Dipole toroidal resonance: vortical properties, anomalous deformation impact, relation to pygmy mode

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COMEX5, Krakow, 14-18.09.2015

To discuss:

- Exotic isoscalar E1 resonances:
  - toroidal (TR),
  - compression (CR)
  - pygmy (PDR)

J. Kvasil, V.O.N., W. Kleinig, P.-G. Reinhard, P.. Vesely,
PRC 84, 034303 (2011)

- TR is the most accurate measure of the nuclear vorticity

P.-G. Reinhard, V.O.N, A. Repko, and J. Kvasil,
PRC 89, 024321 (2014).

- Anomalous deformation splitting of TR as its fingerprint

J. Kvasil, V.O. Nesterenko, W. Kleinig, and P.-G. Reinhard,
Phys. Scr. 89, 054023 (2014).

- PDR is a peripheral manifestation of TR ?

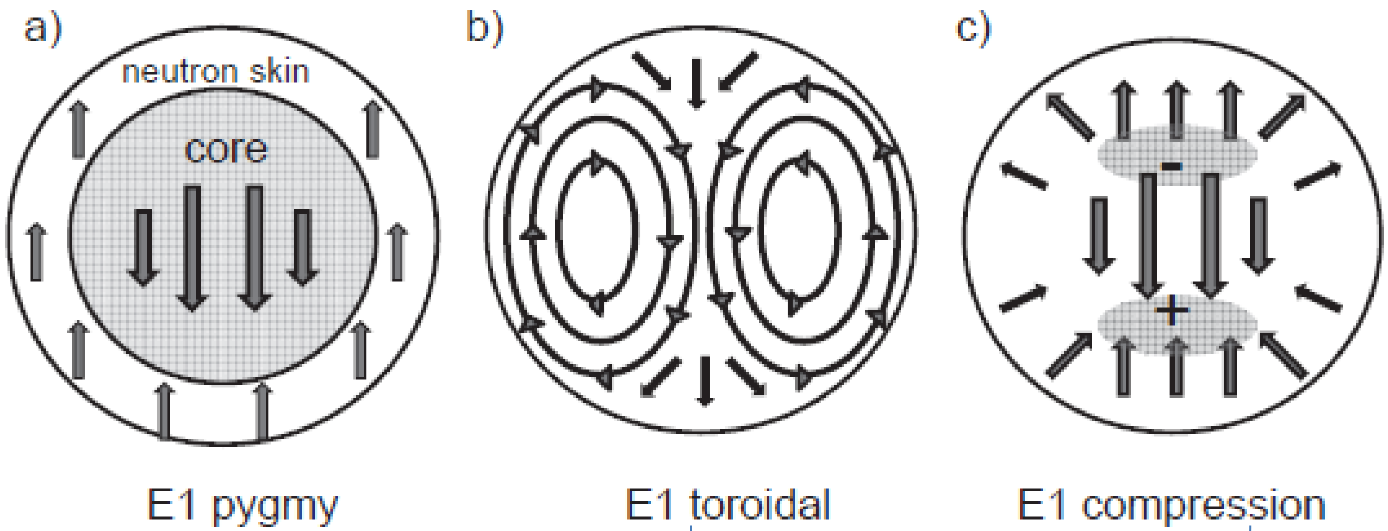
A. Repko, P.-G. Reinhard, V.O.N. and J. Kvasil,
PRC 87, 024305 (2013).

- Experimental perspectives

Exotic dipole resonances

[1] V.M. Dubovik and A.A. Cheshkov, Sov. J. Part. Nucl. v.5, 318 (1975).
[2] S.F. Semenko, Sov. J. Nucl. Phys. v. 34, 356 (1981).

R. Mohan et al (1971), | V.M. Dubovik (1975) S.F. Semenko (1981) | M.N. Harakeh (1977) S. Stringari (1982)



Dominate in E1(T=0) channel
(after exclusion of spurious E1(T=0) c.m. motion)

irrotational | vortical | irrotational

$E = 50 \div 60\ A^{-1/3} MeV$ $\quad$ $E = 50 \div 70\ A^{-1/3} MeV$ $\quad$ $E = 132\ A^{-1/3} MeV$

Reviews:

N. Paar, D. Vretenar, E. Kyan, G. Colo, Rep. Prog. Phys. 70 691 (2007);
D. Savran, T. Aumann, and A. Zilges, Prog. Part. Nucl. Phys. 70, 210 (2013).

TR and CR consitute low- and high-energy ISGDR branches

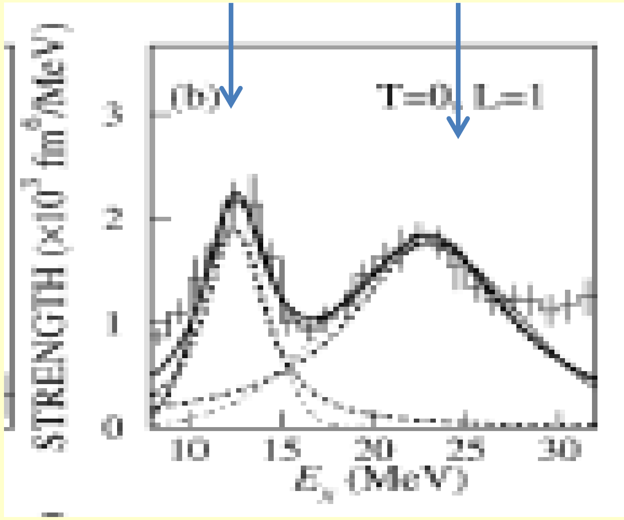
Experiment: (α, α')

LE (toroidal) HE (compression)

^{208}Pb

D.Y. Youngblood et al, 1977
H.P. Morsch et al, 1980
G.S. Adams et al, 1986
B.A. Devis et al, 1997
H.L. Clark et al, 2001
D.Y. Youngblood et al, 2004

M.Uchida et al, PLB 557, 12 (2003),
PRC 69, 051301(R) (2004)



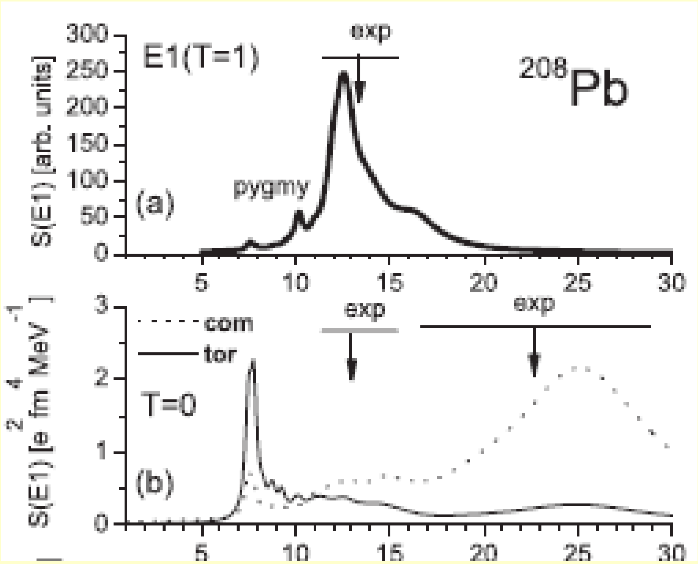
There are also the ISGDR data in

^{56}Fe, ^{58,60}Ni, ^{90}Zr, ^{116}Sn, ^{144}Sm, ...

Theory:

A. Repko, P.-G. Reinhard, V.O.N. and J. Kvasil, PRC 87, 024305 (2013).

Skyrme RPA, SLy6



-discrepancy between theory and experiment for TR
-perhaps, Uchida observed not TR but low-energy CR fraction. Then TR is not still observed?

Toroidal E1 operator:

J. Kvasil, VON, W. Kleinig, P.-G. Reinhard, P. Vesely, PRC, 84, 034303 (2011)

$$\hat{M}_{tor}(E1\mu) = \frac{1}{10\sqrt{2}c}\int d\vec{r}\,[r^3 + \frac{5}{3}r < r^2 >_0]\,\vec{Y}_{11\mu}(\hat{\vec{r}})\cdot[\underbrace{\vec{\nabla}\times\hat{\vec{j}}_{nuc}(\vec{r})}_{\text{vortical flow}}]$$

- second-order part of the electric operator

Compression E1 operator:

$$\hat{M}_{com}(E1\mu) = -\frac{i}{10c}\int d\vec{r}\,[r^3 - \frac{5}{3}r < r^2 >_0]Y_{1\mu}\,[\underbrace{\vec{\nabla}\cdot\hat{\vec{j}}_{nuc}(\vec{r})}_{\text{irrotational flow}}]$$

$$\hat{M}'_{com}(E1\mu) = \int d\vec{r}\,\hat{\rho}(\vec{r})\,[r^3 - \frac{5}{3}r < r^2 >_0]Y_{1\mu} \qquad \hat{M}_{com}(E1\mu) = -k\hat{M}'_{com}(E1\mu)$$

$$\dot{\rho} + \vec{\nabla}\cdot\vec{j}_{nuc} = 0$$

- **TR and CR are ideal examples for the vortical and irrotational motion**

- **to be used below for the tests**

P.-G. Reinhard, V.O. N., A. Repko, and J. Kvasil,
"Nuclear vorticity in isoscalar E1 modes: Skyrme-RPA analysis",
Phys. Rev. C89, 024321 (2014).

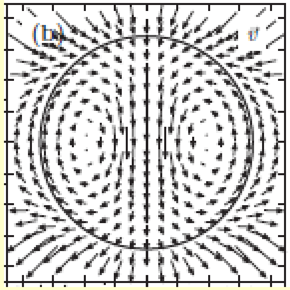
Toroidal motion as the measure of the nuclear vorticity

Two familiar conceptions of nuclear vorticity : HD, RW

1. Hydrodynamical vorticity:

$$\vec{w}(\vec{r}) = \vec{\nabla} \times \vec{v}(\vec{r}) \qquad \delta\vec{v}(\vec{r}) = \frac{\delta\vec{j}_{nuc}(\vec{r})}{\rho_0(\vec{r})}$$

$$(\vec{\nabla} \times \delta\vec{j}_{nuc}) \to \rho_0(\vec{r})(\vec{\nabla} \times \delta\vec{v}) \to \rho_0(\vec{r})\,\vec{w}(\vec{r})$$



2. RW vorticity

D.G.Raventhall, J.Wambach, NPA 475, 468 (1987).

$$\dot{\rho} + \vec{\nabla} \cdot \vec{j}_{nuc} = 0$$ - **continuity equation**

$$\delta\vec{j}_{(fi)}(\vec{r}) = \left\langle j_f m_f \,|\, \hat{\vec{j}}_{nuc}(\vec{r}) \,|\, j_i m_i \right\rangle = \sum_{\lambda\mu} \frac{(j_i m_i \lambda\mu \,|\, j_f m_f)}{\sqrt{2j_f+1}} [j^{(fi)}_{\lambda\lambda-1}(r)\vec{Y}^{*}_{\lambda\lambda-1\mu} + j^{(fi)}_{\lambda\lambda+1}(r)\vec{Y}^{*}_{\lambda\lambda+1\mu}]$$

$$\delta\vec{j}^{\nu}_{1\mu}(\vec{r}) = \left\langle \nu \,|\, \hat{\vec{j}}_{nuc}(\vec{r}) \,|\, 0 \right\rangle = -\frac{i}{\sqrt{3}} [\underbrace{j^{\nu}_{10}(r)}_{j_-}\vec{Y}^{*}_{10\mu} + \underbrace{j^{\nu}_{12}(r)}_{j_+}\vec{Y}^{*}_{12\mu}]$$

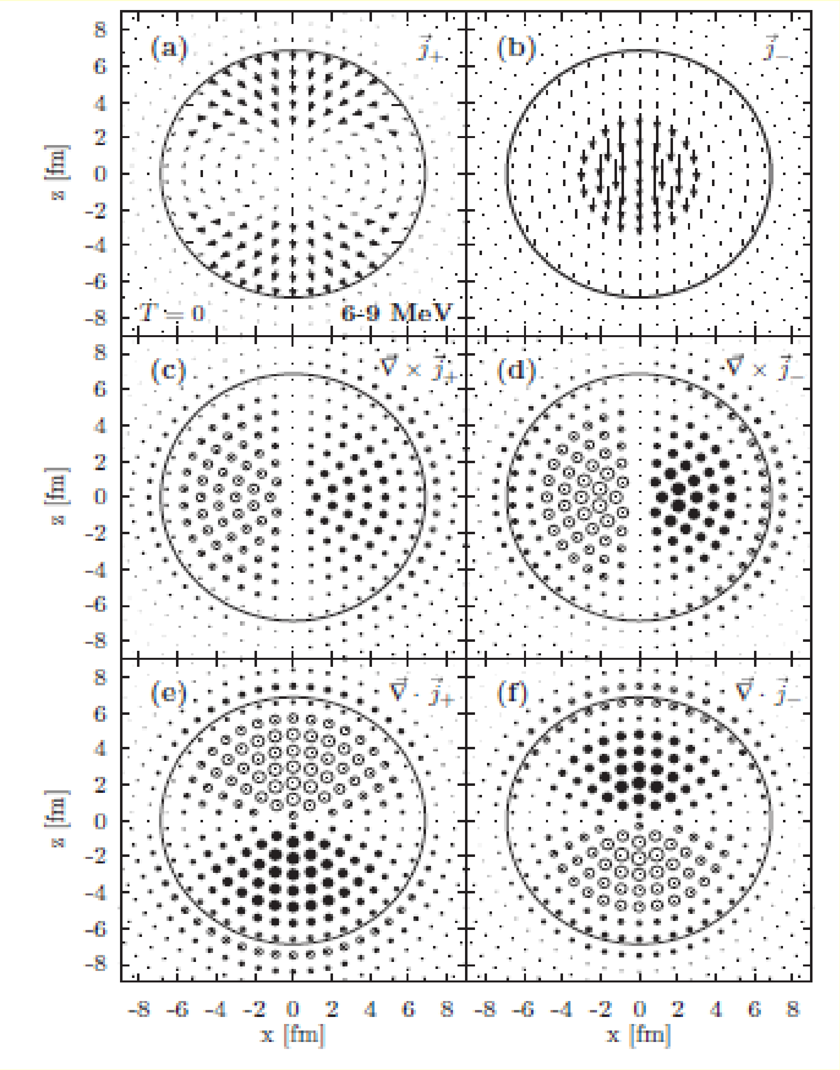
- **current transition density**

$j^{\nu}_{+}(r)$

- independent part of charge-current distribution,
- decoupled from CE in the integral sense
- may be the measure of the vorticity

HD and j+ prescriptions give opposite conclusions on CM vorticity!

j+ j-



208Pb:
all RPA states
at E=6-9 MeV

j+, j-:
- both have strong curl's and div's
- there is no any advantage of j+ over j- to represent the vorticity

The vortical or irrotational character of the flow is provided not by j+ or j- components separately but by their proper superposition.

