Dipole toroidal resonance: vortical properties, anomalous deformation impact, relation to pygmy mode

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To discuss:

★ Exotic isoscalar E1 resonances:
  - toroidal (TR),
  - compression (CR)
  - pygmy (PDR)

J. Kvasil, V.O.N., W. Kleinig, P.-G. Reinhard, P.. Vesely,
PRC 84, 034303 (2011)

★ TR is the most accurate measure of the nuclear vorticity

P.-G. Reinhard, V.O.N, A. Repko, and J. Kvasil,
PRC 89, 024321 (2014).

★ Anomalous deformation splitting of TR as its fingerprint

J. Kvasil, V.O. Nesterenko, W. Kleinig, and P.-G. Reinhard,

★ PDR is a peripheral manifestation of TR?

A. Repko, P.-G. Reinhard, V.O.N. and J. Kvasil,

★ Experimental perspectives
Exotic dipole resonances

Dominate in E1(T=0) channel (after exclusion of spurious E1(T=0) c.m. motion)

irrotational  vortical  irrotational

\[ E = 50 \div 60 \ A^{\frac{1}{3}} \ MeV \]  \[ E = 50 \div 70 \ A^{\frac{1}{3}} \ MeV \]  \[ E = 132 \ A^{\frac{1}{3}} \ MeV \]

Reviews:

TR and CR consist of low- and high-energy ISGDR branches

Experiment: \((\alpha, \alpha')\)

\(^{208}\text{Pb}\)
D.Y. Youngblood et al, 1977
H.P. Morsch et al, 1980
G.S. Adams et al, 1986
B.A. Devis et al, 1997
H.L. Clark et al, 2001
D.Y. Youngblood et al, 2004

There are also the ISGDR data in
\(^{56}\text{Fe}, ^{58,60}\text{Ni}, ^{90}\text{Zr}, ^{116}\text{Sn}, ^{144}\text{Sm}, \ldots\)

Theory:

A. Repko, P.-G. Reinhard, V.O.N. and J. Kvasil,

Skyrme RPA, SLy6

-discrepancy between theory and experiment for TR
-perhaps, Uchida observed not TR but low-energy CR fraction. Then TR is not still observed?
Toroidal E1 operator:

\[
\hat{M}_{\text{tor}}(E1\mu) = \frac{1}{10\sqrt{2}c} \int d\vec{r} \left[ r^3 + \frac{5}{3}r < r^2 >_0 \right] Y_{11\mu}(\vec{r}) \cdot \left[ \hat{\nabla} \times \hat{j}_{\text{nuc}}(\vec{r}) \right]
\]

- second-order part of the electric operator

Compression E1 operator:

\[
\hat{M}_{\text{com}}(E1\mu) = -\frac{i}{10c} \int d\vec{r} \left[ r^3 - \frac{5}{3}r < r^2 >_0 \right] Y_{1\mu} \left[ \hat{\nabla} \cdot \hat{j}_{\text{nuc}}(\vec{r}) \right]
\]

irrotational flow

\[
\hat{M}'_{\text{com}}(E1\mu) = \int d\vec{r} \rho(\vec{r}) \left[ r^3 - \frac{5}{3}r < r^2 >_0 \right] Y_{1\mu} \quad \hat{M}_{\text{com}}(E1\mu) = -k\hat{M}'_{\text{com}}(E1\mu)
\]

\[\rho + \hat{\nabla} \cdot \hat{j}_{\text{nuc}} = 0\]

- TR and CR are ideal examples for the vortical and irrotational motion

- to be used below for the tests

Toroidal motion as the measure of the nuclear vorticity
Two familiar conceptions of nuclear vorticity: HD, RW

1. Hydrodynamical vorticity:

\[ \vec{\omega}(\vec{r}) = \vec{\nabla} \times \vec{v}(\vec{r}) \quad \delta \vec{v}(\vec{r}) = \frac{\delta \vec{j}_{\text{nuc}}(\vec{r})}{\rho_0(\vec{r})} \]

\[ (\vec{\nabla} \times \delta \vec{j}_{\text{nuc}}) \rightarrow \rho_0(\vec{r})(\vec{\nabla} \times \delta \vec{v}) \rightarrow \rho_0(\vec{r}) \vec{\omega}(\vec{r}) \]

2. RW vorticity

\[ \dot{\rho} + \vec{\nabla} \cdot \vec{j}_{\text{nuc}} = 0 \quad - \text{continuity equation} \]

\[ \delta \vec{j}_{(n)}(\vec{r}) = \left\langle j_f m_f \mid \hat{\vec{j}}_{\text{nuc}}(\vec{r}) \mid j_i m_i \right\rangle = \sum_{\lambda,\mu} \frac{(j_i m_i \lambda \mu \mid j_f m_f)}{\sqrt{2 j_f + 1}} \left[ j_{\lambda \lambda - 1}(r) Y_{\lambda \lambda - 1 \mu}^* + j_{\lambda \lambda + 1}(r) Y_{\lambda \lambda + 1 \mu}^* \right] \]

\[ \delta \vec{j}_{1 \mu}^\nu(\vec{r}) = \left\langle \nu \mid \hat{\vec{j}}_{\text{nuc}}(\vec{r}) \mid 0 \right\rangle = -\frac{1}{\sqrt{3}} \left[ j_{10}^\nu(r) Y_{10 \mu}^* + j_{12}^\nu(r) Y_{12 \mu}^* \right] \quad - \text{current transition density} \]

\[ j_+^\nu(r) \quad - \text{independent part of charge-current distribution,} \]

\[ \text{- decoupled from CE in the integral sense} \]

\[ \text{- may be the measure of the vorticity} \]

