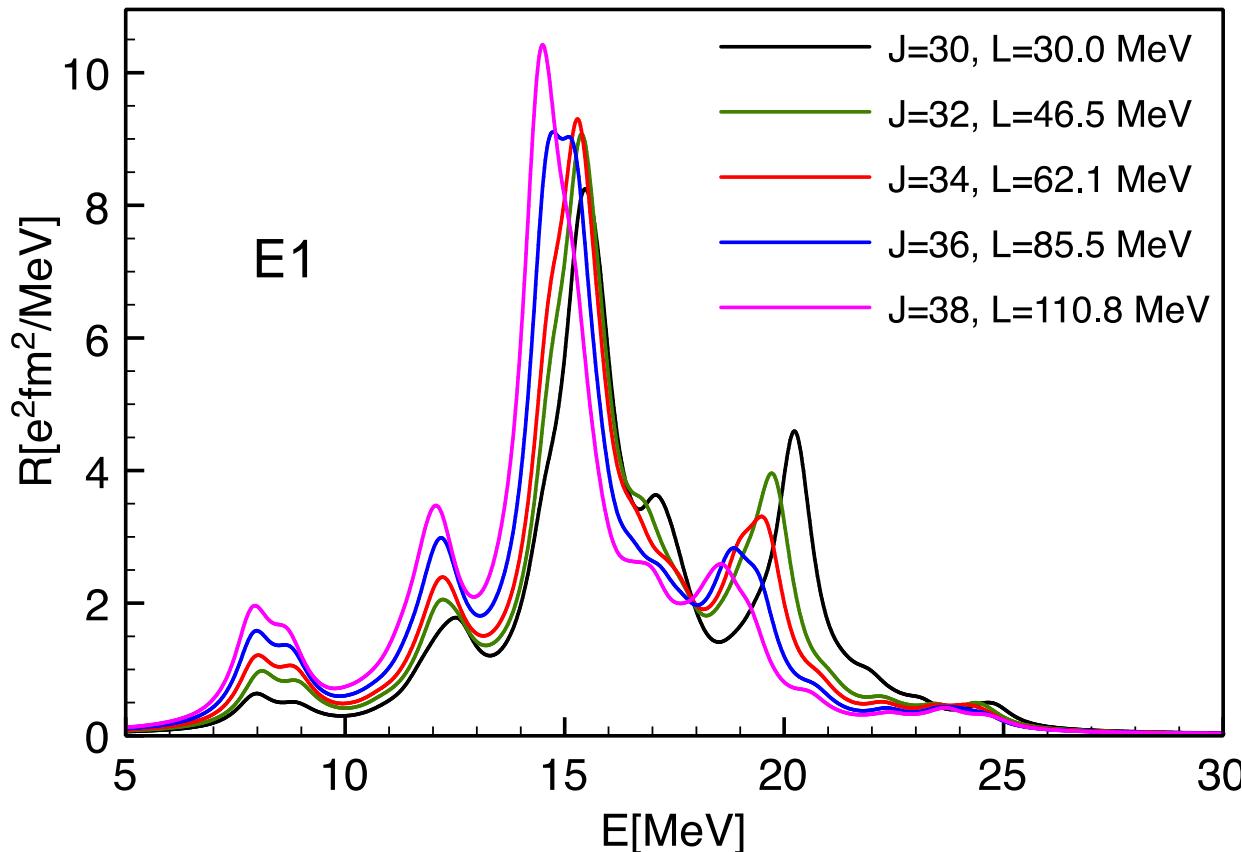


Density ρ/ρ_0

B. Tsang, NSCL

CONSTRAINING THE SYMMETRY ENERGY

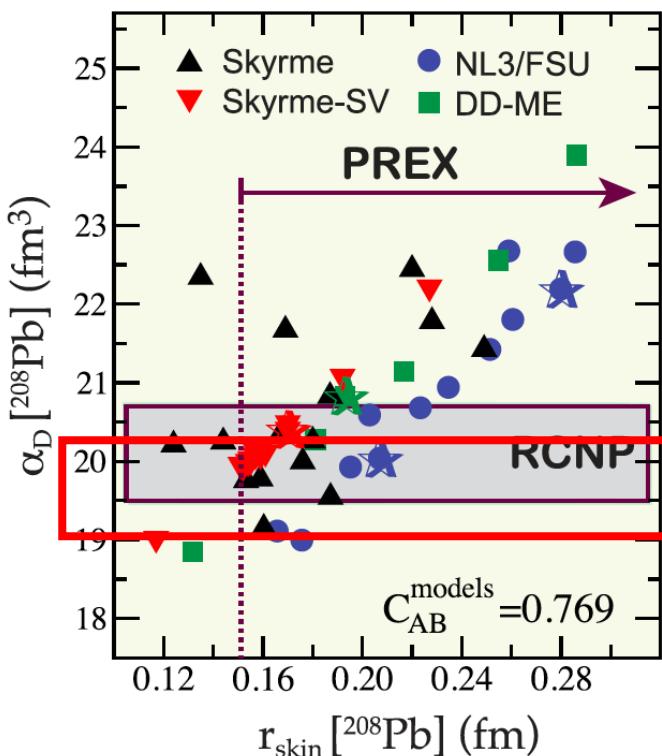
- Isovector dipole transition strength is sensitive to the symmetry energy at saturation density (J) / slope of the symmetry energy (L).
- Set of effective interactions constrained on the same data, but with different constraint on J (30,32,...,38 MeV).



CONSTRAINING THE SYMMETRY ENERGY

- Electric dipole polarizability α_D is strongly correlated with the neutron skin thickness and symmetry energy parameters

$$\alpha_D = \frac{8\pi}{9} e^2 m_{-1}$$



J. Piekarewicz, et al.,
PRC 85, 041302 (R) (2012)

- A word of caution: the value

$$\alpha_D(^{208}\text{Pb}) = 20.1 \pm 0.6 \text{ fm}^3$$

(A.Tamii et al., PRL 107, 062502 (2011))