So just the toroidal current but not j+ is the relevant measure of the nuclear vorticity .

$$\langle \nu / \hat{M}_{tor}(E1\mu)/0\rangle = -\frac{1}{6c}\int dr\, r^2\, [\frac{\sqrt{2}}{5} r^2 j^{\nu}_{+}(r) + (r^2 - \langle r^2\rangle_0) j^{\nu}_{-}(r)]$$

$$\langle \nu / \hat{M}_{com}(E1\mu)/0\rangle = -\frac{1}{6c}\int dr\, r^2\, [\frac{2\sqrt{2}}{5} r^2 j^{\nu}_{+}(r) - (r^2 - \langle r^2\rangle_0) j^{\nu}_{-}(r)]$$

Anomalous deformation effect in the toroidal resonance

To be used as TR fingerprint?

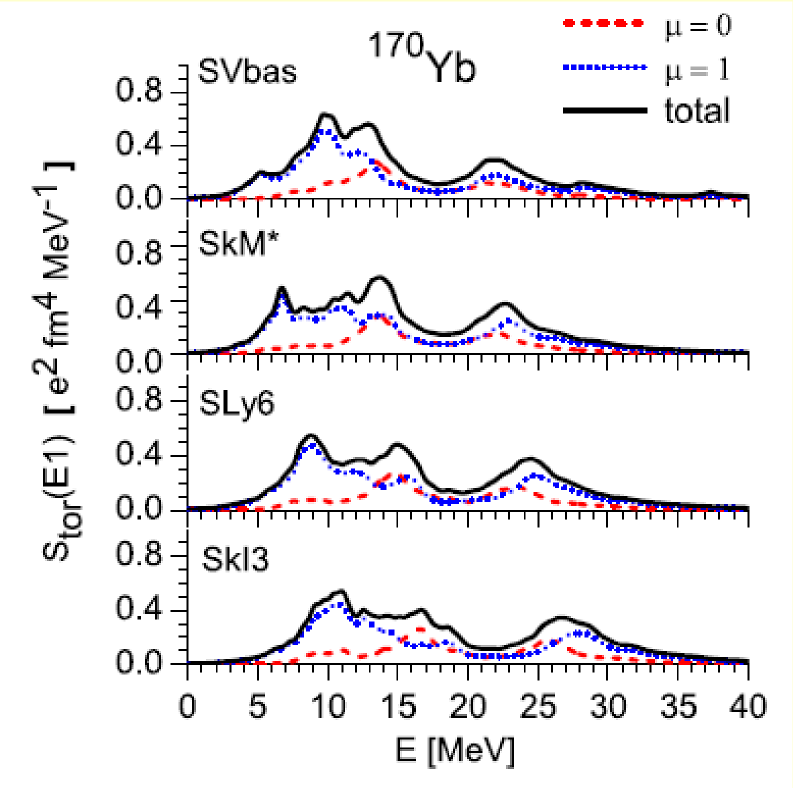
J. Kvasil, V.O. Nesterenko, W. Kleinig, D. Bozik, P.-G. Reinhard, and N. Lo Iudice,
"Toroidal, compression, and vortical dipole strengths in {144-154}Sm: Skyrme-RPA exploration of deformation effect",
Eur. Phys. J. A, v.49, 119 (2013).

J. Kvasil, V.O. Nesterenko, W. Kleinig, and P.-G. Reinhard,
"Deformation effects in toroidal and compression dipole excitations of 170Yb: Skyrme-RPA analysis",
Phys. Scri., v.89, n.5, 054023 (2014).

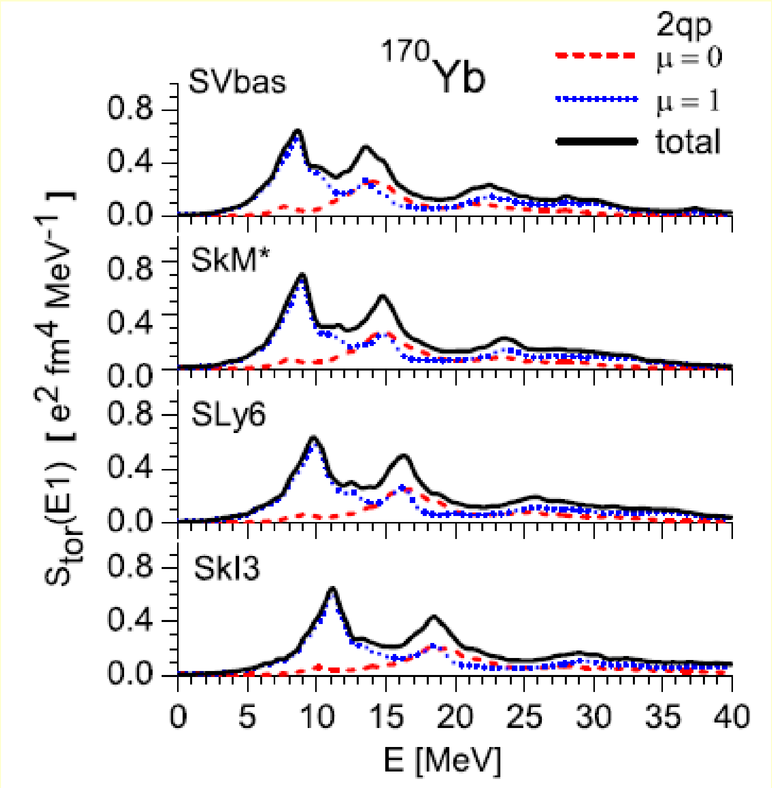
Deformation effects in the toroidal mode

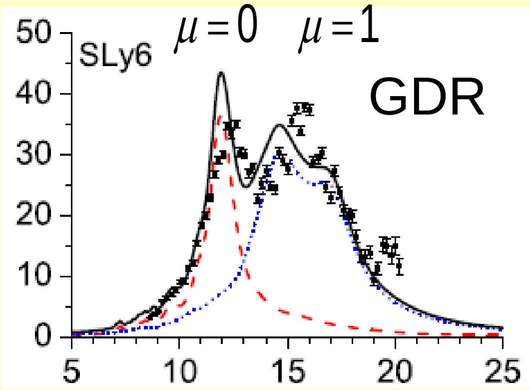
J. Kvasil, VON, W. Kleinig and P.-G. Reinhard,
Phys. Scr. 89, 054023 (2014)

RPA



2qp





GDR: $E(\mu=0) < E(\mu=1)$

TM: $E(\mu=0) > E(\mu=1)$

Unusual sequence of $\mu=0$ and $\mu=1$ branches

Deformation (not resid. Interaction) effect

Non-Tassie mode!

Should affect PDR properties

$$\nabla \times \vec{F} = 0, \nabla \cdot \vec{F} = 0$$

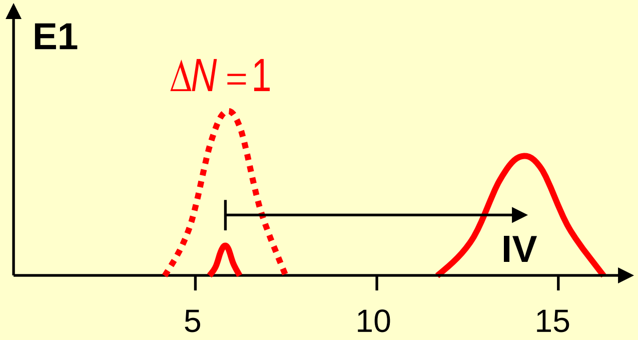
$$\vec{F} = \nabla \Phi, \Phi = r^{\lambda} Y_{\lambda\mu}$$

$$\nabla \times \vec{F} = 0, \nabla \cdot \vec{F} = 0$$

$$\vec{F} = \nabla \Phi, \Phi = r^{\lambda} Y_{\lambda\mu}$$

GDR: $\Phi = r Y_{1\mu}$ - Tassie mode

TR: $\Phi = r^3 Y_{1\mu}$ - non-Tassie mode



IV residual interaction upshifts the Tassie-like dipole strength.
Perhaps the remaining small strength is basically of non-Tassie character (toroidal).

The deformation effect can be used:

- as a direct experimental fingerprint of the toroidal flow,
- can be observed in $(\alpha, \alpha' \gamma)$ reaction where μ -branches can be discriminated.

Relation of E1 toroidal and pygmy resonances

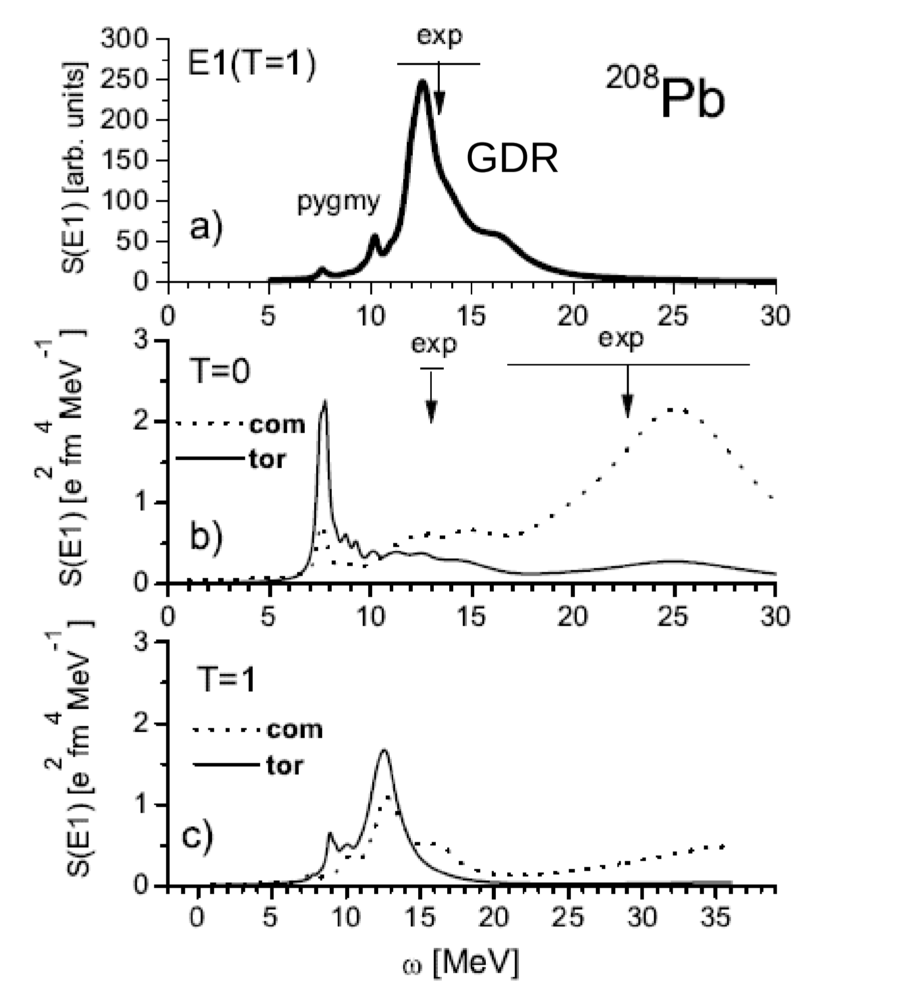
Is PDR a local (peripheral) part of TR?

A. Repko, P.-G. Reinhard, V.O. Nesterenko, and J. Kvasil,
"Toroidal nature of the low-energy E1 mode",
Phys. Rev. C87, 024305 (2013).