208Pb: all RPA states at \( E=6-9 \) MeV

\( j^+, j^- : \)
- both have strong curl’s and div’s
- there is no any advantage of \( j^+ \) over \( j^- \) to represent the vorticity

The vortical or irrotational character of the flow is provided not by \( j^+ \) or \( j^- \) components separately but by their proper superposition.

So just the toroidal current but not \( j^+ \) is the relevant measure of the nuclear vorticity.

\[
\langle \nu / \hat{M}_{\text{tor}}(E1\mu)/0 \rangle = -\frac{1}{6c} \int dr \, r^2 \left[ \frac{\sqrt{2}}{5} r^2 j_+^\nu(r) + (r^2 - \langle r^2 \rangle_0) j_-^\nu(r) \right]
\]

\[
\langle \nu / \hat{M}_{\text{com}}(E1\mu)/0 \rangle = -\frac{1}{6c} \int dr \, r^2 \left[ \frac{2\sqrt{2}}{5} r^2 j_+^\nu(r) - (r^2 - \langle r^2 \rangle_0) j_-^\nu(r) \right]
\]
Anomalous deformation effect in the toroidal resonance

To be used as TR fingerprint?


Deformation effects in the toroidal mode


GDR: $E(\mu = 0) < E(\mu = 1)$
TM: $E(\mu = 0) > E(\mu = 1)$

Unusual sequence of $\mu = 0$ and $\mu = 1$ branches
Deformation (not resid. Interaction) effect
Non-Tassie mode!
Should affect PDR properties
The deformation effect can be used:
- as a direct experimental fingerprint of the toroidal flow,
- can be observed in \((\alpha, \alpha', \gamma)\) reaction where \(\mu\)-branches can be discriminated.

\[ \nabla \times \vec{F} = 0, \nabla \cdot \vec{F} = 0 \]
\[ \vec{F} = \nabla \Phi, \Phi = r^\lambda Y_{\lambda \mu} \]

GDR: \(\Phi = rY_{1\mu}\) - Tassie mode
TR: \(\Phi = r^3 Y_{1\mu}\) - non-Tassie mode

IV residual interaction upshifts the Tassie-like dipole strength.
Perhaps the remaining small strength is basically of non-Tassie character (toroidal).
Relation of E1 toroidal and pygmy resonances

Is PDR a local (peripheral) part of TR?

A. Repko, P.-G. Reinhard, V.O. Nesterenko, and J. Kvasil,
"Toroidal nature of the low-energy E1 mode",

V.O. Nesterenko, A. Repko, P.-G. Reinhard, and J. Kvasil,
"Relation of E1 pygmy and toroidal resonances",
EPJ Web of Conferences, 93, 01020 (2015); arXiv:1410.5634[nucl-th],
Strength functions

A. Repko, P.G. Reinhard, VON, J. Kvasil, PRC, 87, 024305 (2013)

Two peaks at 7.5 and 10.3 MeV in agreement to RMF calculations (D. Vretenar, N. Paar, P. Ring, PRC, 63, 047301 (2001))

(\(\alpha, \alpha'\)) experiment of Uchida et al (2003)

PDR region hosts TR and CR!

Typical PDR transition density:
- n
- p
Benchmark examples

Current fields:

\[ \hat{j}(\vec{r}) = -i \sum_{q=n,p} e_{eff}^q \sum_{k \in q} (\delta(\vec{r} - \vec{r}_k) \hat{\nabla}_k - \hat{\nabla}_k \delta(\vec{r} - \vec{r}_k)) \]

\[ \delta \vec{j}_\nu(\vec{r}) = \langle \nu | \vec{j}(\vec{r}) | 0 \rangle \]

- Transition density of the convection current for the RPA state \( \nu \)

\[
T=0: \quad e_{eff}^n = e_{eff}^p = 1 \\
T=1: \quad e_{eff}^p = \frac{N}{A}, \quad e_{eff}^n = -\frac{Z}{A} \\
p: \quad e_{eff}^p = 1, \quad e_{eff}^n = 0 \\
n: \quad e_{eff}^p = 0, \quad e_{eff}^n = 1
\]

- Good reproduction of known fields,
- Justifies accuracy of our model
RPA vs 1ph

1ph

- both isoscalar and isovector
- toroidal flow mainly from neutrons

RPA

- mainly isoscalar
- toroidal flow from both n/p

So the toroidal flow is basically formed already by the mean-field. But residual interaction makes it collective and more impressive.
Does the toroidal flow contradict the familiar PRD picture?
- PDR can be viewed as a local peripheral part of TR and CR
- Our calculations demonstrate the TR motion in PDR energy region for other nuclei: Ni, Zr, Sn, …


132Sn, SVbas, with PDR
40Ca, SVbas

![Graphs for 40Ca, SVbas with energy distributions and angular distributions for protons (p) and neutrons (n).]