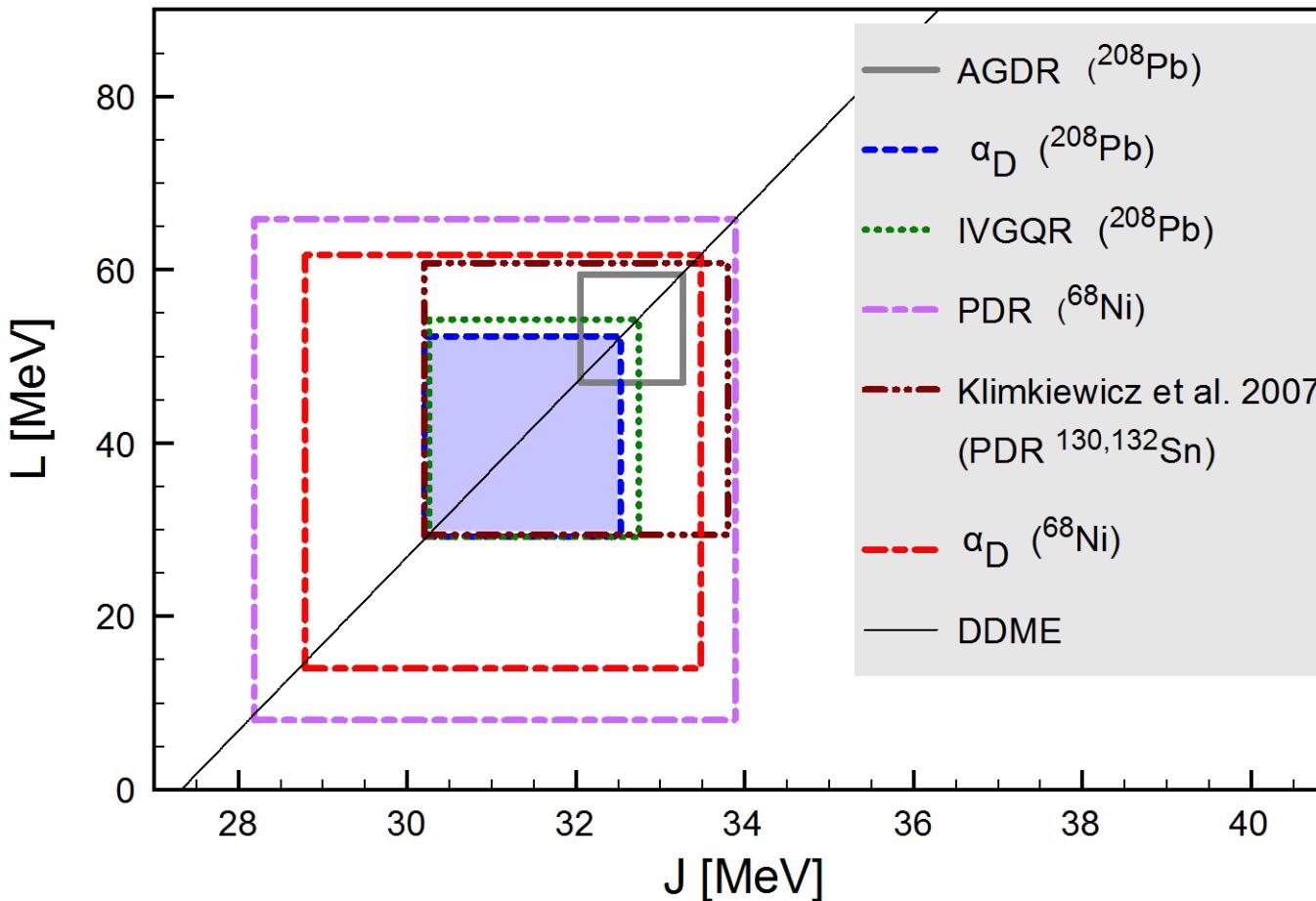
contains nonresonant contributions at higher energies (quasideuteron excitations) that should be removed before comparison with the RPA calculations

- Correct value for comparison with the RPA
A. Tamii,T. Hashimoto, private communications (2015).

$$\alpha_D(^{208}\text{Pb}) = 19.6 \pm 0.6 \text{ fm}^3$$

CONSTRAINING THE SYMMETRY ENERGY

N. P., Ch. C. Moustakidis, et al PRC 90, 011304(R) (2014) + update on α_D



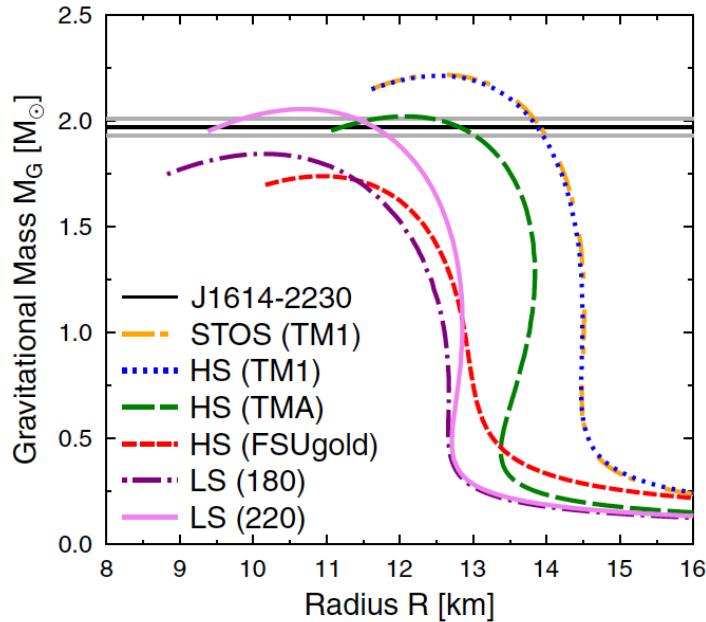
- The same set of DD-ME interactions used in the analysis based of various giant resonances and pygmy strengths (consistent theory !)
- Excellent agreement, except for the AGDR – new measurements are needed for the AGDR (→ talk A. Krasznahorkay)

Exp. data
for various
excitations:

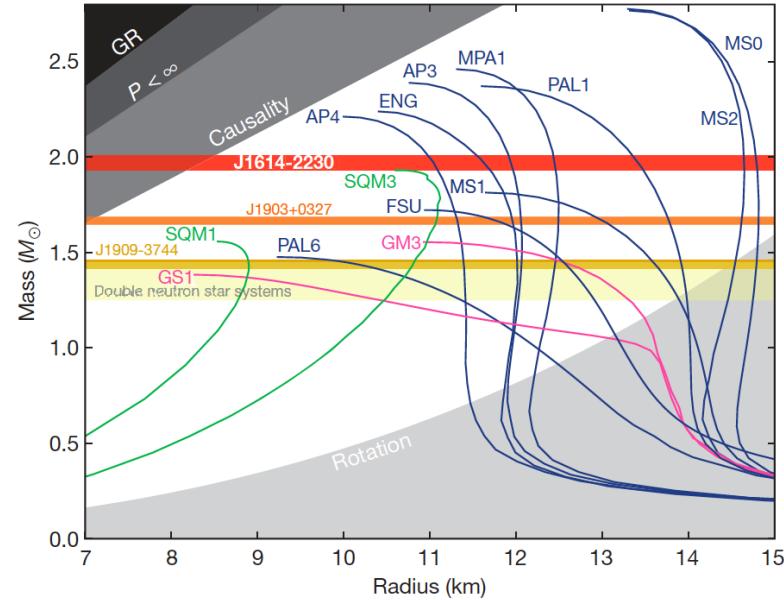
- $\alpha_D ({}^{208}\text{Pb}) \rightarrow$ A. Tamii et al., PRL 107, 062502 (2011) – update A. Tamii et al. (2015) (no q-deuteron).
- $\alpha_D ({}^{68}\text{Ni}) \rightarrow$ K. Boretzky, D. Rossi, T. Aumann, et al., (2015).
- $\text{PDR} ({}^{68}\text{Ni}) \rightarrow$ O. Wieland, A. Bracco, F. Camera et al., PRL 102, 092502 (2009).
- $({}^{130,132}\text{Sn}) \rightarrow$ A. Klimkiewicz et al., PRC 76, 051603(R) (2007).
- $\text{IVGQR} ({}^{208}\text{Pb}) \rightarrow$ S. S. Henshaw, M. W. Ahmed G. Feldman et al, PRL 107, 222501 (2011).
- $\text{AGDR} ({}^{208}\text{Pb}) \rightarrow$ A. Krasznahorkay et al., arXiv:1311.1456 (2013)

NEUTRON STAR PROPERTIES

- Mass-radius relations of cold neutron stars for different EOS – observational constraints on the neutron star mass rule out many models for EOS.



M. Hempel et al., Astr.J. 748,70 (2012)



P. B. Demorest et al., Nature 467, 1081 (2010)

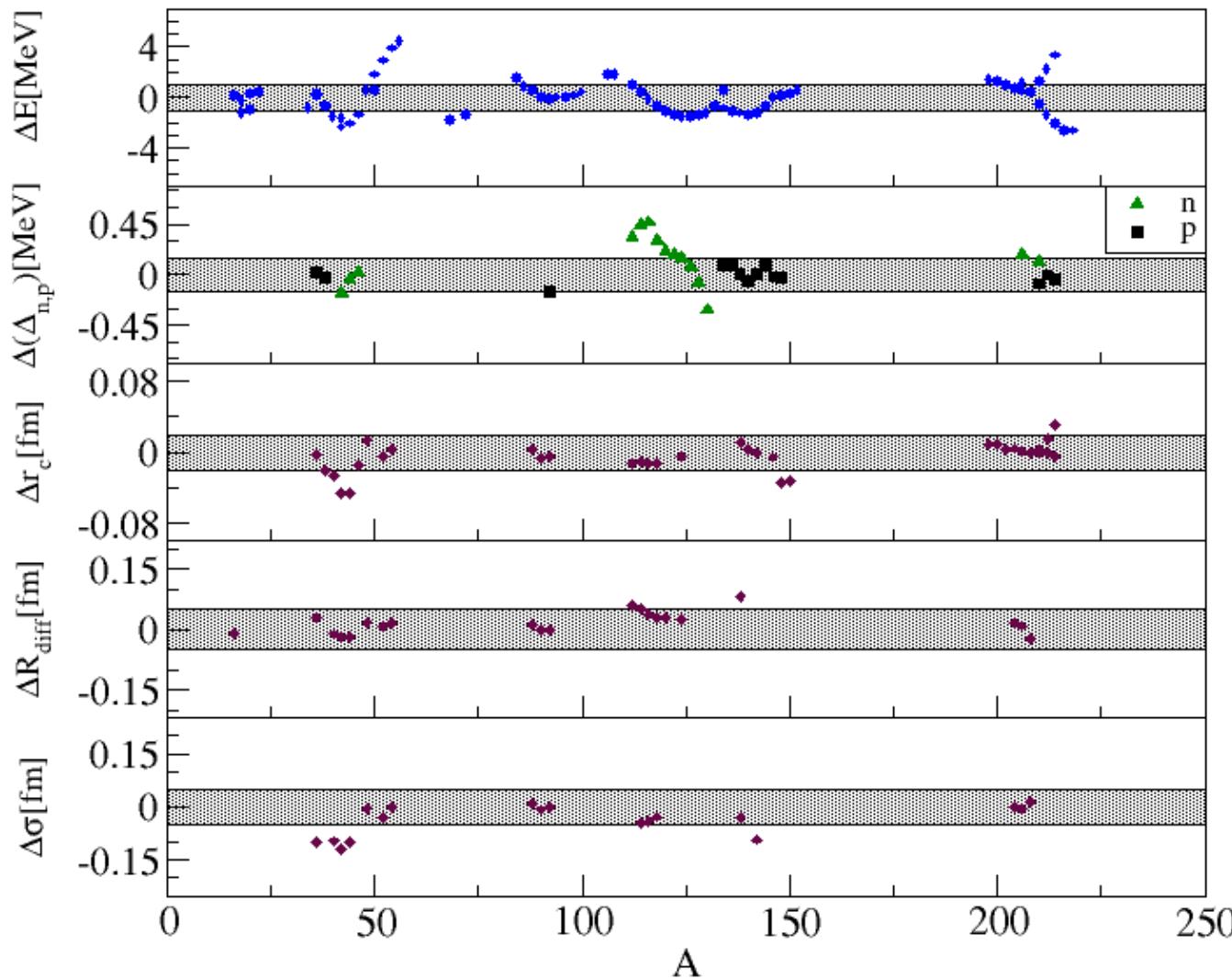
- Building relativistic EDFs for finite nuclei and neutron stars
Wei-Chia Chen and J. Piekarewicz, PRC 90, 044305 (2014).
J. Erler, C.J. Horowitz, W. Nazarewicz et al., PRC 87, 044320 (2013).
- Constraints on the maximal neutron star mass from observation:
J. Antoniadis, P. C. C. Freire, N. Wex et al. Science 340, 448 (2013) → 2.01(4) Msun
P. B. Demorest et al., Nature 467, 1081 (2010) → 1.97(4) Msun

Towards a universal relativistic nuclear energy density functional for astrophysical applications – RNEDF1 (N.P., M. Hempel et al. 2015)

The strategy to constrain the functional (relativistic point coupling model)