V.O. Nesterenko, A. Repko, P.-G. Reinhard, and J. Kvasil,
"Relation of E1 pygmy and toroidal resonances",
EPJ Web of Conferences, 93, 01020 (2015); arXiv:1410.5634[nucl-th],

Strength functions SLy6

A. Repko, P.G. Reinhard, VON, J. Kvasil,
PRC, 87, 024305 (2013)

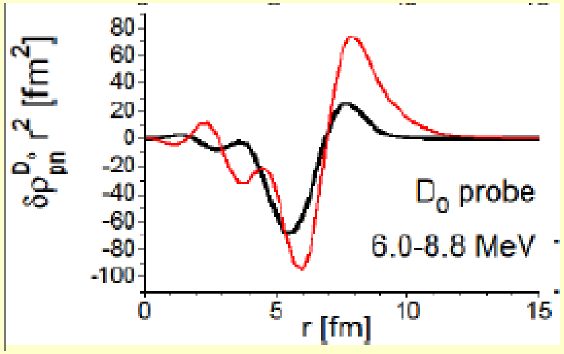


Two peaks at 7.5 and 10.3 MeV in agreement to RMF calculations
(D. Vretenar, N. Paar, P. Ring, PRC, **63**, 047301 (2001))

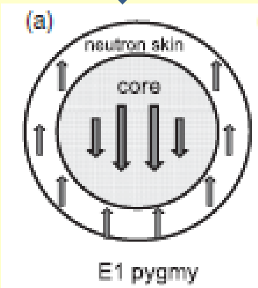
(α,α') experiment
of Uchida et al (2003)

PDR region hosts TR and CR!

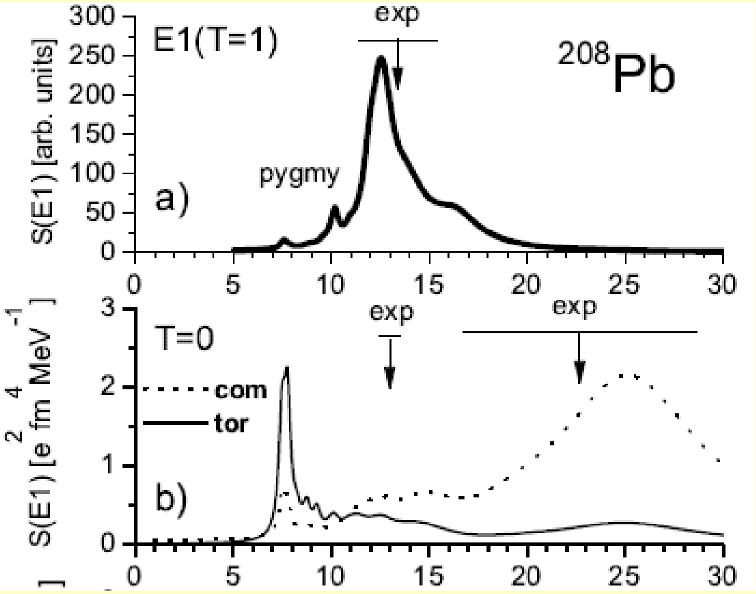
Typical PDR transition density:



- n
- p



Benchmark examples



Current fields:

$$\hat{\vec{j}}(\vec{r}) = -i \sum_{q=n,p} e^q_{eff} \sum_{k \ni q} \left(\delta(\vec{r}-\vec{r}_k)\vec{\nabla}_k - \overleftarrow{\nabla}_k \delta(\vec{r}-\vec{r}_k)\right)$$

$$\delta\vec{j}_\nu(\vec{r}) = \left\langle \nu \,|\, \hat{\vec{j}}(\vec{r}) \,|\, 0 \right\rangle$$

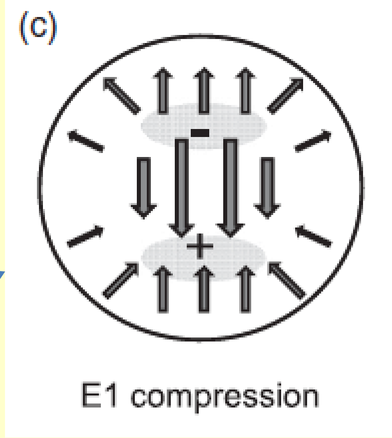
← transition density of the convection current for the RPA state ν

T=0: $e^n_{eff} = e^p_{eff} = 1$

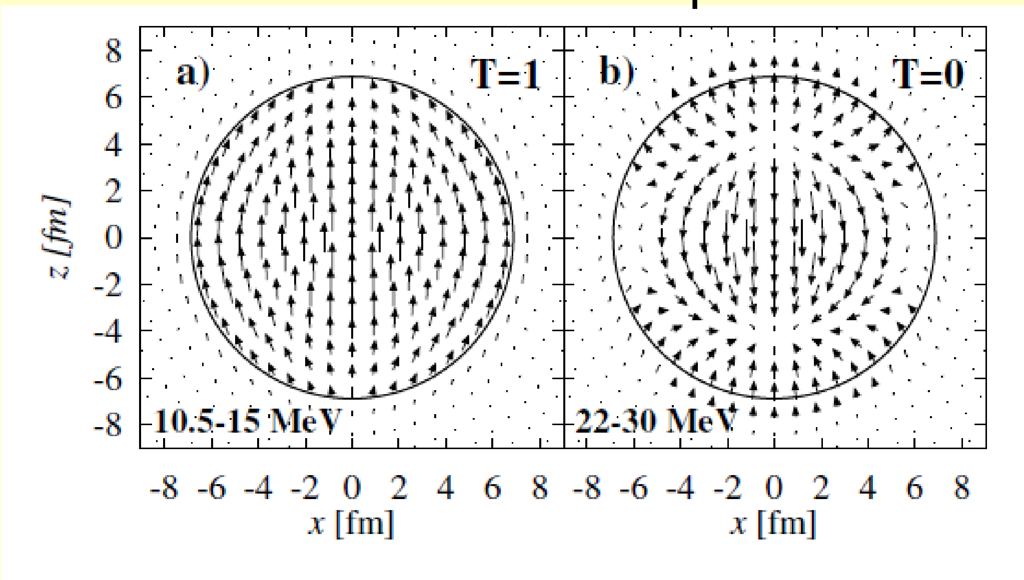
T=1: $e^p_{eff} = \frac{N}{A}, e^p_{eff} = -\frac{Z}{A}$

p: $e^p_{eff} = 1, \quad e^n_{eff} = 0$

n: $e^p_{eff} = 0, \quad e^n_{eff} = 1$



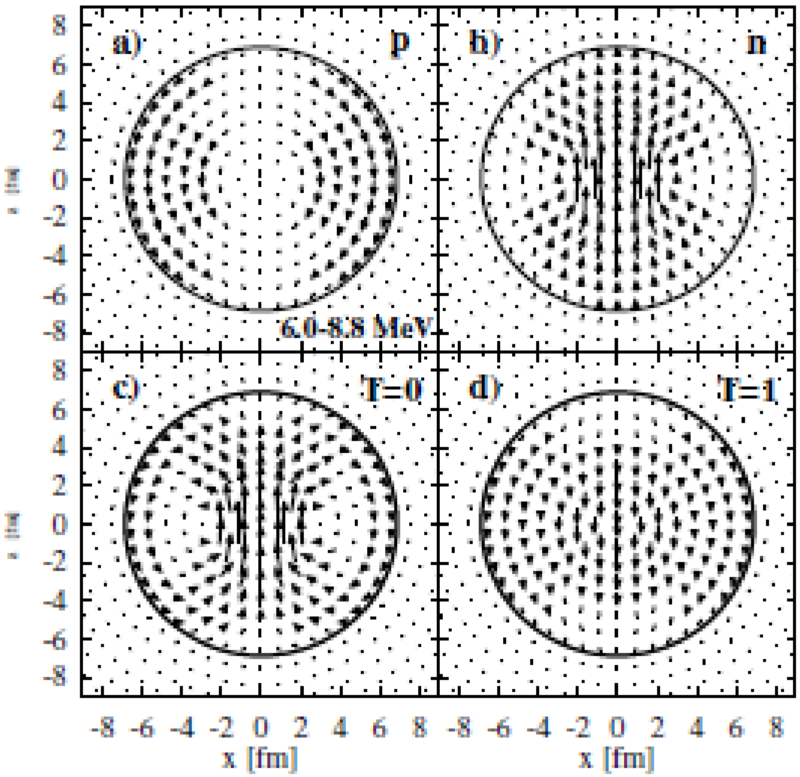
GDR compression



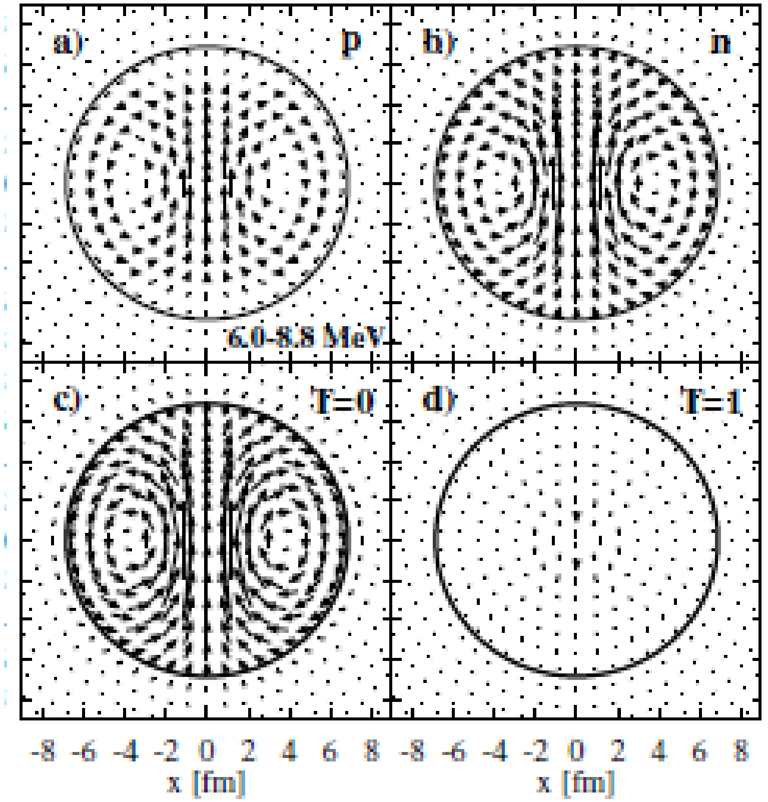
- good reproduction of known fields,
- justifies accuracy of our model

RPA vs 1ph

1ph



RPA

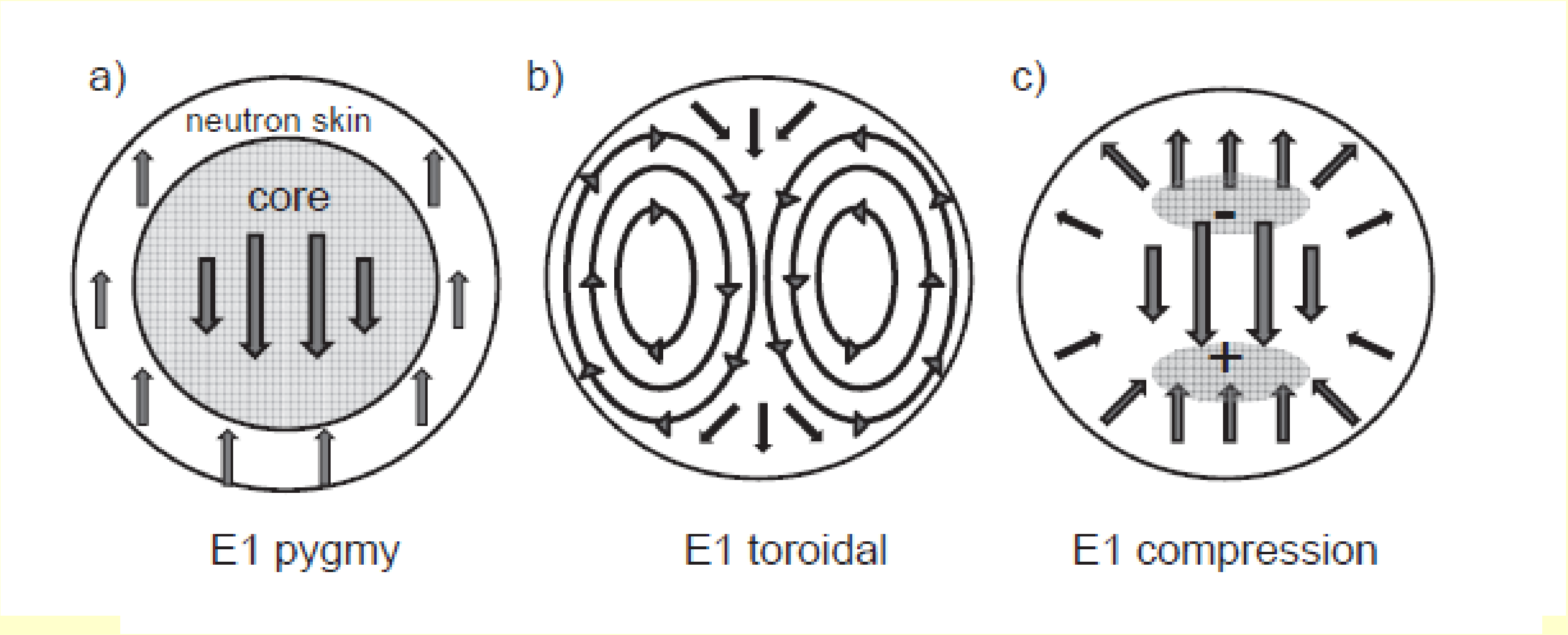


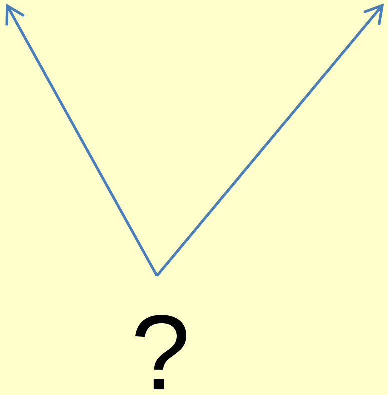
-both isoscalar and isovector
- toroidal flow mainly fom neutrons