48Ca, SVbas

![Graphs for 48Ca, SVbas with energy distributions and angular distributions for protons (p) and neutrons (n).]
So it is quite possible that PDR is a peripheral part of the dipole toroidal flow!
TR: experimental perspectives -1

Experiment: \((\alpha, \alpha')\)


\((e,e')\), - both IS/IV, strong magnetic form-factor
\((p,p')\) - both IS/IV
photoabsorption, \((\gamma, \gamma')\) - both IS/IV

Peripheral IS reactions \((\alpha, \alpha')\) and \((^{16,17}O, ^{16,17}O')\) seem to be the best options:
To use \((\alpha, \alpha' \gamma)\) in deformed nuclei.
TR can be excited though its peripheral part (together with IS PDR and CR).

What we actually observe in \((\alpha, \alpha')\)? Isoscalar PDR or TR?
This is yet unclear … .

124Sn, \((\alpha, \alpha' \gamma)\)
A. Bracco: \((^{17}O, ^{17}O' \gamma)\)

LE HE
(toroidal) (compression)
It would be interesting to observe:

1) Deformation splitting (sequence of K-branches) in TR/PDR energy region by using \((\alpha, \alpha' \gamma)\) - direct fingerprint of TR!

2) To look for TR in \(N \approx Z\) nuclei where the PDR is absent.

   There are preliminary data on TR in \(^{27}\text{Al}\) \((^{16}\text{O},^{16}O', \text{F. Cappuzzello et al})\)

   It would be interesting to inspect the deformed \(^{28}\text{Si}\).

3) Comparison of photoabsorption and \((\alpha, \alpha')\) data in nuclei with \(N=Z\) and \(N>Z\). For example:

\[
\begin{array}{cccc}
\text{40Ca} & \text{48Ca} \\
(\alpha, \alpha') & \text{IS}, \ r^3Y_{1\mu} & \text{TR} & \text{TR, PDR} \\
\text{Photoabsorp.} & \text{IS/IV}, \ rY_{1\mu} & \text{--} & \text{PDR}
\end{array}
\]

To compare TR, PDR and GDR \((\alpha, \alpha')\) formfactors. The TR formfactors should have maxima at higher transfer mom. (talk of P.G. Reinhard)
Conclusions

★ Toroidal current (strength) is the most relevant fingerprint and measure of the nuclear vorticity.
- It is more convenient and relevant than RW and HD prescriptions.
- TR is the only known example of the vortical collective electric motion.

★ Anomalous deformation effect as TR specific feature.

★ PDR could be:
- local surface part of the toroidal motion.
- or oscillations of the neutron excess, coupled to TR and CR

PDR is a complex mixture of:
- IS/IV,
- collective/s-p,
- irrotational/vortical,
- TM / CM / GDR,
- complex configurations

★ IS reactions $(\alpha, \alpha')$, $(\alpha, \alpha' \gamma)$, $(^{16}\text{O}, ^{16}\text{O}')$ are best.

★ Outlook:
- TR in deformed nuclei: $(\alpha, \alpha' \gamma)$ to observe anomalous deformation effect as the TR fingerprint,
- comparative measurements of TR and PDR at about the same conditions

Response depends on the probe!
Thank you for attention!
Previous studies

D. Vretenar et al, relativistic mean field RPA


QPM calculations taking into account complex configurations

**Summed** QPM velocity fields in 6.5-10.5 MeV region

\[ \delta \vec{V} = \frac{N}{A} \delta \vec{V}_p - \frac{Z}{A} \delta \]

However none of these studies has clamed the toroidal origin of PDR
$^{208}\text{Pb}$:
all RPA states
at $E=6$-$9$ MeV

$j^+$, $j^-$:
- both have strong curls and divs
- Both locally vortical and irrotational
- no any curl-advantage of $j^+$ over $j^-$ to

- $j^+$ has no any strong advantage over $j^-$
to represent the vortical flow.
j+ and j- contributions to TR and CR

- Both j+ and j- are peaked at low-energy and high-energy regions. They are equally active in vortical TR and irrotational CR.

- TR and CR are formed by constructive interference of the current components while in other regions there is the destructive interference.