- Adjust the properties of 72 nuclei to exp. data (binding energies ($\Delta=1$ MeV), charge radii (0.02 fm), diffraction radii (0.05 fm), surface thickness (0.05 fm))
- Improve description of open-shell nuclei by adjusting the pairing strength parameter to empirical paring gaps (n,p) (0.14 MeV)
- constrain the symmetry energy $S_2(\rho_0)=J$ (2%) from experimental data on dipole polarizability (^{208}Pb) A. Tamii et al., PRL 107, 062502 (2011) + update (2015).
- constrain the nuclear matter incompressibility K_{nm} (2%) from exp. data on ISGMR modes (^{208}Pb) D.H. Youngblood et al., PRC 69, 034315 (2004); D. Patel et al., PLB 726, 178 (2013).
- constrain the equation of state using the saturation point (ρ_0) and point at twice the saturation density ($2\rho_0$) from heavy ion collisions (FOPI-IQMD) (10%) A. Le Fevre et al., arXiv:1501.05246v1 (2015)
- constrain the maximal neutron star mass by solving the Tolman-Oppenheimer-Volkov (TOV) equations and using observational data (slightly larger value) $M_{\text{max}}=2.2M_\odot$ (5%) J. Antoniadis, et al. Science 340, 448 (2013); P. B. Demorest et al., Nature 467, 1081 (2010)
- The fitting protocol is supplemented by the covariance analysis

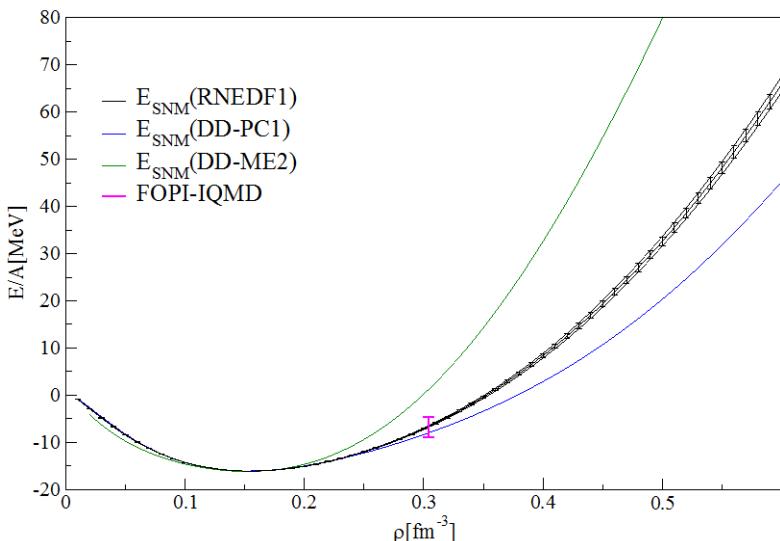
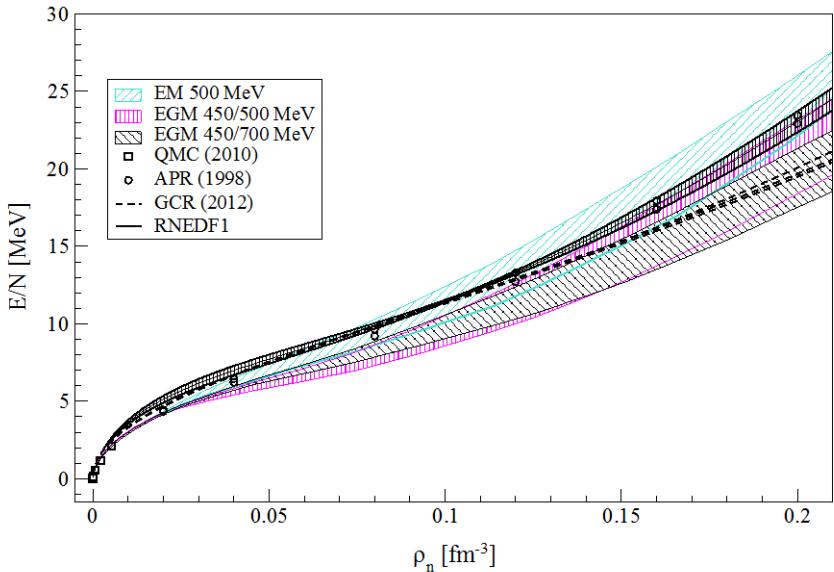
RNEDF1: DEVIATIONS FROM THE EXP. DATA



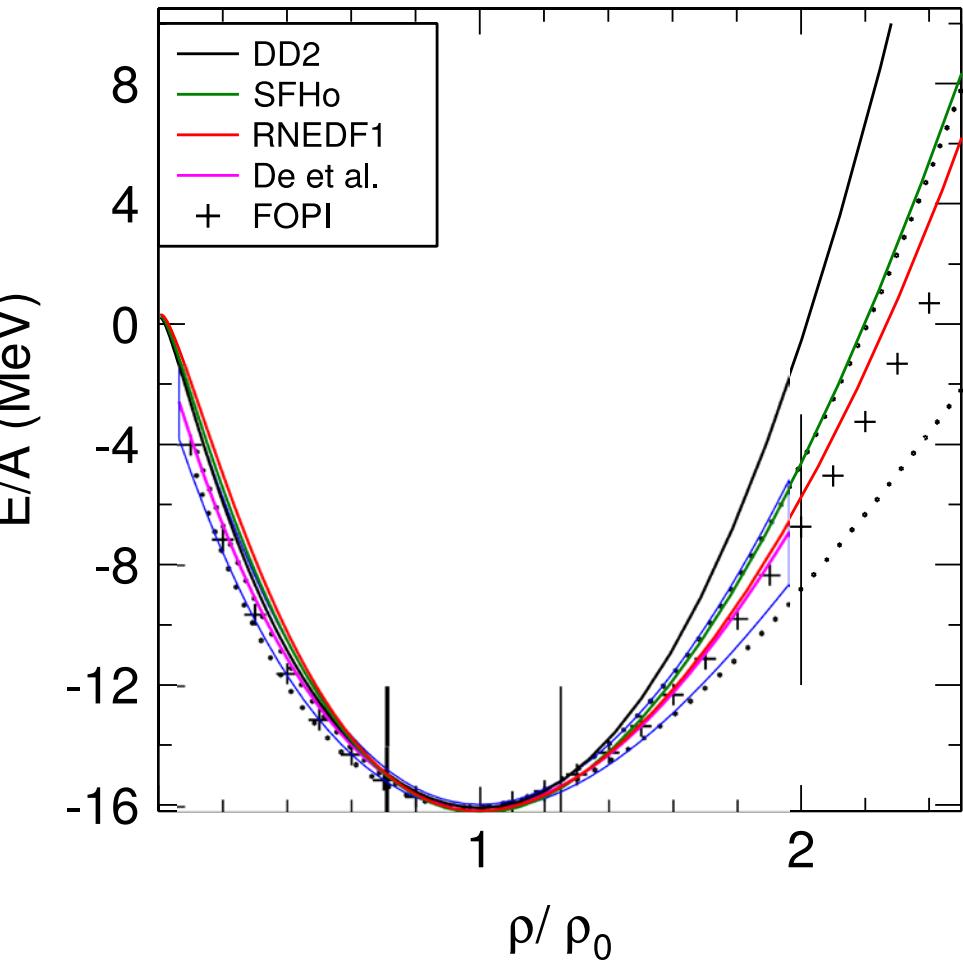
- binding energies
- pairing gaps
- charge radii
- diffraction radii
- surface thickness