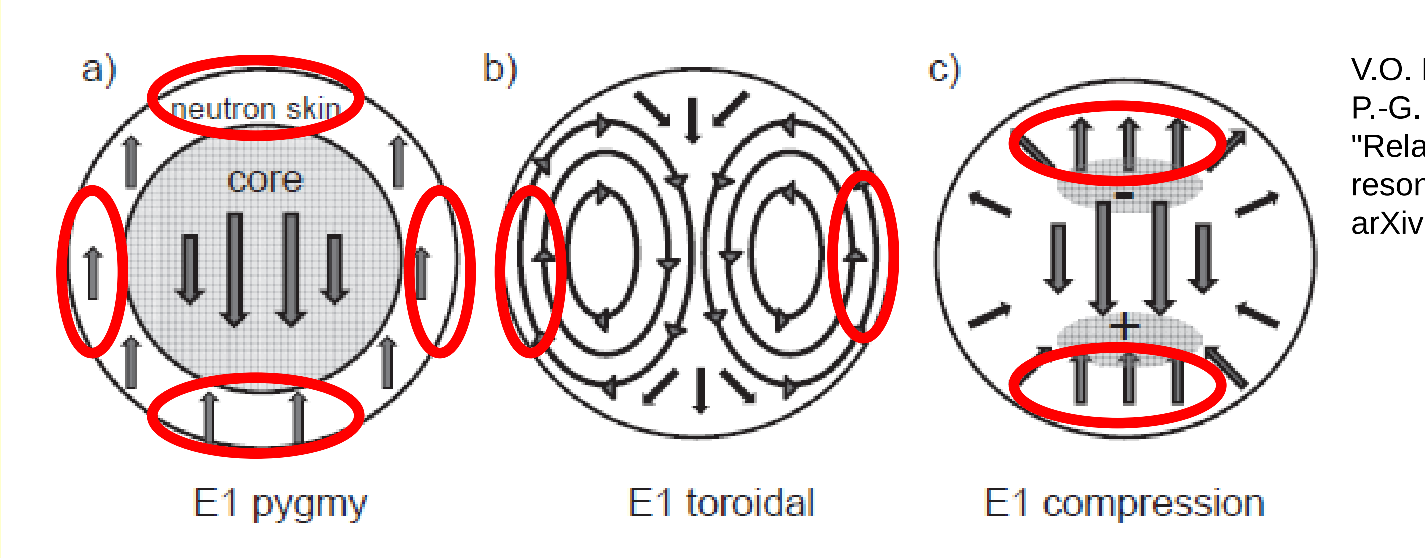
- mainly isoscalar
- toroidal flow from both n/p

So the toroidal flow is basically formed already by the mean-field.
But residual interaction makes it collective and more impressive.

Does the toroidal flow contradicts the familiar PRD picture?





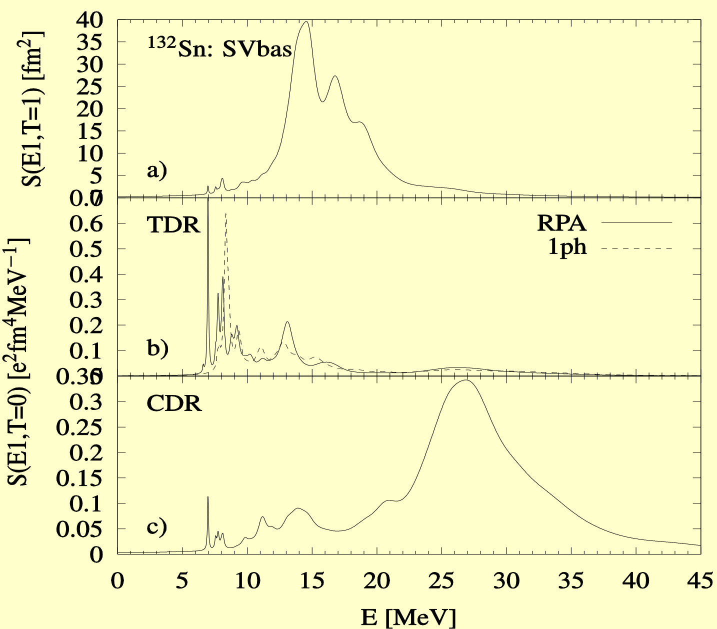


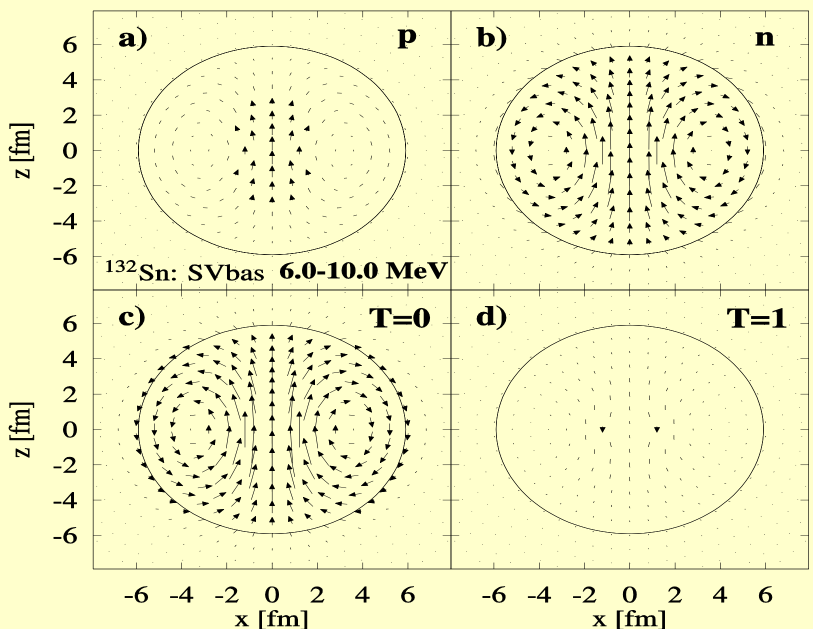
V.O. Nesterenko, A. Repko, P.-G. Reinhard, and J. Kvasil, "Relation of E1 pygmy and toroidal resonances", arXiv:1410.5634[nucl-th],

- PDR can be viewed as a local peripheral part of TR and CR

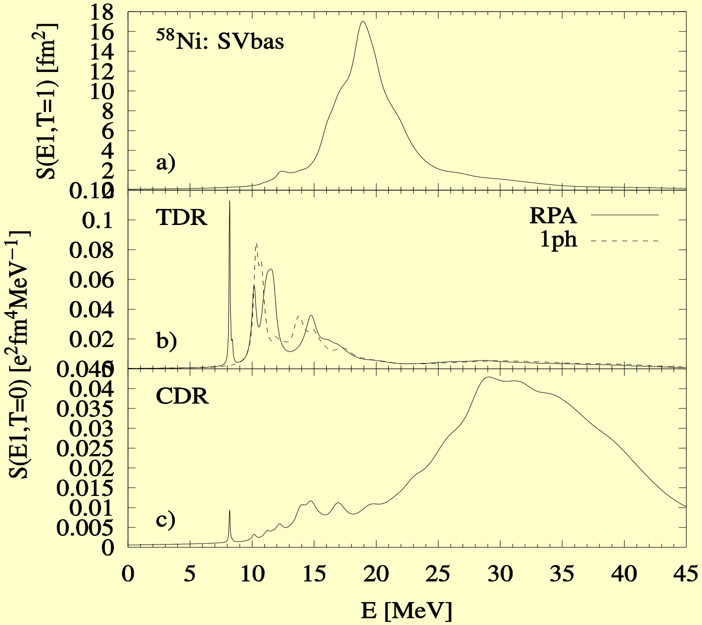
- Our calculations demonstrate the TR motion in PDR energy region for other nuclei: Ni, Zr, Sn, …