- j+ has no any strong advantage to be a vortical descriptor!

\[
\left< \nu / \hat{M}_{tor} (E1\mu) / 0 \right> = -\frac{1}{6c} \int dr^2 \left[ \frac{\sqrt{2}}{5} r^2 j_+^{\nu}(r) + (r^2 - \left< r^2 \right>_0) j_-^{\nu}(r) \right]
\]

\[
\left< \nu / \hat{M}_{com} (E1\mu) / 0 \right> = -\frac{1}{6c} \int dr^2 \left[ \frac{2\sqrt{2}}{5} r^2 j_+^{\nu}(r) - (r^2 - \left< r^2 \right>_0) j_-^{\nu}(r) \right]
\]

The vortical or irrotational character of the flow is provided not by j+ or j- components alone but by their proper superposition.
Current fields

\[ \hat{j}(\vec{r}) = -i \sum_{q=n,p} e_{\text{eff}}^{q} \sum_{k\in q} (\delta(\vec{r} - \vec{r}_{k}) \vec{\nabla}_{k} - \vec{\nabla}_{k} \delta(\vec{r} - \vec{r}_{k})) \]

\[ \delta \hat{j}_{\nu}(\vec{r}) = \left\langle \nu | \hat{j}(\vec{r}) | 0 \right\rangle \]

Transition density of the convection current for the RPA state \( \nu \)

Tests for GDR and CR:

T=0: \( e_{\text{eff}}^{n} = e_{\text{eff}}^{p} = 1 \)

T=1: \( e_{\text{eff}}^{p} = \frac{N}{A}, e_{\text{eff}}^{p} = -\frac{Z}{A} \)

p: \( e_{\text{eff}}^{p} = 1, \ e_{\text{eff}}^{n} = 0 \)

n: \( e_{\text{eff}}^{p} = 0, \ e_{\text{eff}}^{n} = 1 \)

The current fields are OK.
Finally:

- RW conception of the vorticity is not relevant:
  - CE-unrestricted in integral sense,
  - failure for CM,
  - $j^+$ has no advantages over $j^-$.  

- TR conception is more correct:
  - vortical by construction,
  - locally CE-unrestricted,
  - close to HD conception,
  - gives visually vortical image,
  - correct for both TR and CR.

So just the toroidal strength/current is the best measure of the nuclear vorticity.
Speculations with toroidal stuff:
- Robert Scherrer and Chiu Man Ho (2013): attempt to explain dark matter by existence of Majorana fermions with the anapole moment

\[
\tilde{T} = \frac{1}{10c} \int d\vec{r} \left[ (\vec{j} \cdot \vec{r})\hat{r} - 2r^2 \hat{j} \right]
\]
\[
T = \frac{\pi}{2c} jR_0^2 \sqrt{\frac{\delta}{b_n}}
\]

- No electric and magnetic moments but the toroidal (anapole) moment

Recent publications on TR/CR:

J. Kvasil, V.O. Nesterenko, W. Kleinig, P.-G. Reinhard, and P. Vesely,
"General treatment of vortical, toroidal, and compression modes",

A. Repko, P.-G. Reinhard, V.O. Nesterenko, and J. Kvasil,
"Toroidal nature of the low-energy E1 mode",

J. Kvasil, V.O. Nesterenko, W. Kleinig, D. Bozik, P.-G. Reinhard, and N. Lo Iudice,
"Toroidal, compression, and vortical dipole strengths in \{144-154\}Sm: Skyrme-RPA exploration of deformation effect",

J. Kvasil, V.O. Nesterenko, A. Repko, W. Kleinig, P.-G. Reinhard, and N. Lo Iudice,
"Toroidal, compression, and vortical dipole strengths in 124Sn",

P.-G. Reinhard, V.O. Nesterenko, A. Repko, and J. Kvasil,
"Nuclear vorticity in isoscalar E1 modes: Skyrme-RPA analysis",

J. Kvasil, V.O. Nesterenko, W. Kleinig, and P.-G. Reinhard,
"Deformation effects in toroidal and compression dipole excitations of 170Yb: Skyrme-RPA analysis",

V.O. Nesterenko, A. Repko, P.-G. Reinhard, and J. Kvasil,
"Relation of E1 pygmy and toroidal resonances",
arXiv:1410.5634[nucl-th],
Toroidal and compression operators

\[ \hat{M}_{\text{tor}} (E1\mu) = \frac{1}{10\sqrt{2}c} \int d\vec{r} \left[ r^3 + \frac{5}{3} r < r^2 >_0 \right] \hat{Y}_{11\mu} (\vec{r}) \cdot [\vec{\nabla} \times \hat{j}_{\text{nuc}} (\vec{r})] \]

- second-order part of the electric operator

\[ \hat{M} (E\kappa\lambda\mu) = \frac{(2\lambda + 1)!!}{ck^{\lambda+1}} \sqrt{\frac{\lambda}{\lambda + 1}} \int d\vec{r} \ j_\lambda (kr) \hat{Y}_{\lambda\lambda\mu} \cdot [\vec{\nabla} \times \hat{j}_{\text{nuc}} (\vec{r})] \]

\[ j_\lambda (kr) = \frac{(kr)^2}{(2\lambda + 1)!!} \left[ 1 - \frac{(kr)^2}{2(2\lambda + 3)} + \ldots \right] \]

\[ \hat{M} (E\kappa\lambda\mu) = \hat{M} (E\lambda\mu) + k\hat{M}_{\text{tor}} (E\lambda\mu) \]

\[ \hat{M} (E\kappa\lambda\mu) = \int d\vec{r} \ \rho (\vec{r}) r^\lambda Y_{\lambda\mu} \]

\[ \hat{M}_{\text{com}} (E1\mu) = -\frac{i}{10c} \int d\vec{r} \left[ r^3 - \frac{5}{3} r < r^2 >_0 \right] Y_{1\mu} \cdot [\vec{\nabla} \cdot \hat{j}_{\text{nuc}} (\vec{r})] \]