RNEDF1: NUCLEAR MATTER PROPERTIES

NEUTRON MATTER

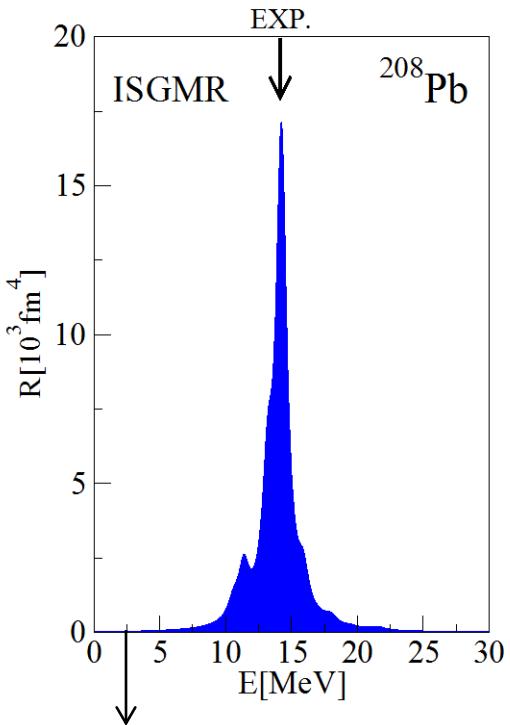


SYMMETRIC NUCLEAR MATTER



GIANT RESONANCES, COMPRESSIBILITY, SYMMETRY ENERGY

ISOSCALAR GIANT MONOPOLE RESONANCE

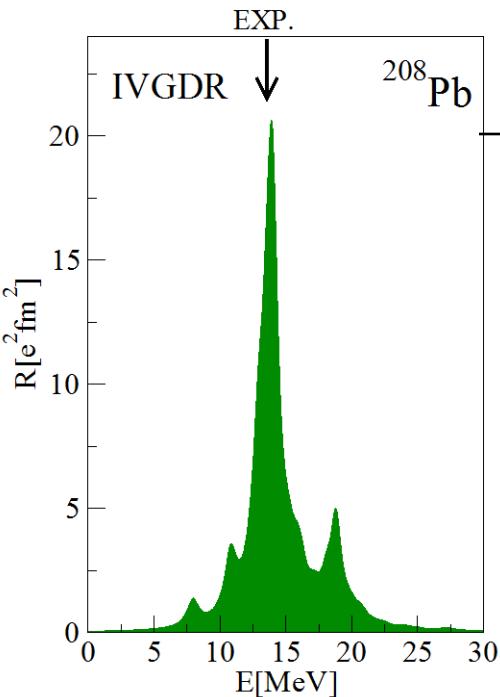


- ISGMR energy determines the nuclear matter incompressibility:
 $K_{\text{nm}} = 232.4 \text{ MeV}$

$$E(\text{Exp.}) = (13.91 \pm 0.11) \text{ MeV (TAMU)}$$

$$E(\text{Exp.}) = (13.7 \pm 0.1) \text{ MeV (RCNP)}$$

ISOVECTOR GIANT DIPOLE RESONANCE



Dipole polarizability:
 $\alpha_D = (19.68 \pm 0.21) \text{ fm}^3$

Exp.
 $\alpha_D = (19.6 \pm 0.6) \text{ fm}^3$

A.Tamii et al., PRL 107, 062502
(2011). + update (2015).

- IVGDR – α_D constrain the symmetry energy of the interaction

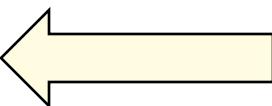
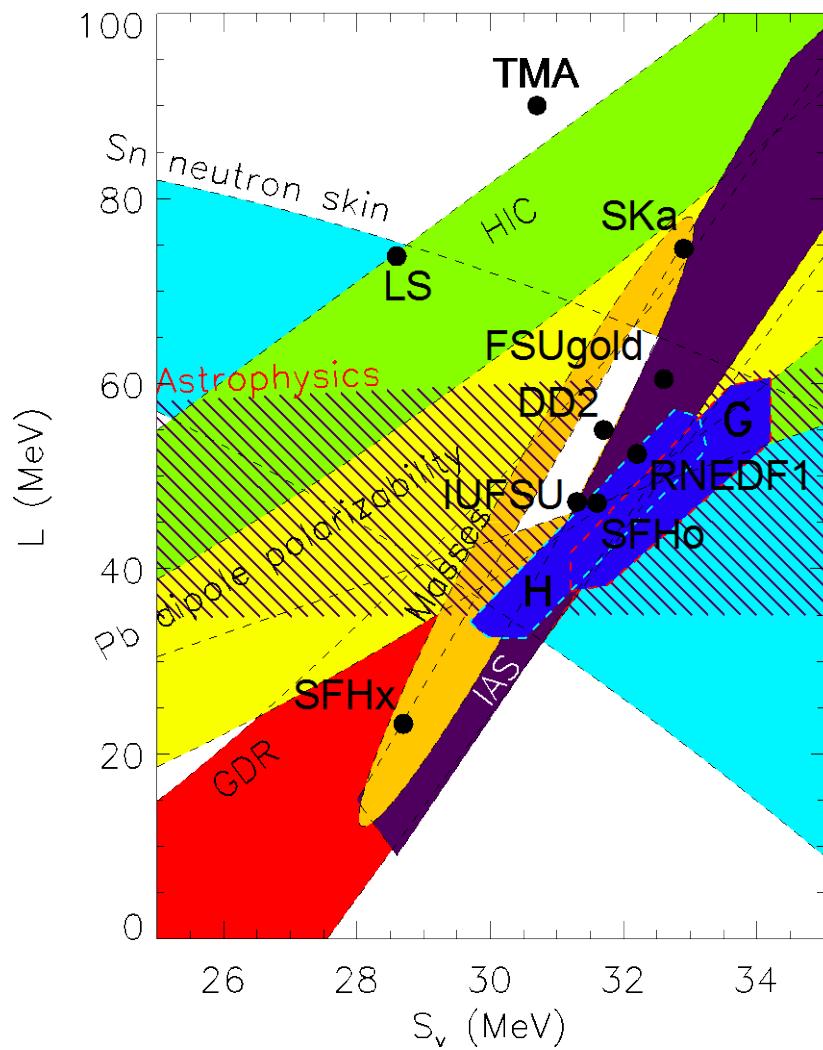
$$\boxed{\begin{aligned} J &= 31.89 \text{ MeV} \\ L &= 51.48 \text{ MeV} \end{aligned}}$$