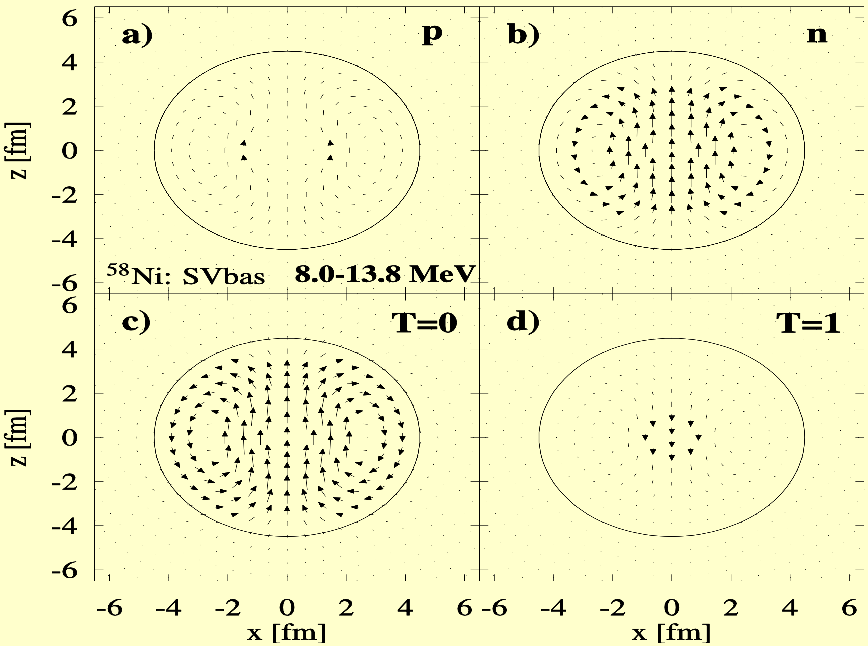
132Sn, SVbas, with PDR



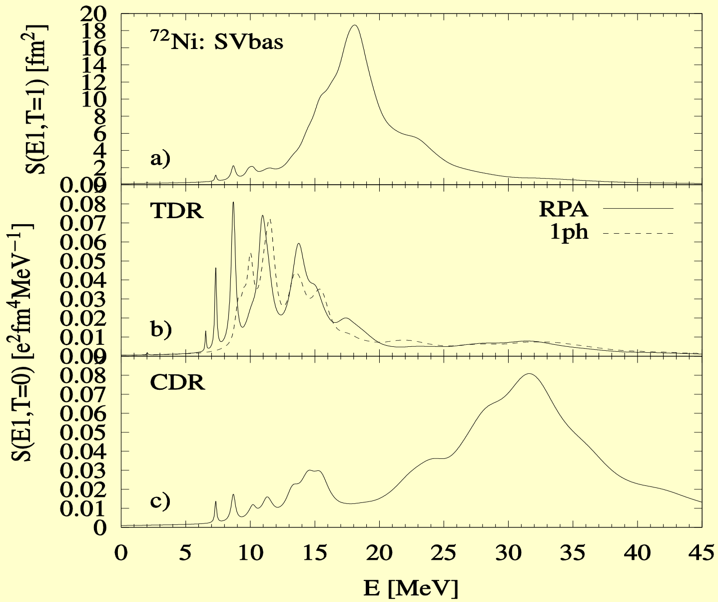


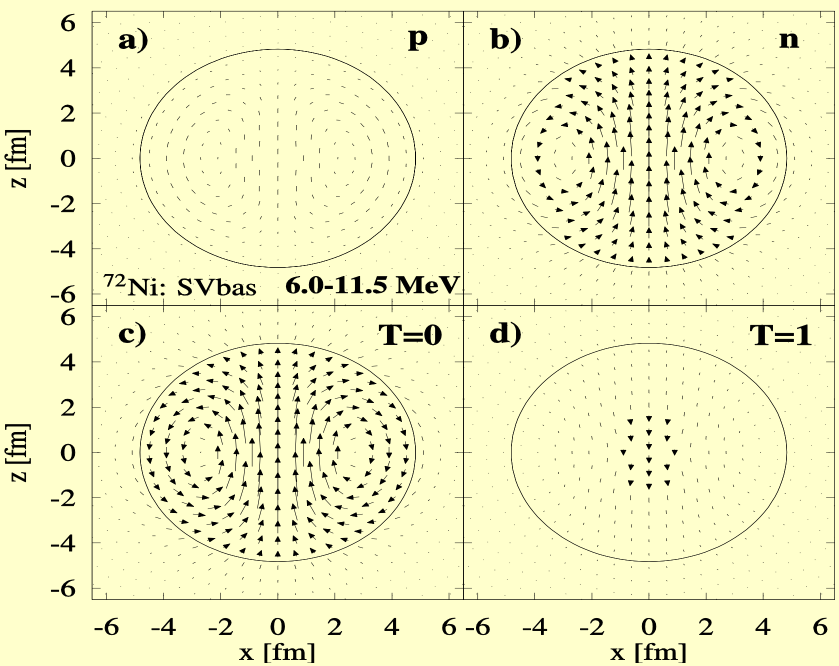
58Ni, SVbas



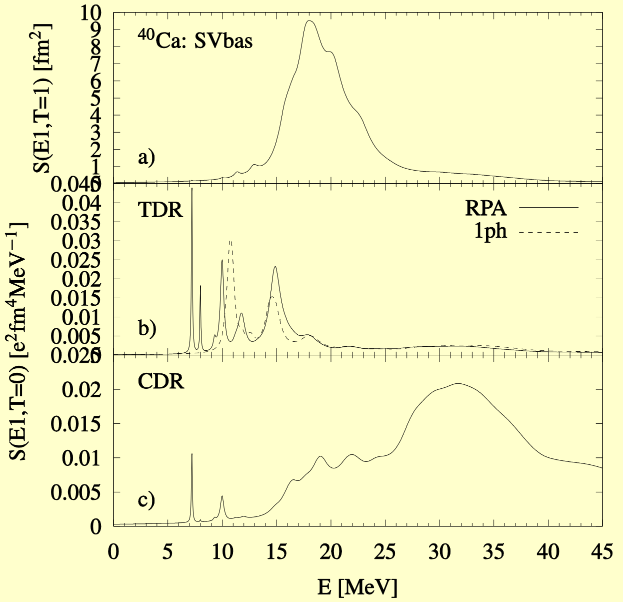


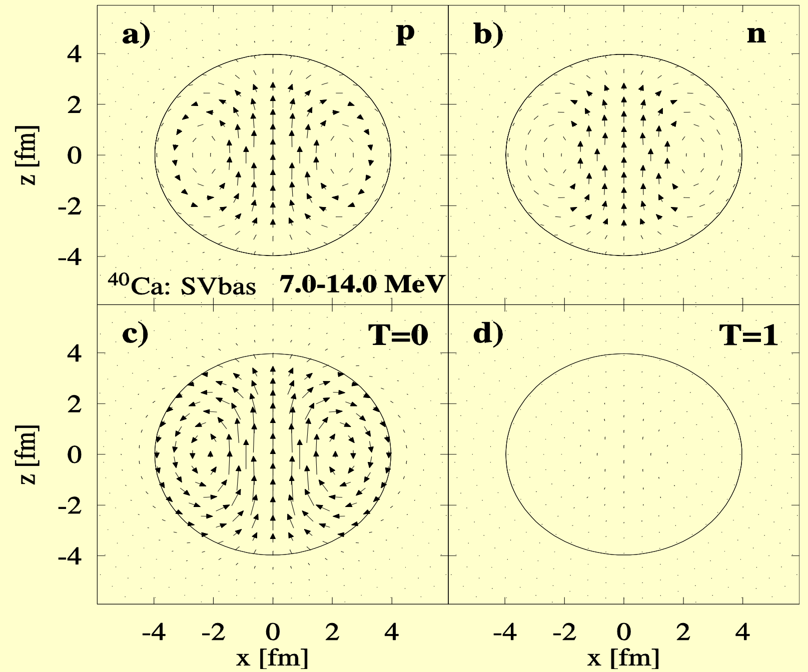
72Ni, SVbas



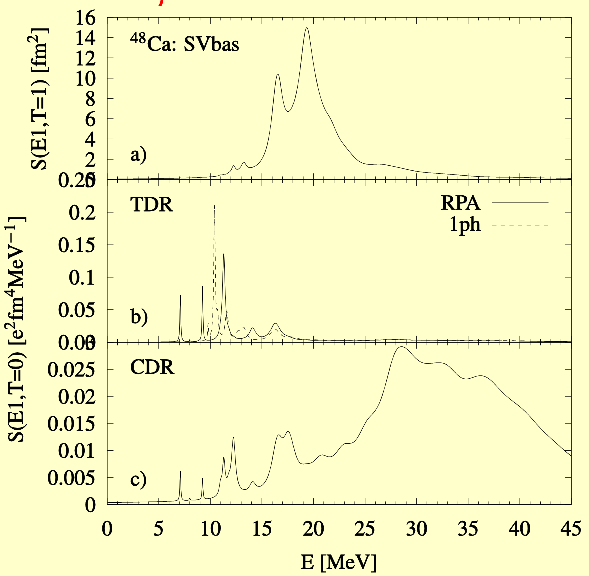


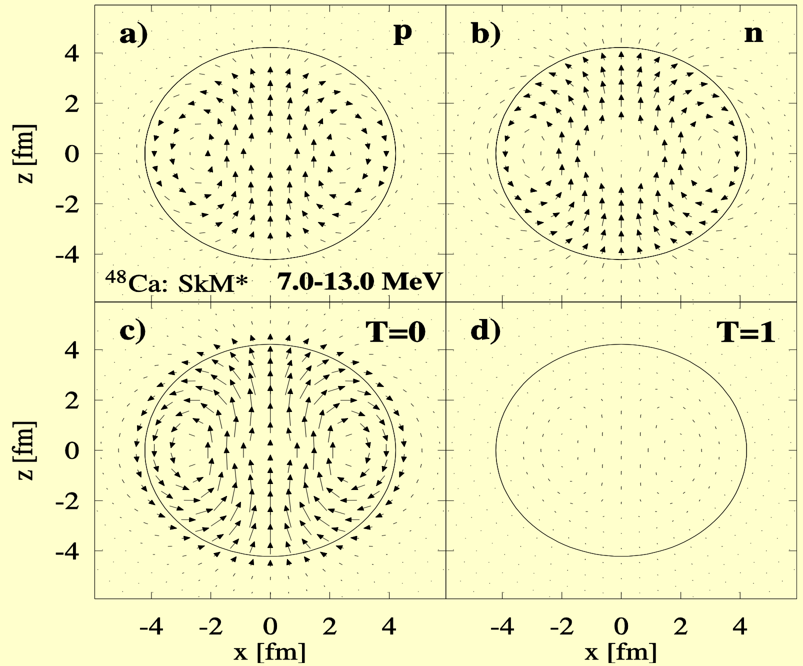
40Ca, SVbas





48Ca, SVbas





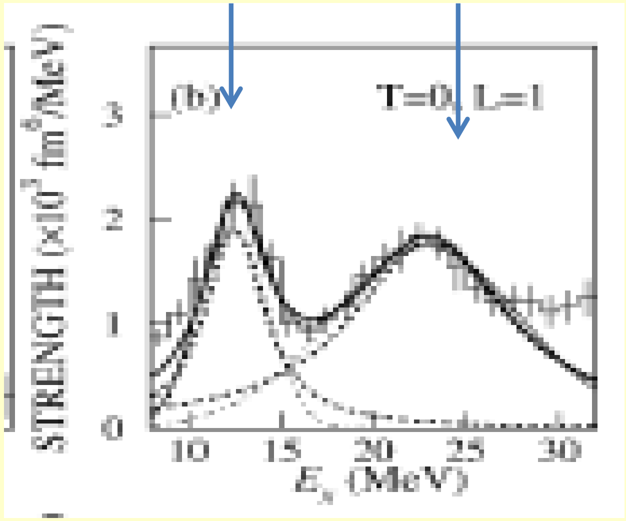
So it is quite possible that PDR
is a peripheral part
of the dipole toroidal flow!

TR: experimental perspectives -1

Experiment: (α, α')

M.Uchida et al, PLB 557, 12 (2003),
PRC 69, 051301(R) (2004)

LE (toroidal) HE (compression)



(e,e'), - both IS/IV, strong magnetic form-factor
(p,p') - both IS/IV
photoabsorption, (γ, γ') - both IS/IV

} not good for IS-TR

Peripheral IS reactions (α, α') and $(^{16,17}O, ^{16,17}O')$ seem to be the best options:
To use $(\alpha, \alpha'\gamma)$ in deformed nuclei.
TR can be excited though its peripheral part (together with IS PDR and CR).

What we actually observe in (α, α')? Isoscalar PDR or TR?
This is yet unclear … .

J. Endres, et al, PRL, 105, 212503 (2010)
124Sn,$(\alpha, \alpha'\gamma)$
A. Bracco: $(^{17}O, ^{17}O'\gamma)$

?

TR: experimental perspectives -2

It would be interesting to observe:

1) Deformation splitting (sequence of K-branches) in TR/PDR energy region by using $(\alpha,\alpha'\gamma)$.. - direct fingerprint of TR!

2) To look for TR in $N \approx Z$ nuclei where the PDR is absent .

There are preliminary data on TR in ^{27}Al $((^{16}O,^{16}O')$, F. Cappuzzello et al)

It would be interesting to inspect the deformed ^{28}Si .

3) Comparison of photoabsorption and (α,α') data in nuclei with N=Z and N>Z. For example:

			^{40}Ca	^{48}Ca
(α,α')	IS,	$r^3Y_{1\mu}$	TR	TR, PDR
Photoabsorp.	IS/IV,	$rY_{1\mu}$	--	PDR

To compare TR, PDR and GDR (α,α') formfactors. The TR formfactors should have maxima at higher transfer mom. (talk of P.G. Reinhard)

Conclusions

- **Toroidal current (strength) is the most relevant fingerprint and measure of the nuclear vorticity.**
  - It is more convenient and relevant than RW and HD prescriptions.
  - TR is the only known example of the vortical collective electric motion.

- **Anomalous deformation effect as TR specific feature.**

- **PDR could be:**
  - local surface part of the toroidal motion.
  - or oscillations of the neutron excess, coupled to TR and CR

Response depends on the probe!

PDR is a complex mixture of:
- IS/IV,
- collective/s-p,
- irrotational/vortical,
- TM / CM / GDR,
- complex configurations

But the vortical TM seems to dominate!

- **IS reactions** (α,α'), $(\alpha,\alpha'\gamma)$, $({}^{16}O,{}^{16}O')$ **are best.**
- **Outlook:**
  - TR in deformed nuclei: $(\alpha,\alpha'\gamma)$ **to observe anomalous deformation effect as the TR fingerprint,**
  - comparative measurements of TR and PDR at about the same conditions

Thank you for attention!

Previous studies

D. VRETENAR, N. PAAR, P. RING, AND T. NIKŠIĆ PHYSICAL REVIEW C **65** 021301(R)

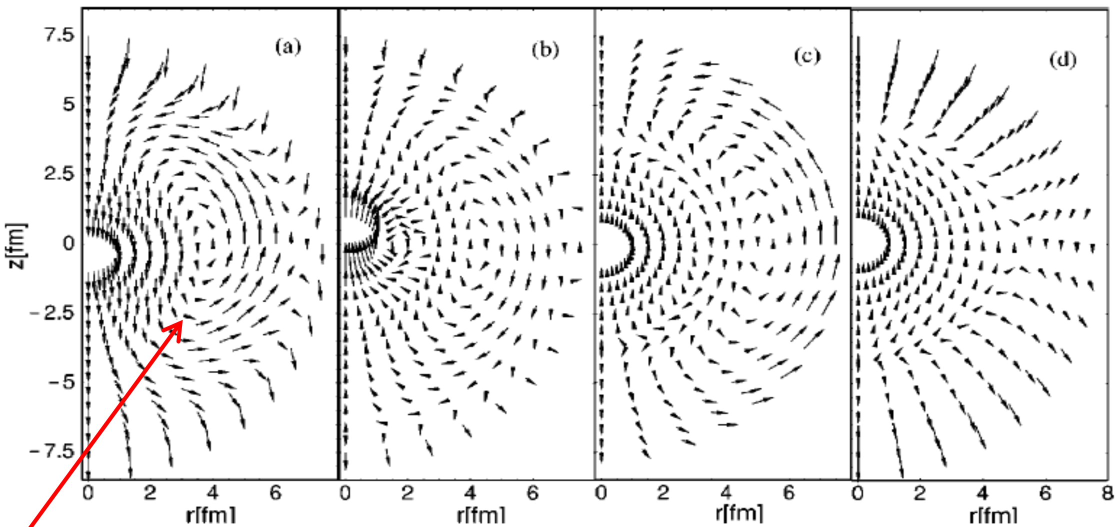
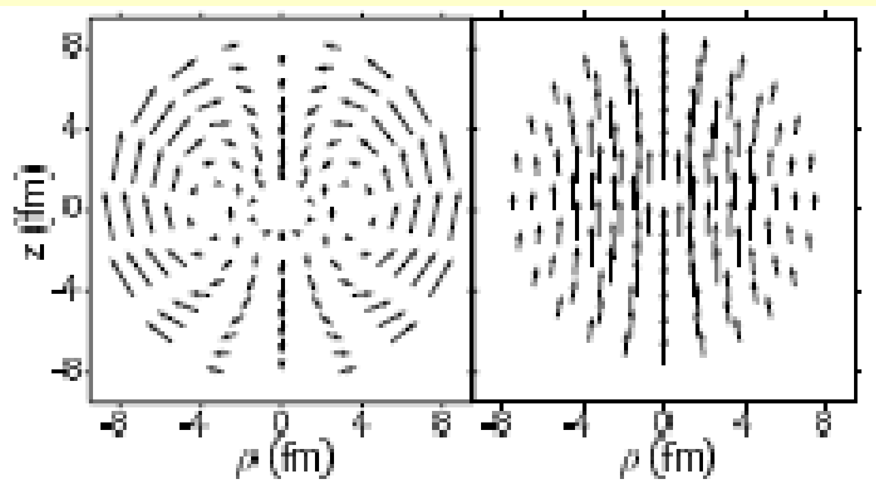


FIG. 3. Velocity distributions for the most pronounced dipole peaks in ^{116}Sn (see Fig. 2). The velocity fields correspond to the peaks at 8.82 MeV (a), 10.47 MeV (b), 17.11 MeV (c), and 30.97 MeV (d).