- probe operator of the compression mode

- c.m. corrections, \( r^3 \) -dependence

- relation of TR and CR

- main IS-E1 vortical and irrotational flow

\[ \hat{M}'_{\text{com}} (E1\mu) = \int d\vec{r} \ \hat{\rho} (\vec{r}) \left[ r^3 - \frac{5}{3} r < r^2 >_0 \right] Y_{1\mu} \]

\[ \hat{M}_{\text{com}} (E1\mu) = -k\hat{M}'_{\text{com}} (E1\mu) \]

\[ \hat{\rho} + \vec{\nabla} \cdot \hat{j}_{\text{nuc}} = 0 \]
\[
\omega \rho_{\lambda}(r) = \sqrt{\frac{\lambda}{2\lambda + 1}} \left( \frac{d}{dr} - \frac{\lambda - 1}{\lambda} \right) j_{\lambda \lambda - 1}(r) - \sqrt{\frac{\lambda + 1}{2\lambda + 1}} \left( \frac{d}{dr} + \frac{\lambda + 2}{\lambda} \right) j_{\lambda \lambda + 1}(r)
\]

- to integrate left and right parts of CE with the weight \( r^{\lambda + 2} \)

\[
\omega \gamma_{\lambda} = \omega \int_{0}^{\infty} dr \, r^{\lambda + 2} \rho_{\lambda}(r) = \sqrt{\lambda(2\lambda + 1)} \int_{0}^{\infty} dr \, r^{\lambda + 1} j_{\lambda \lambda - 1}(r)
\]

\[
\int_{0}^{\infty} dr \, \frac{d}{dr} \left( r^{\lambda - 1} j_{\lambda \lambda + 1}(r) \right) = \lim_{r \to \infty} r^{\lambda - 1} j_{\lambda \lambda + 1}(r) \to 0
\]

So just \( j^{(fi)}_{\lambda \lambda \lambda + 1}(r) \)

- is decoupled to CE in the integral sense
- has to be chosen as measure of vorticity
- convenient because it is obtained in the familiar basis of vector harmonics

To be shown that RW-conception:
- incorrect locally
- fails for CM.
3) Toroidal current

\[ \delta \vec{j}(\vec{r}) = \delta \vec{j}_L(\vec{r}) + \delta \vec{j}_T(\vec{r}) + \delta \vec{j}_E(\vec{r}) \]

\[ \vec{j}(\vec{r}) = \vec{\nabla} \phi(\vec{r}) + \vec{\nabla} \times (\vec{r} \eta(\vec{r})) + \vec{\nabla} \times \vec{\nabla} \times (\vec{r} \chi(\vec{r})) \]

\[
\delta \vec{j}(\vec{r}) = \frac{1}{(2\pi)^3} \sum_{\lambda \mu k} F_{\lambda k} \left\{ \sqrt{\frac{\lambda}{\lambda + 1}} \vec{J}^{(-)}_{\lambda \mu k}(\vec{r}) \dot{Q}_{\lambda \mu}(k^2) + k \vec{J}^{(0)}_{\lambda \mu k}(\vec{r}) M_{\lambda \mu}(k^2) \right. \\
+ \vec{J}^{(+)}_{\lambda \mu k}(\vec{r})[\dot{Q}_{\lambda \mu}(k = 0) + k^2 T_{\lambda \mu}(k^2)] \}
\]

\[ \vec{J}^{(-)}_{\lambda \mu k}(\vec{r}) = \frac{-i}{k} \vec{\nabla} J_{\lambda \mu k}(\vec{r}) \quad E\lambda\mu\text{-longitudinal} \]

\[ \vec{J}^{(0)}_{\lambda \mu k}(\vec{r}) = \frac{-i}{k} \vec{\nabla} \times [\vec{r} J_{\lambda \mu k}(\vec{r})] \quad M\lambda\mu\text{-transversal} \]

\[ \vec{J}^{(+)}_{\lambda \mu k}(\vec{r}) = \frac{i}{k} \vec{\nabla} \times \vec{\nabla} \times [\vec{r} J_{\lambda \mu k}(\vec{r})] \quad E\lambda\mu\text{-transversal} \]

Formfactors \( Q_{\lambda \mu}(k^2) \), \( M_{\lambda \mu}(k^2) \), \( T_{\lambda \mu}(k^2) \) form the complete set to determine the full current. \( T_{\lambda \mu}(k^2) \) delivers independent, vortical, CE-unrestricted current and so the toroidal current can serve as a measure of the vorticity.
Divergence-curl analysis: \( \nabla \times \vec{J}(\vec{r}), \quad \nabla \cdot \vec{J}(\vec{r}) \)

GDR, center of mass motion:
- are characterized by the operator \( rY_{1\mu} \) with \( \vec{V}(\vec{r}) \propto \nabla \cdot (rY_{1\mu}(\vec{r})) \)
- are Tassie modes \( (\nabla \times \vec{V}(\vec{r}) = \nabla \cdot \vec{V}(\vec{r}) = 0) \)
- do not contribute to \( \nabla \times \vec{V}(\vec{r}), \quad \nabla \cdot \vec{V}(\vec{r}) \)