- Lattimer & Lim, ApJ. 771, 51 (2013)

$$\boxed{\begin{aligned} J &= 29.0\text{--}32.7 \text{ MeV} \\ L &= 40.5\text{--}61.9 \text{ MeV} \end{aligned}}$$

NL3●

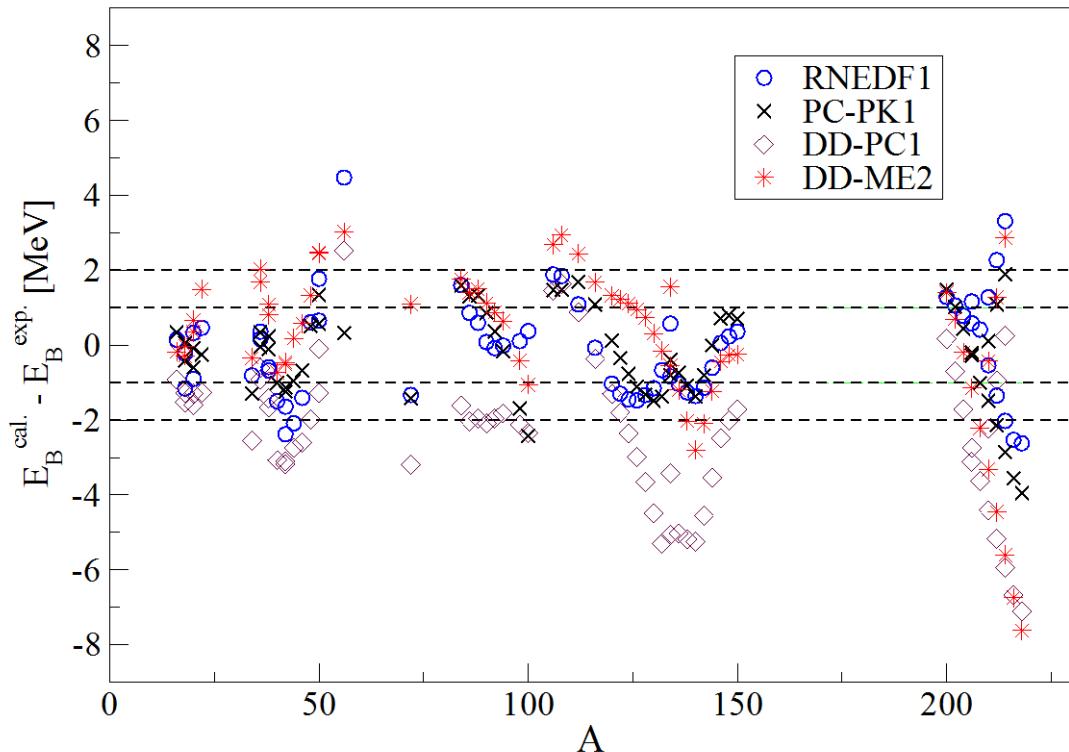
TM1●



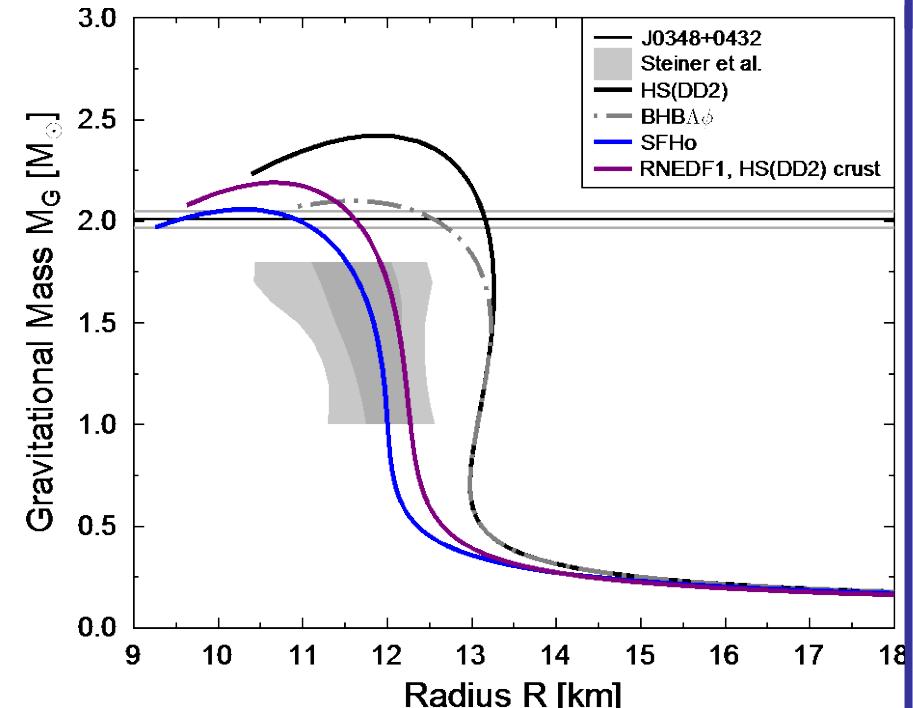
Also see: Lattimer & Lim, ApJ. 771, 51 (2013)

RNEDF1: FROM FINITE NUCLEI TOWARD THE NEUTRON STAR

Nuclear binding energies (calc. – exp.)