D. Vretenar et al,
relativistic mean field RPA

Toroidal-like flow in T=1 channel .

PDR GDR



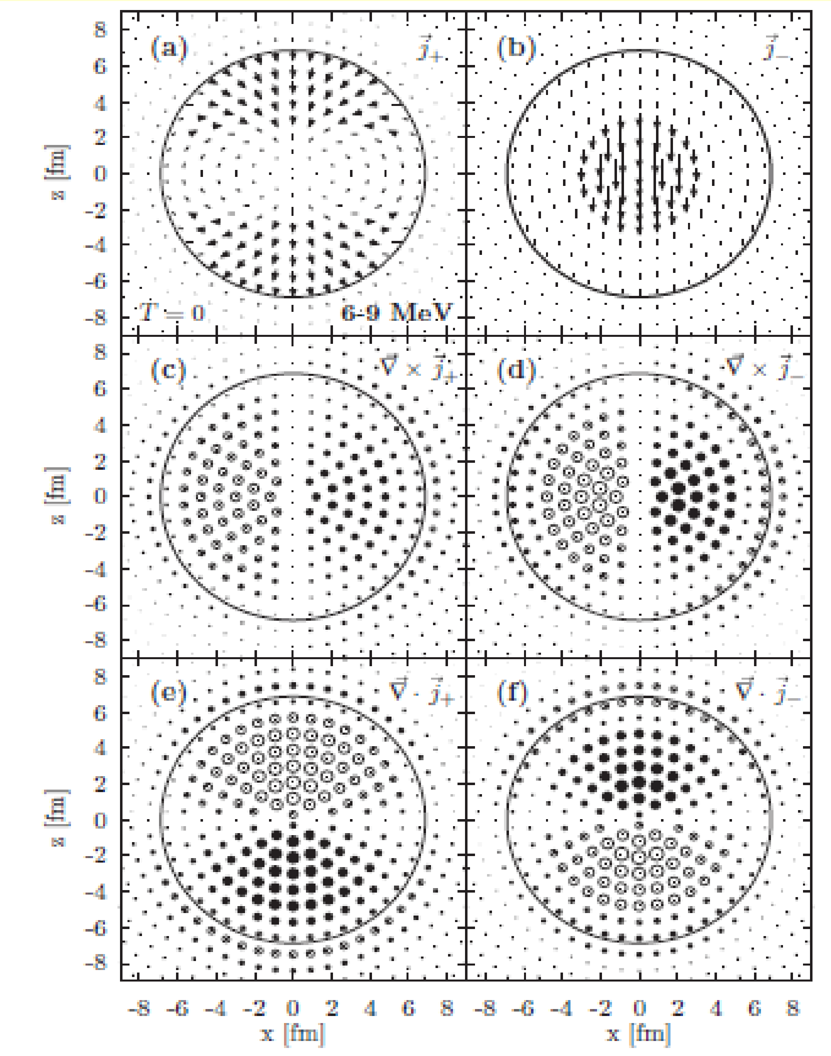
N.Ryezayeva et al, PRL 89, 272502 (2002).

QPM calculations taking into account complex configurations

Summed QPM velocity fields in 6.5-10.5 MeV region

$$\delta\vec{v} = \frac{N}{A}\delta\vec{v}_p - \frac{Z}{A}\delta$$

However none of these studies has clamed the toroidal origin of PDR j+ j-

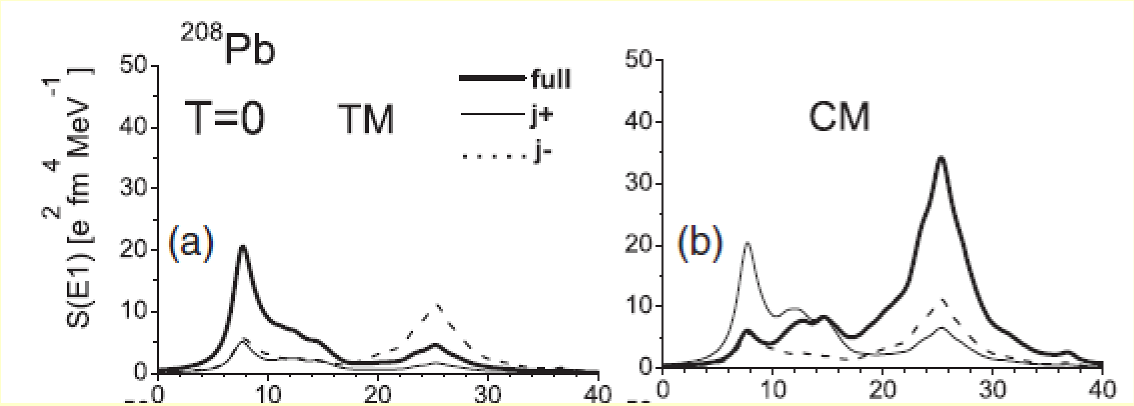


208Pb:
all RPA states
at E=6-9 MeV

j+, j-:
- both have strong curls and divs
- Both locally vortical and irrotational
- no any curl-advantge of j+ over j- to

-j+ has no any strong advantage over j- to represent the vortical flow.

j+ and j- contributions to TR an CR



MeV MeV

-**Both j+ and j- are peaked at low-energy and high-energy regions**
They are equally active in vortical TR and irrotational CR.

-**TR and CR are formed by constructive interference of the current components while in other regions there is the destructive interference .**

-**j+ has no any strong advantage to be a vortical descriptor!**

$$\langle \nu / \hat{M}_{tor}(E1\mu) / 0 \rangle = -\frac{1}{6c}\int dr\, r^2\, [\frac{\sqrt{2}}{5} r^2 j_{+}^{\nu}(r) + (r^2 - \langle r^2 \rangle_0) j_{-}^{\nu}(r)]$$

$$\langle \nu / \hat{M}_{com}(E1\mu) / 0 \rangle = -\frac{1}{6c}\int dr\, r^2\, [\frac{2\sqrt{2}}{5} r^2 j_{+}^{\nu}(r) - (r^2 - \langle r^2 \rangle_0) j_{-}^{\nu}(r)]$$

The vortical or irrotational character of the flow is provided not by j+ or j- components alone but by their proper superposition.

Current fields

$$\hat{\vec{j}}(\vec{r}) = -i \sum_{q=n,p} e^{q}_{eff} \sum_{k \ni q} \left(\delta(\vec{r}-\vec{r}_k) \vec{\nabla}_k - \overleftarrow{\nabla}_k \delta(\vec{r}-\vec{r}_k) \right)$$

$$\delta \vec{j}_{\nu}(\vec{r}) = \langle \nu | \hat{\vec{j}}(\vec{r}) | 0 \rangle$$

← Transition density of the convection current for the RPA state ν

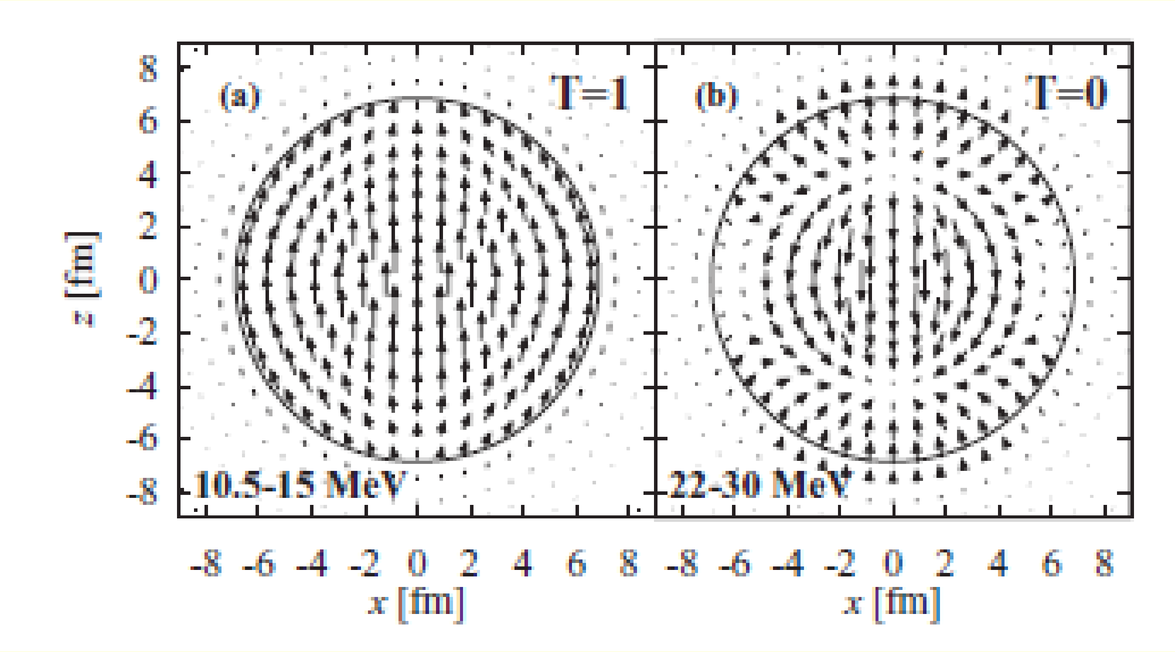
T=0: $e^{n}_{eff} = e^{p}_{eff} = 1$

T=1: $e^{p}_{eff} = \frac{N}{A}, \; e^{p}_{eff} = -\frac{Z}{A}$

p: $e^{p}_{eff} = 1, \quad e^{n}_{eff} = 0$

n: $e^{p}_{eff} = 0, \quad e^{n}_{eff} = 1$

Tests for GDR and CR:



The current fields are OK.

Finally:

- RW conception of the vorticity is not relevant:

- CE-unrestricted in integral sense,

- failure for CM,

- j+ has no advantages over j-.

-TR conception is more correct:

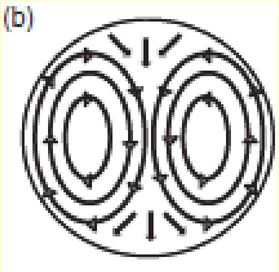
- vortical by construction,

- locally CE-unrestricted,

- close to HD conception,

- gives visually vortical image,

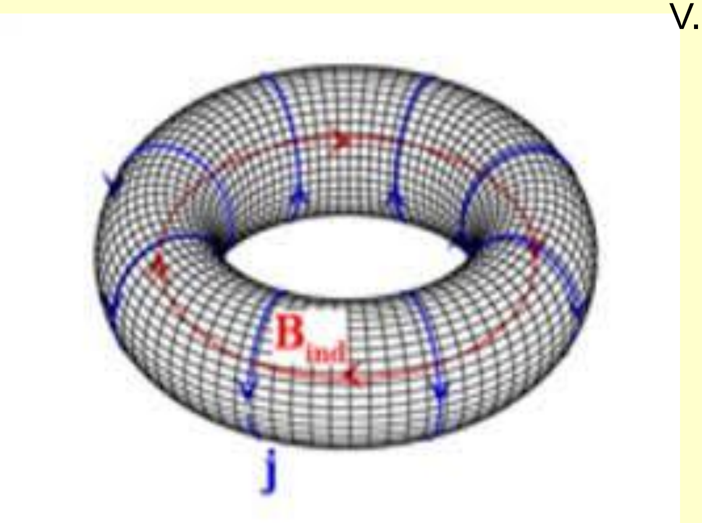
- correct for both TR and CR.



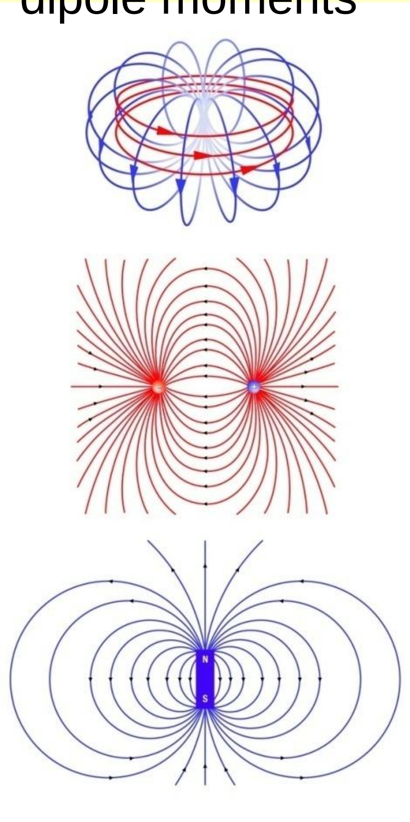
So just the toroidal strength/current is the best measure of the nuclear vorticity .