TR, CR:
- are characterized by the operator \( r^3Y_{1\mu} \)
- are not Tassie modes
- do contribute to \( \nabla \times \vec{V}(\vec{r}), \quad \nabla \cdot \vec{V}(\vec{r}) \)

So the div-curl analysis is just suitable for TR-CR exploration

\[
\nabla \times \delta \vec{J}_\nu(\vec{r}) = i[rot \vec{J}]_\nu(r)\vec{Y}_{11}^*, \quad \nabla \cdot \vec{J}(\vec{r}) = i[div \vec{J}]_\nu(r)Y_1^*
\]

to be plotted
Average $r^2$-weighted transition densities (TD) for two parts of PDR region: 6-8.8 MeV and 8.8-10.5 MeV

Bin 6-8.8 MeV:
- typical TD structure used to justify the PDR picture:
  neutron excess (7-10 fm) oscillates against the nuclear core (4-7 fm)

The flow in nuclear interior ($r< 4$ fm) is damped though it may be important for disclosing the true PDR origin.

TD loses angular dependence of the flow.

More detailed characteristics (velocity fields) are necessary.

Bins 6-8.8 MeV and 8.8-10.5 MeV:
- different scales of IS DT $\rightarrow$ the bin 6-8.8 MeV id more IS than 8.8-10.5 MeV

Bin 8.8-10.5 MeV: mixed IS/IV structure
Flow patterns: 8.8-10.5 MeV

- mainly $T=1$ in interior and $T=0$ at the surface,

- TR: $(n, T=0)$
  CR: $(n)$
  linear dipole: $(p, T=1)$

- complex structure with mixed is/iv, TR/CR/dipole

- More significant $T=1$ contribution than at 6-8.8 MeV

in accordance to:
- experiment for 124Sn, $(\alpha, \alpha', \gamma')$
  (Enders et al, PRL, 2010)
Comparison of \( (D_1) \) and \( (D_0) \) patterns: 6.0-.8.8 MeV

\[
\hat{D}_1 = \frac{N}{A} \sum_i^Z (rY_i)_i - \frac{Z}{A} \sum_i^n (rY_1)_i
\]

\[
\hat{D}_0 = \sum_i^A (r^3 Y_1)_i
\]

Up to the general sign, the \( D_0 \) and \( D_1 \) flows are about the same. Moreover, they are very similar to flows with normalized weights:

\[
D_1 / |D_1| \quad D_0 / |D_0|
\]
The model

Strength function

\[ S(E1; \omega) = \sum_{\nu \neq 0} \omega^L \left| <\Psi_\nu | \hat{M}_\alpha | 0 > \right|^2 \zeta(\omega - \omega_\nu) \]

\[ \zeta(\omega - \omega_\nu) = \frac{1}{2\pi} \frac{\Delta(\omega_\nu)}{[(\omega - \omega_\nu)^2 + \frac{[\Delta(\omega_\nu)]^2}{4}]^2} \]

\[ \Delta(\omega_\nu) = \max\{0.4, (\omega_\nu - 8 \text{ MeV}) / 3\} \]

Ornary dipole, toroidal, compression operators

\[ \alpha = \{E1, \text{com, tor}\} \]

Lorentz weight with

\[ L = \begin{cases} 1 \text{ for E1} \\ 0 \text{ for com, tor} \end{cases} \]

Toroidal and compression operators

\[ \hat{M}_{\text{tor}}(E1\mu) = \frac{1}{10\sqrt{2}c} \int d\vec{r} \left[ (r^3 - \frac{5}{3} r < r^2 >_0) \right] \tilde{Y}_{11\mu}(\vec{r}) \cdot [\vec{\nabla} \times j_{\text{nuc}}(\vec{r})] \]

\[ \hat{M}_{\text{com}}(E1\mu) = -\frac{i}{10c} \int d\vec{r} \left[ (r^3 - \frac{5}{3} r < r^2 >_0) Y_{1\mu} \right] [\vec{\nabla} \cdot j_{\text{nuc}}(\vec{r})] \]

\[ \hat{M}_{\text{com}}'(E1\mu) = \int d\vec{r} \rho(\vec{r}) \left[ r^3 - \frac{5}{3} r < r^2 >_0 \right] Y_{1\mu} \]

J. Kvasil, VON, W. Kleinig, P.-G. Reinhard, P. Vesely, PRC, 84, 034303 (2011)
Summed RPA transition densities and currents

\[ \delta \rho_{\nu}(\vec{r}) = \langle \nu | \hat{\rho}(\vec{r}) | 0 \rangle \]

- are determined up to the general sign of RPA state \( \nu \),
- being summed by \( \nu \) may give ambiguous results

\[ \delta \vec{j}_{\nu}(\vec{r}) = \langle \nu | \vec{j}(\vec{r}) | 0 \rangle \]