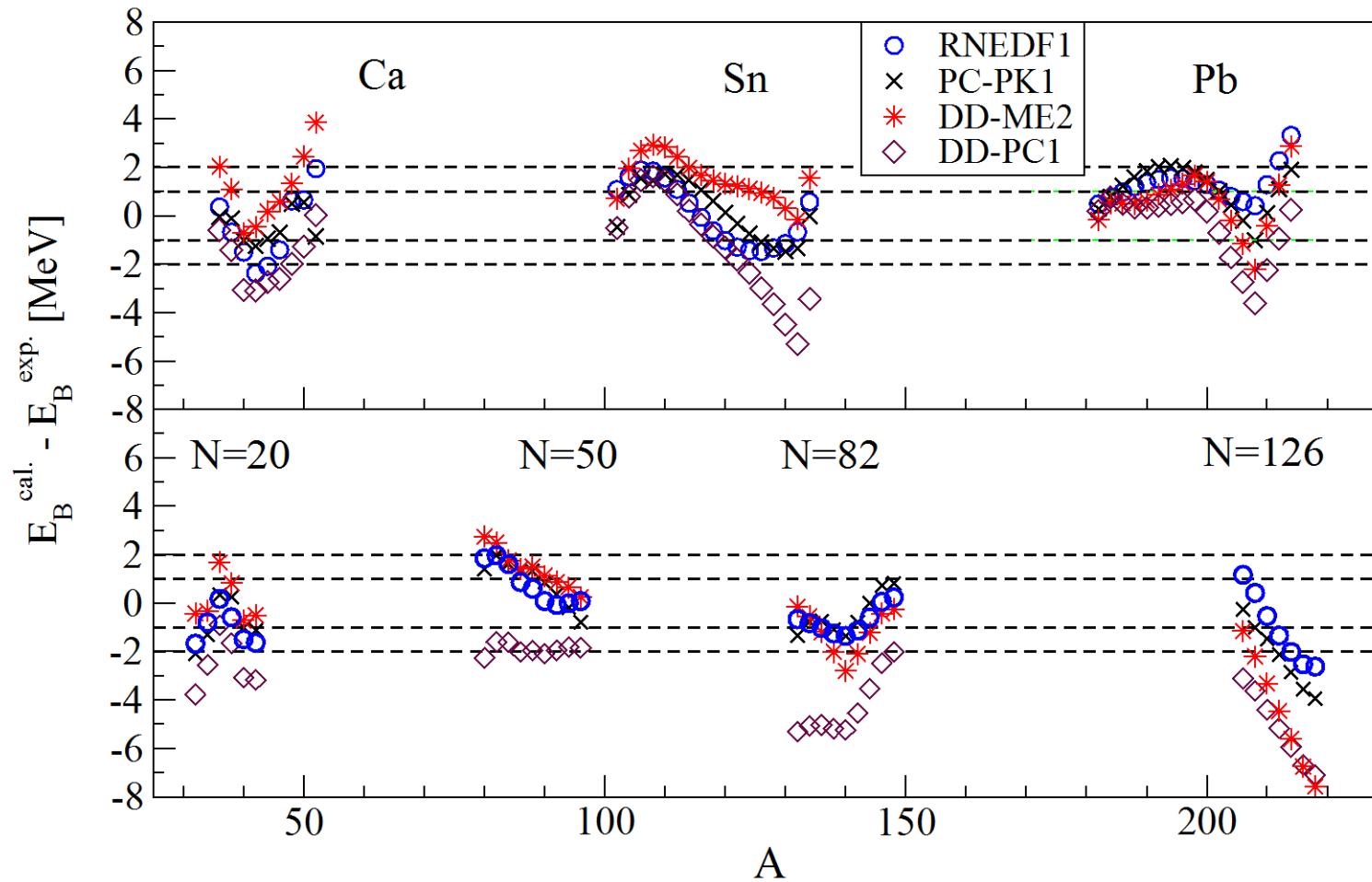


Neutron star mass-radius

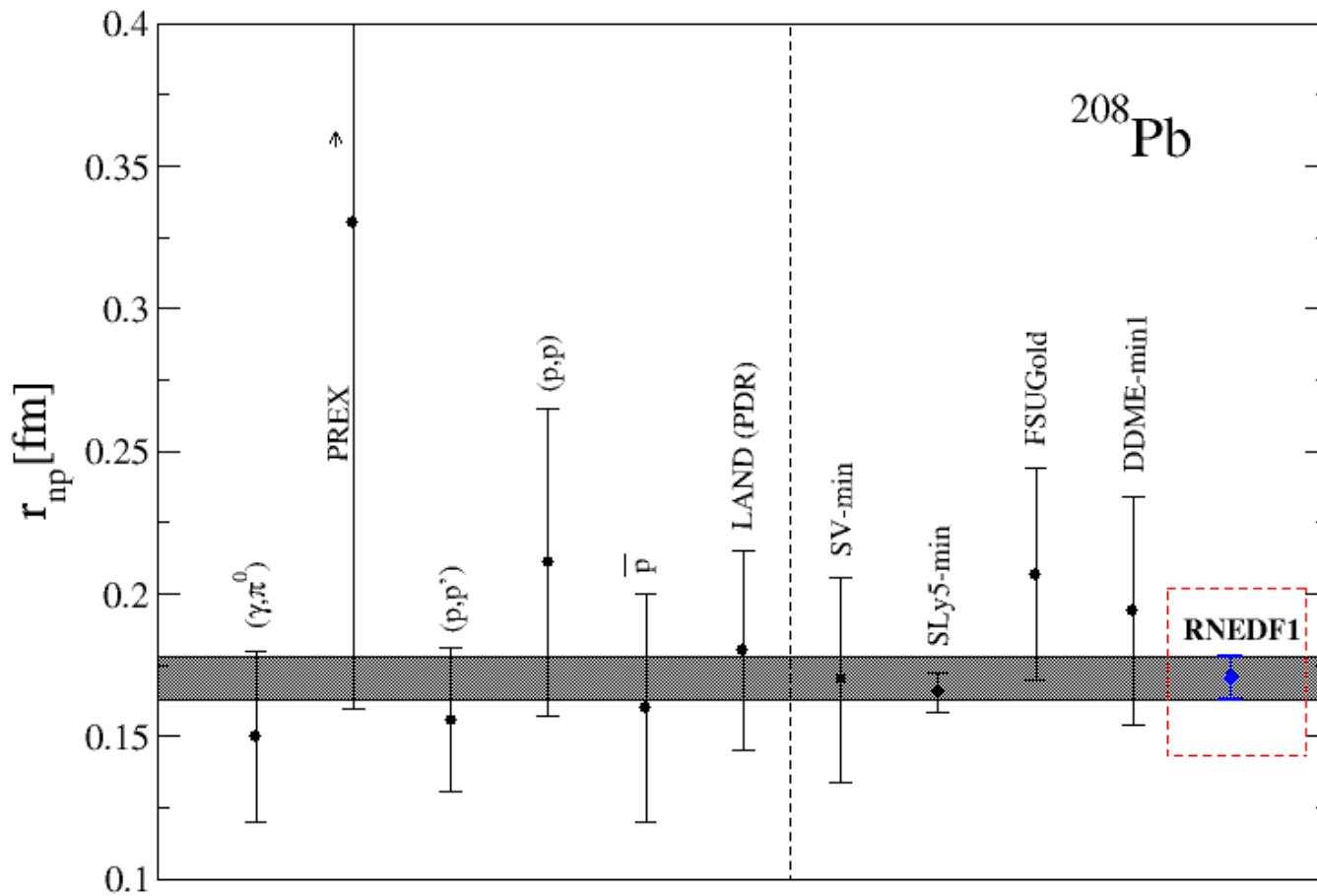


PC-PK1: P.W.Zhao et al., PRC 82, 054319 (2010)
 - (K=238 MeV, J=35.6, L = 113 MeV)