Toroidal moment

Ya. B. Zel'dovich, Zh. Eksp. Teor. Fiz. 33, 1531 (1957)
V.M. Dubovik and L.A. Tosunyan, Part. Nucl., 14, 1193 (1983)



dipole moments



anapole

electric

magnetic

- No electric and magnetic moments but the toroidal (anapole) moment

$$\vec{T} = \frac{1}{10c}\int d\vec{r}\ [(\vec{j}\cdot\vec{r})\vec{r} - 2r^2\vec{j}]$$

$$T = \frac{\pi}{2c} jR_0^2 bn$$

Speculations with toroidal stuff:

- Robert Scherrer and Chiu Man Ho (2013): attepmt to explain dark matter by existence of Maiorana fermions with the anapole moment

Recent publications on TR/CR:

J. Kvasil, V.O. Nesterenko, W. Kleinig, P.-G. Reinhard, and P. Vesely,
"General treatment of vortical, toroidal, and compression modes",
Phys. Rev. C84, n.3, 034303 (2011)

A. Repko, P.-G. Reinhard, V.O. Nesterenko, and J. Kvasil,
"Toroidal nature of the low-energy E1 mode",
Phys. Rev. C87, 024305 (2013).

J. Kvasil, V.O. Nesterenko, W. Kleinig, D. Bozik, P.-G. Reinhard, and N. Lo Iudice,
"Toroidal, compression, and vortical dipole strengths in {144-154}Sm: Skyrme-RPA exploration of deformation effect",
Eur. Phys. J. A, v.49, 119 (2013).

J. Kvasil, V.O. Nesterenko, A. Repko, W. Kleinig, P.-G. Reinhard, and N. Lo Iudice,
"Toroidal, compression, and vortical dipole strengths in 124Sn",
Phys. Scr., T154, 014019 (2013).

P.-G. Reinhard, V.O. Nesterenko, A. Repko, and J. Kvasil,
"Nuclear vorticity in isoscalar E1 modes: Skyrme-RPA analysis",
Phys. Rev. C89, 024321 (2014).

J. Kvasil, V.O. Nesterenko, W. Kleinig, and P.-G. Reinhard,
"Deformation effects in toroidal and compression dipole excitations of 170Yb: Skyrme-RPA analysis",
Phys. Scri., v.89, n.5, 054023 (2014).

V.O. Nesterenko, A. Repko, P.-G. Reinhard, and J. Kvasil,
"Relation of E1 pygmy and toroidal resonances",
arXiv:1410.5634[nucl-th],

Toroidal and compression operators

J. Kvasil, VON, W. Kleinig, P.-G. Reinhard, P. Vesely, PRC, 84, 034303 (2011)

$$\hat{M}_{tor}(E1\mu) = \frac{1}{10\sqrt{2}c}\int d\vec{r}\,[r^3 + \frac{5}{3}r < r^2 >_0]\,\vec{Y}_{11\mu}(\hat{\vec{r}})\cdot[\vec{\nabla}\times\hat{\vec{j}}_{nuc}(\vec{r})]$$

vortical flow $\vec{\nabla}\times\vec{j}(\vec{r})\neq 0$

- second-order part of the electric operator

$$\vec{j}(\vec{r}) = \vec{\nabla}\phi + \vec{\nabla}\times(\vec{r}\upsilon) + \vec{\nabla}\times\vec{\nabla}\times(\vec{r}\chi)$$

$$\hat{M}(Ek\lambda\mu) = \frac{(2\lambda+1)!!}{ck^{\lambda+1}}\sqrt{\frac{\lambda}{\lambda+1}}\int d\vec{r}\; j_\lambda(kr)\vec{Y}_{\lambda\lambda\mu}\cdot[\vec{\nabla}\times\hat{\vec{j}}_{nuc}(\vec{r})]$$

$$j_\lambda(kr) = \frac{(kr)^\lambda}{(2\lambda+1)!!}[1 - \frac{(kr)^2}{2(2\lambda+3)} + \ldots]$$

$$\hat{M}(Ek\lambda\mu) = \hat{M}(E\lambda\mu) + k\hat{M}_{tor}(E\lambda\mu)$$

$$\hat{M}(E\lambda\mu) = \int d\vec{r}\,\rho(\vec{r})r^\lambda Y_{\lambda\mu}$$

$$\hat{M}_{com}(E1\mu) = -\frac{i}{10c}\int d\vec{r}\,[r^3 - \frac{5}{3}r < r^2 >_0]Y_{1\mu}\,[\vec{\nabla}\cdot\hat{\vec{j}}_{nuc}(\vec{r})]$$

irrotational flow

- probe operator of the compression mode

$$\hat{M}'_{com}(E1\mu) = \int d\vec{r}\,\hat{\rho}(\vec{r})\,[r^3 - \frac{5}{3}r < r^2 >_0]Y_{1\mu}$$

$$\hat{M}_{com}(E1\mu) = -k\hat{M}'_{com}(E1\mu)$$

$$\dot{\rho} + \vec{\nabla}\cdot\vec{j}_{nuc} = 0$$

- c.m. corrections, r^3-dependence
- relation of TR and CR
- main IS-E1 vortical and irrotational flow

RW- prescription

D.G.Raventhall, J.Wambach, NPA 475, 468 (1987).

$$i\omega\rho_{\lambda}(r)=\sqrt{\frac{\lambda}{2\lambda+1}}\left(\frac{d}{dr}-\frac{\lambda-1}{\lambda}\right)j_{\lambda\lambda-1}(r)-\sqrt{\frac{\lambda+1}{2\lambda+1}}\left(\frac{d}{dr}+\frac{\lambda+2}{\lambda}\right)j_{\lambda\lambda+1}(r)$$

- to integrate left and right parts of CE with the weight $r^{\lambda+2}$

$$\omega\gamma_{\lambda}=\omega\int_0^\infty dr\, r^{\lambda+2}\rho_{\lambda}(r)=\sqrt{\lambda(2\lambda+1)}\int_0^\infty dr\, r^{\lambda+1}j_{\lambda\lambda-1}(r)$$

$$\int_0^\infty dr\,\frac{d}{dr}\left(r^{\lambda-1}j_{\lambda\lambda+1}(r)\right)=\lim_{r\to\infty} r^{\lambda-1}j_{\lambda\lambda+1}(r)\to 0$$

So just $j^{(fi)}_{\lambda\lambda+1}(r)$:
- is decoupled to CE in the integral sense
- has to be chosen as measure of vorticity
- convenient because it is obtained in the familiar basis of vector harmonics

To be shown that RW-conception:
- incorrect locally
- fails for CM.

3) Toroidal current

V.M. Dubovik and A.A. Cheshkov, SJPN 5, 318 (1975).

$$\delta\vec{j}(\vec{r}) = \delta\vec{j}_L(\vec{r}) + \delta\vec{j}_T^M(\vec{r}) + \delta\vec{j}_T^E(\vec{r})$$

$$\vec{j}(\vec{r}) = \vec{\nabla}\phi(\vec{r}) + \vec{\nabla}\times(\vec{r}\eta(\vec{r})) + \vec{\nabla}\times\vec{\nabla}\times(\vec{r}\chi(\vec{r}))$$

$$\delta\vec{j}(\vec{r}) = \frac{1}{(2\pi)^3}\sum_{\lambda\mu k} F_{\lambda k}\{\sqrt{\frac{\lambda}{\lambda+1}}\vec{J}^{(-)}_{\lambda\mu k}(\vec{r})\dot{Q}_{\lambda\mu}(k^2) \; + \; k\vec{J}^{(0)}_{\lambda\mu k}(\vec{r})M_{\lambda\mu}(k^2)$$

$$+ \vec{J}^{(+)}_{\lambda\mu k}(\vec{r})[\dot{Q}_{\lambda\mu}(k=0) + k^2 T_{\lambda\mu}(k^2)]\}$$

$$\vec{J}^{(-)}_{\lambda\mu k}(\vec{r}) = \frac{-i}{k}\vec{\nabla}J_{\lambda\mu k}(\vec{r})$$ $E\lambda\mu$ -longitudinal

$$\vec{J}^{(0)}_{\lambda\mu k}(\vec{r}) = \frac{-i}{k}\vec{\nabla}\times[\vec{r}\,J_{\lambda\mu k}(\vec{r})]$$ $M\lambda\mu$-transversal

$$\vec{J}^{(+)}_{\lambda\mu k}(\vec{r}) = \frac{i}{k}\vec{\nabla}\times\vec{\nabla}\times[\vec{r}\,J_{\lambda\mu k}(\vec{r})]$$ $E\lambda\mu$ -transversal

$J_{\lambda\mu k}$ -eigenfunction of Helmholtz equation $(\Delta + k^2)J_{\lambda\mu k}(\vec{r}) = 0$

basis vectors

Similar to be used in the book of Aisenberg and Greiner

Formfactors $Q_{\lambda\mu}(k^2), M_{\lambda\mu}(k^2), T_{\lambda\mu}(k^2)$ form the complete set to determine the full current.

$T_{\lambda\mu}(k^2)$ delivers independent, vortical, CE-unrestricted current and so the toroidal current can serve as a measure of the vorticity .

Divergence-curl analysis: $\vec{\nabla}\times\vec{j}(\vec{r}), \quad \vec{\nabla}\cdot\vec{j}(\vec{r})$

GDR, center of mass motion:

- are characterized by the operator $rY_{1\mu}$ with $\vec{v}(\vec{r}) \propto \vec{\nabla}\cdot(rY_{1\mu}(\vec{r}))$
- areTassie modes $(\vec{\nabla}\times\vec{v}(\vec{r}) = \vec{\nabla}\cdot\vec{v}(\vec{r}) = 0)$
- do <u>not</u> contribute to $\vec{\nabla}\times\vec{v}(\vec{r}), \quad \vec{\nabla}\cdot\vec{v}(\vec{r})$

TR, CR:

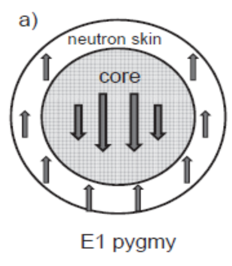
- are characterized by the operator $r^3Y_{1\mu}$
- are not Tassie modes
- do contribute to $\vec{\nabla}\times\vec{v}(\vec{r}), \quad \vec{\nabla}\cdot\vec{v}(\vec{r})$

So the div-curl analysis is just suitable for TR-CR exploration

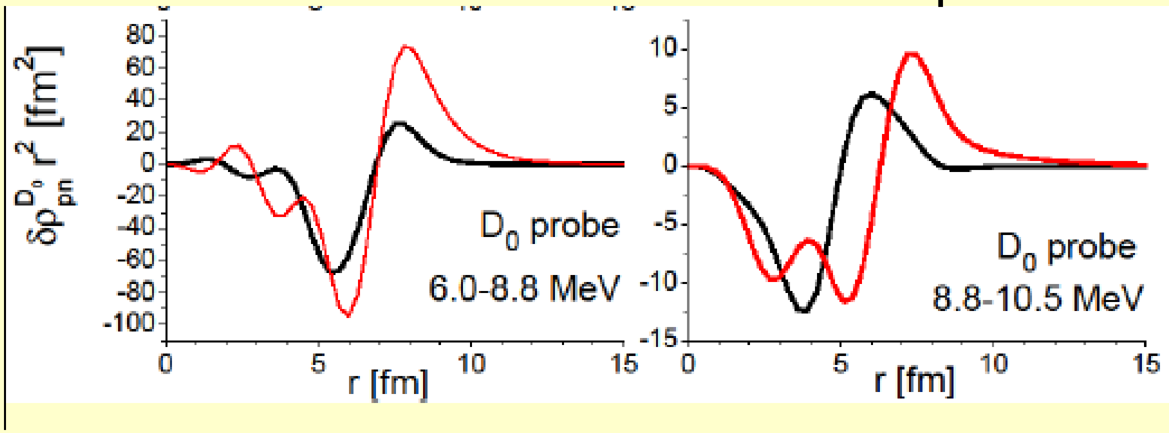
$$\vec{\nabla}\times\delta\vec{j}_{\nu}(\vec{r}) = i[rot\ j]_{\nu}(r)\vec{Y}^{*}_{11}, \quad \vec{\nabla}\cdot\vec{j}(\vec{r}) = i[div\ j]_{\nu}(r)Y^{*}_{1}$$

to be plotted

Average r^2-weighted transition densities (TD) for two parts of PDR region: 6-8.8 MeV and 8.8-10.5 MeV



-- n
-- p



Bin 6-8.8 MeV:

- typical TD structure used to justify the PDR picture: neutron excess (7-10 fm) oscillates against the nuclear core (4-7 fm)

The flow in nuclear interior (r< 4 fm) is damped though It may be important for disclosing the true PDR origin.

TD loses angular dependence of the flow .