The problem may be cured by weighting TD and CTD by matrix elements

\[ D_{Tv} = \langle \nu | \hat{D}(E1) | 0 \rangle \]

of a probe operator \( \hat{D}_{Tv} \)

\[ \delta \rho_{\beta}^{(D)}(\vec{r}) = \langle \nu | \hat{\rho}(\vec{r}) | 0 \rangle = \sum_{\nu \in [\omega_1, \omega_2]} \sum_{q=n,p} D_{Tv}^* \sum e^q_{\beta} \delta \rho^q_{\nu}(\vec{r}) \]

- bilinear combinations of \( \nu \)
- for the energy interval \([\omega_1, \omega_2]\)

\[ \delta \vec{j}_{\beta}^{(D)}(\vec{r}) = \langle \nu | \vec{j}(\vec{r}) | 0 \rangle = \sum_{\nu \in [\omega_1, \omega_2]} \sum D_{Tv}^* \sum e^q_{\beta} \delta \vec{j}^q_{\nu}(\vec{r}) \]

\[ \hat{D}_1 = \frac{N}{A} \sum_i^Z (rY_1)_i - \frac{Z}{A} \sum_i^N (rY_1)_i \]

- relevant for photoabsorption and (e,e')

\[ \hat{D}_0 = \sum_i^A (r^3Y_1)_i \]

- relevant for isoscalar \((\alpha, \alpha')\)

\[ e^q_{\beta} \]

- effective charge

- The contributions of RPA states with a large D strength is enhanced
- There may be normalized weight
Nuclear current

\[ \hat{j}_{\text{nuc}}(\vec{r}) = \hat{j}_{\text{con}}(\vec{r}) + \hat{j}_{\text{mag}}(\vec{r}) = \frac{e\hbar}{m} \sum_{q=n,p} (\hat{j}^q_{\text{con}}(\vec{r}) + \hat{j}^q_{\text{mag}}(\vec{r})) \]

\[ \hat{j}^q_{\text{con}}(\vec{r}) = -ie^q_{\text{eff}} \sum_{k \in q} \left( \delta(\vec{r} - \vec{r}_k) \vec{\nabla}_k - \vec{\nabla}_k \delta(\vec{r} - \vec{r}_k) \right) \]

\[ \hat{j}^q_{\text{mag}}(\vec{r}) = \frac{g_s}{2} \sum_{k \in q} \vec{\nabla}_k \times \vec{s}_{qk} \delta(\vec{r} - \vec{r}_k) \]

\[ T=0: \quad e^n_{\text{eff}} = e^p_{\text{eff}} = 1 \]

\[ T=1: \quad e^p_{\text{eff}} = \frac{N}{A}, \quad e^n_{\text{eff}} = -\frac{Z}{A} \]

p: \quad e^p_{\text{eff}} = 1, \quad e^n_{\text{eff}} = 0

n: \quad e^p_{\text{eff}} = 0, \quad e^n_{\text{eff}} = 1

used in the present calculations
Center of mass corrections

\[ \hat{O} = \sum_{k=1}^{A} o(\vec{r}_k) \rightarrow \hat{O} = \frac{1}{A} \sum_{k=1}^{A} z_k \]

\[ \delta \langle \hat{O} \rangle = \int d\vec{r} \delta \rho(\vec{r}) o(\vec{r}) = \int d\vec{r} \delta \vec{j}(\vec{r}) \cdot \vec{\nabla} o(\vec{r}) = 0 \]

\[ \sum_{\nu} \left\langle 0 \left| \hat{j}(\vec{r}) \right| \nu \right\rangle \left\langle 0 \left| \hat{F} \right| \nu \right\rangle = \frac{1}{2mi} \rho_0(\vec{r}) \vec{\nabla} f(\vec{r}) \]

\[ \sum_{\nu} \omega_{\nu} \left\langle 0 \left| \hat{\rho}(\vec{r}) \right| \nu \right\rangle \left\langle 0 \left| \hat{F} \right| \nu \right\rangle = -\frac{1}{2m} \vec{\nabla} \cdot \left[ \rho_0(\vec{r}) \vec{\nabla} f(\vec{r}) \right] \]

\[ \delta j_{\nu}(\vec{r}) = \left\langle 0 \left| \hat{j}(\vec{r}) \right| \nu \right\rangle \propto \rho_0(\vec{r}) \vec{\nabla} f(\vec{r}) \propto \rho_0(\vec{r}) \vec{v}(\vec{r}) \]

\[ \delta \rho_{\nu}(\vec{r}) = \left\langle 0 \left| \hat{\rho}(\vec{r}) \right| \nu \right\rangle \propto \vec{\nabla} \cdot \left[ \rho_0(\vec{r}) \vec{\nabla} f(\vec{r}) \right] \propto \vec{\nabla} \cdot \left[ \rho_0(\vec{r}) \vec{v}(\vec{r}) \right] \]

\[ \vec{V}_{vor} = r^2 \vec{Y}_{12\mu} + \eta \vec{Y}_{10\mu} \]

\[ \vec{V}_{tor} = \frac{\sqrt{2}}{5} r^2 \vec{Y}_{12\mu} + (r^2 - \eta) \vec{Y}_{10\mu} \]

\[ \vec{V}_{com} = \frac{\sqrt{2}}{5} r^2 \vec{Y}_{12\mu} - (r^2 - \eta) \vec{Y}_{10\mu} \]

\[ \eta_{vor} = 0 \]

\[ \eta_{tor} = \eta_{com} = \left\langle r^2 \right\rangle_0 \]

\[ \eta_{com}' = \frac{5}{3} \left\langle r^2 \right\rangle_0 \]
TR: experimental status

Experiment: \((\alpha, \alpha')\)

M.Uchida et al, PLB 557, 12 (2003),
PRC 69, 051301(R) (2004)

Looks reasonable since the theory predicts
only TR to form the low-energy part of ISGDR.