RNEDF1: ISOTOPE AND ISOTONE CHAINS



RNEDF1: NEUTRON SKIN THICKNESS IN ^{208}Pb



(γ, π^0) : C.M. Tarbert et al., PRL 112, 242502 (2014)

PREX: S. Abrahamyan et al., PRL. 108, 112502 (2012)

(p, p') : A. Tamii et al., PRL 107, 062502 (2011)

(p, p) : J. Zenihiro et al., PRC 82, 044611 (2010)

Antipr. at.: B. Kłos et al., Phys. Rev. C 76, 014311 (2007).

LAND (PDR): A. Klimkiewicz et al., PRC 76, 051603 (2007).

SV-min: P.G. Reinhard et al.

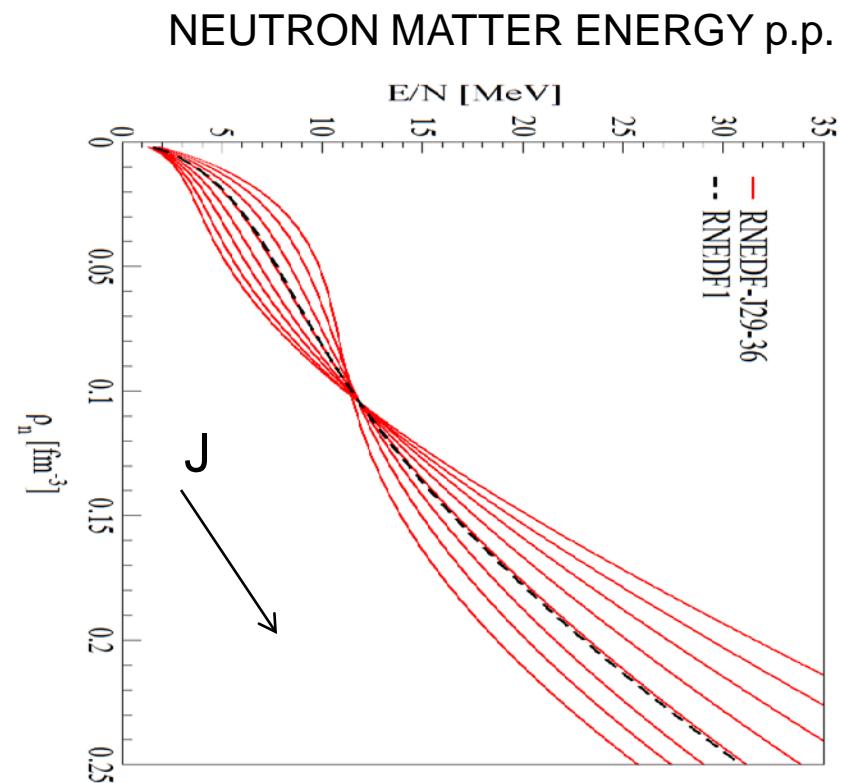
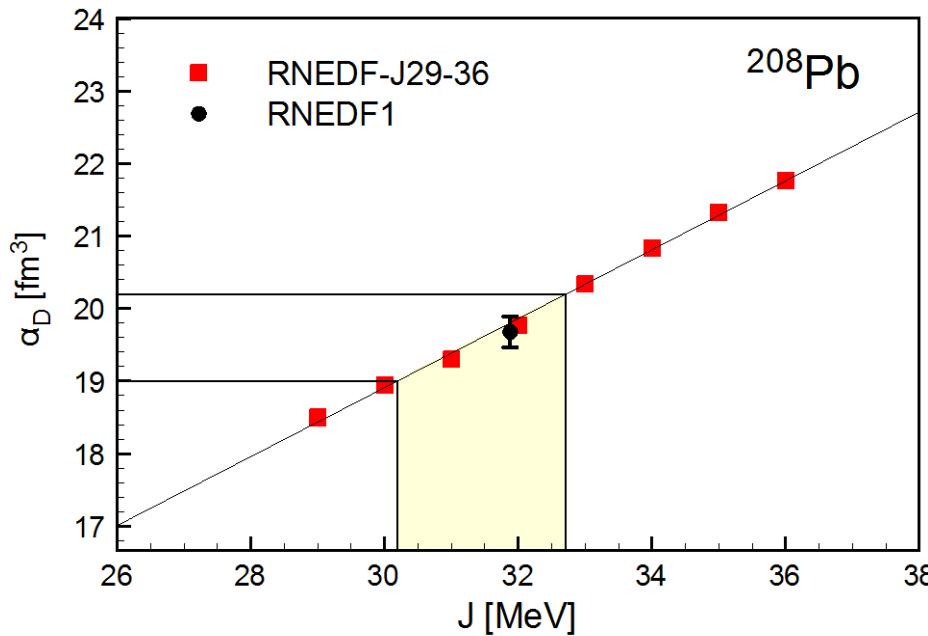
SLy5-min: X. Roca-Maza, G. Colò et al.

FSUGold: J. Piekarewicz et al.

DDME-min1: N.P. et al.

VARIATION OF THE SYMMETRY ENERGY IN CONSTRAINING THE EDF

- New set of 8 relativistic point coupling interactions that span the range of values of the symmetry energy at saturation density: $J=29,30,\dots,36$ MeV
- Each interaction is determined independently with strong constraint on J



CONCLUDING REMARKS

- ✓ Towards a universal self-consistent framework based on the relativistic nuclear energy density functional from finite nuclei toward neutron stars
- Accurate measurements of collective excitations in finite nuclei have important implications in constraining the EDFs, properties of the symmetry energy and neutron star properties



APPLICATIONS IN PROGRESS

- neutrino-nucleus cross sections, both for neutral-current and charged current reactions
- modeling neutrino response in neutrino detectors – constraining neutrino mass hierarchy from supernova neutrino signal
- systematic calculations of presupernova electron capture rates at finite temperature
- neutron star properties – mass/radius relationship, liquid-to-solid core-crust transition density and pressure

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