Anyway is it possible to propose a reaction where TR:
- could be observed alone or
- could demonstrate a particular fingerprint?

The reaction should be:
- IS (to suppress the effect of the dominant E1(T=1) modes)
- transversal but not polluted by magnetic form-factors
- sensitive to nuclear interior

\((e,e'), \text{ both IS/IV, strong magnetic forf-mactor}\)
\((\alpha, \alpha'), \text{ peripheral, not sensitive to nuclear interior,}\)
\((p,p'), \text{ both IS/IV}\)

Reactions with polarized beams/targets?

So far \((\alpha, \alpha')\) is the best option where TR can be excited:
- not directly but through the coupling with CR or PDR
- through peripheral part of TR
TR and CR constitute low- and high-energy ISGDR branches

**Experiment:** \((\alpha, \alpha')\)

\[
\begin{align*}
208 \text{Pb} & \quad \text{D.Y. Youngblood et al, 1977} \\
& \quad \text{H.P. Morsch et al, 1980} \\
& \quad \text{G.S. Adams et al, 1986} \\
& \quad \text{B.A. Devis et al, 1997} \\
& \quad \text{H.L. Clark et al, 2001} \\
& \quad \text{D.Y. Youngblood et al, 2004} \\
& \quad \text{M.Uchida et al, PLB 557, 12 (2003), PRC 69, 051301(R) (2004)}
\end{align*}
\]

There are also the ISGDR data in

\[
\begin{align*}
56 \text{Fe}, 58, 60 \text{Ni}, 90 \text{Zr}, 116 \text{Sn}, 144 \text{Sm}, \ldots
\end{align*}
\]

Preliminary results on TR (F. Guerelly) \((16\text{O}, 16\text{O}')\) in \(27\text{Al}\)

**Theory:**


Skyrme RPA, SLy6
Speculations with toroidal stuff:
- Robert Scherrer and Chiu Man Ho (2013): attempt to explain dark matter by existence of Majorana fermions with the anapole moment.

\[ T = \frac{\pi}{2c} jR_0^2 b n \]

- No electric and magnetic moments but the toroidal (anapole) moment

\[ \vec{T} = \frac{1}{10c} \int d\vec{r} \left[ (\vec{j} \cdot \vec{r})\vec{r} - 2r^2 \vec{j} \right] \]
TR: experimental perspectives

Experiment: \( (\alpha,\alpha') \)


Looks reasonable since the theory predicts only TR to form the low-energy part of ISGDR.

The reaction should be:
- IS (to suppress the dominant E1(T=1) modes)
- (partly) transversal but not polluted by magnetic form-factors

\[
\begin{align*}
(e,e') , & \text{ both IS/IV, strong magnetic form-mactor} \\
(p,p') , & \text{ both IS/IV} \\
\end{align*}
\]

Peripheral IS reactions \( (\alpha,\alpha') \) and \( (^{16}\text{O},^{16}\text{O}') \) seem to be the best options:

TR can be excited though its peripheral part (together with IS PDR and CR).
TR: experimental perspectives -2

The response depends on the probe:

\[(\alpha, \alpha') \quad r^3Y_{1\mu} \rightarrow TR\]

Photoabsorp. \[rY_{1\mu} \rightarrow PDR\]

Different conditions of \((\alpha, \alpha')\)-experiments:

It would be interesting to observe:

1) Deformation splitting (sequence of K-branches) in TR/PDR energy region by using \((\alpha, \alpha')\).

2) Comparison of photoabsorption and \((\alpha, \alpha')\) data in nuclei with \(N=Z\) and \(N>Z\). For example:

\[
\begin{array}{ccc}
40\text{Ca} & 48\text{Ca} \\
(\alpha, \alpha') & \text{IS}, \quad r^3Y_{1\mu} & \text{TR} \\
\text{Photoabsorp.} & \text{IS/IV}, \quad rY_{1\mu} & \text{--} \\
\end{array}
\]

Always small angles but different \(E_\alpha\)

- PDR, Endres, \(E_\alpha = 136 \text{ MeV}\)
- TR+CR, Uchida \(E_\alpha = 400 \text{ MeV}\)
- TR+CR, Youngblood \(E_\alpha = 240 \text{ MeV}\)

To compare TR, PDR and GDR \((\alpha, \alpha')\) formfactors.
The TR formfactors should have maxima at higher transfer mom.
(talk of P.G. Reinhard)