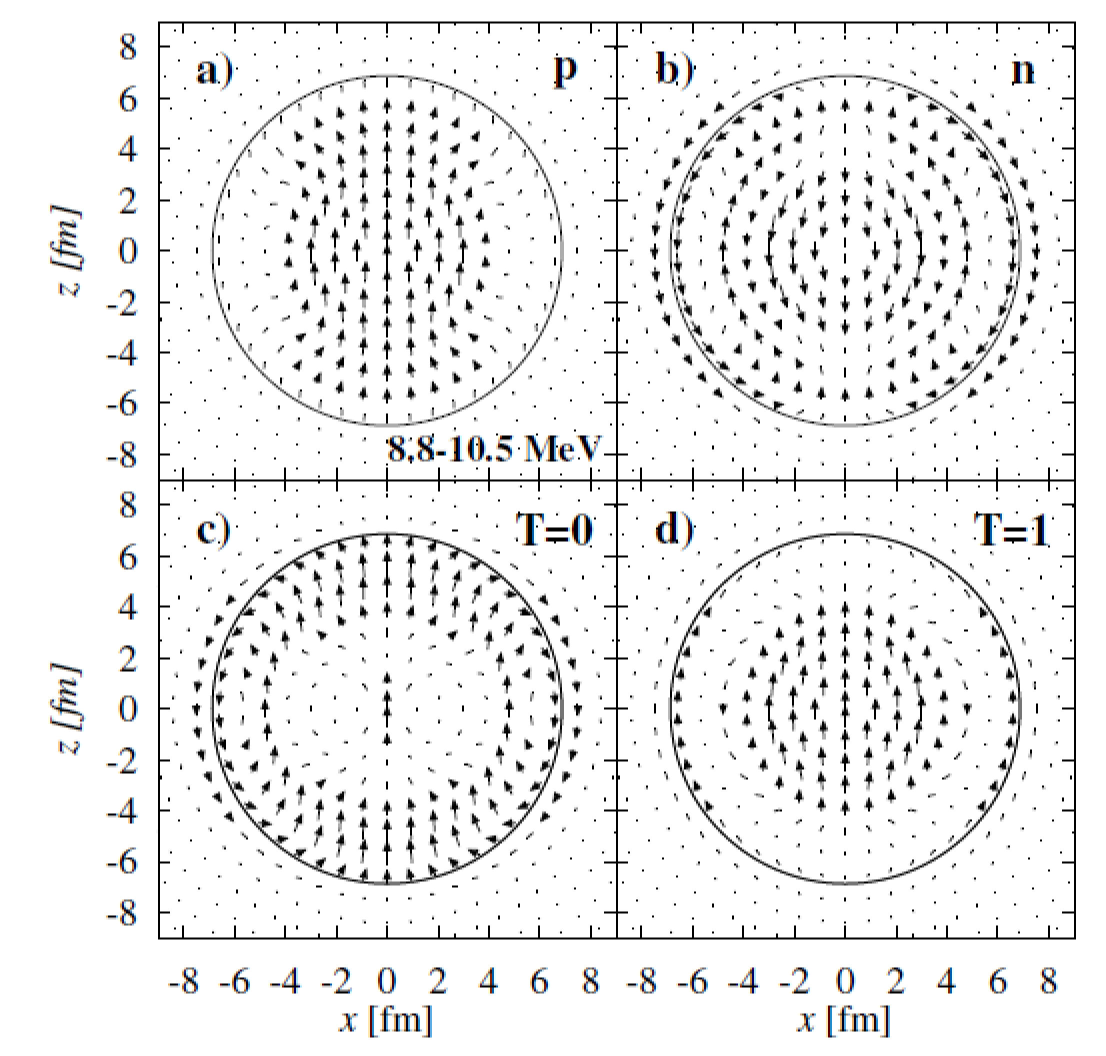
More detailed characteristics (velocity fields) are necessary.

Bins 6-8.8 MeV and 8.8-10.5 MeV :

- different scales of IS DT ⟶ the bin 6-8.8 MeV id more IS than 8.8-10.5 MeV

Bin 8.8-10.5 MeV: mixed IS/IV structure

Flow patterns : 8.8-10.5 MeV



- **mainly T=1 in interior and T=0 at the surface,**

-TR: (n, T=0)
CR: (n)
linear dipole: (p, T=1)

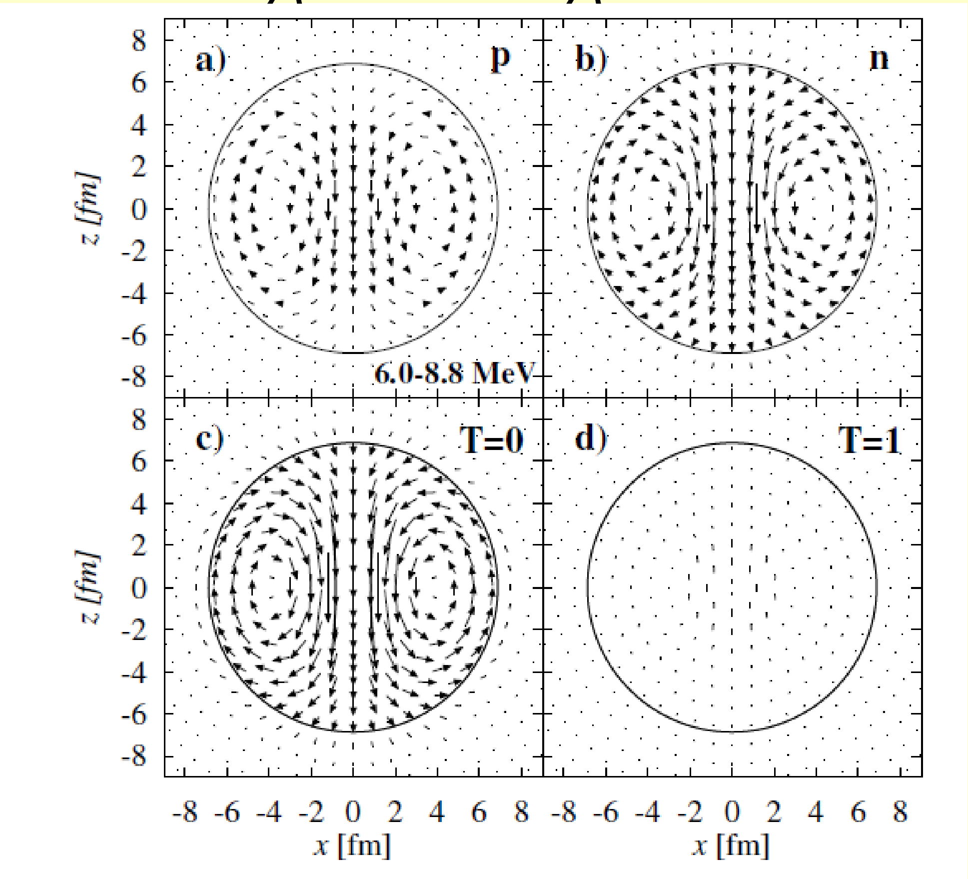
- **complex structure with mixed is/iv, TR/CR/dipole**

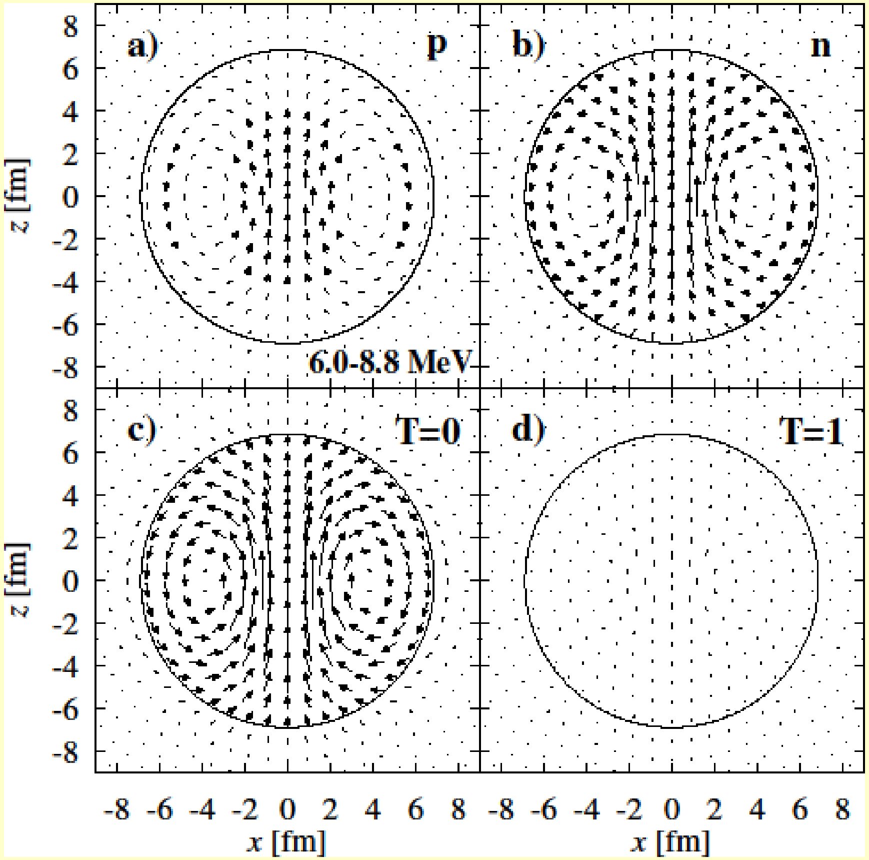
- **More significant T=1 contribution than at 6-8.8 MeV**

in accordance to:
- experiment for 124Sn, $(\alpha,\alpha'\gamma')$
(Enders et al, PRL, 2010)

Comparison of (D_1) and (D_0) patterns: 6.0-.8.8 MeV

$$\hat{D}_1 = \frac{N}{A}\sum_i^Z (rY_1)_i - \frac{Z}{A}\sum_i^N (rY_1)_i \qquad \hat{D}_0 = \sum_i^A (r^3 Y_1)_i$$





Up to the general sign, the D_0 and D_1 flows are about the same.
Moreover, they are very similar to flows with normalized weights: $D_1/|D_1|$ $D_0/|D_0|$

The model

Ordinary dipole, toroidal, compression operators

$$\alpha = \{E1, com, tor\}$$

Strength function

$$S(E1;\omega) = \sum_{\nu \neq 0} \omega^L \, |< \Psi_\nu \,|\, \hat{M}_\alpha \,|\, 0 >|^2 \, \varsigma(\omega - \omega_\nu)$$

L= { 1 for E1; 0 for com, tor }

$$\varsigma(\omega - \omega_\nu) = \frac{1}{2\pi} \frac{\Delta(\omega_\nu)}{[(\omega - \omega_\nu)^2 + \frac{[\Delta(\omega_\nu)]^2}{4}]}$$

← Lorentz weight with

$$\Delta(\omega_\nu) = \max\{0.4, (\omega_\nu - 8\,MeV)/3\}$$

Toroidal and compression operators

J. Kvasil, VON, W. Kleinig, P.-G. Reinhard, P. Vesely, PRC, 84, 034303 (2011)

$$\hat{M}_{tor}(E1\mu) = \frac{1}{10\sqrt{2}c} \int d\vec{r} \, [(r^3 - \frac{5}{3} r < r^2 >_0)] \, \vec{Y}_{11\mu}(\hat{\vec{r}}) \cdot [\vec{\nabla} \times \vec{j}_{nuc}(\vec{r})]$$

$$\hat{M}_{com}(E1\mu) = -\frac{i}{10c} \int d\vec{r} \, [(r^3 - \frac{5}{3} r < r^2 >_0) Y_{1\mu}] \, [\vec{\nabla} \cdot \vec{j}_{nuc}(\vec{r})]$$

$$\hat{M}'_{com}(E1\mu) = \int d\vec{r} \, \hat{\rho}(\vec{r}) \, [r^3 - \frac{5}{3} < r^2 >_0 \, r] Y_{1\mu} \qquad \hat{M}_{com}(E\lambda\mu) = -k \hat{M}'_{com}(E\lambda\mu)$$

Summed RPA transition densities and currents

$$\delta\rho_\nu(\vec{r}) = \langle \nu \,|\, \hat{\rho}\,(\vec{r})\,|\, 0 \rangle$$

$$\delta\vec{j}_\nu(\vec{r}) = \langle \nu \,|\, \vec{j}\,(\vec{r})\,|\, 0 \rangle$$

- are determined up to the general sign of RPA state ν ,
- being summed by ν may give ambiguous results

The problem may be cured by weighting TD and CTD by matrix elements

$D_{T\nu} = \langle \nu \,|\, \hat{D}(E1) \,|\, 0 \rangle$ of a probe operator $\hat{D}_{T\nu}$

$$\delta\rho_\beta^{(D)}(\vec{r}) = \langle \nu \,|\, \hat{\rho}\,(\vec{r})\,|\, 0 \rangle = \sum_{\nu\in[\omega_1,\omega_2]} D^*_{T\nu} \sum_{q=n,p} e^q_\beta\, \delta\rho^q_\nu(\vec{r})$$

$$\delta\vec{j}_\beta^{(D)}(\vec{r}) = \langle \nu \,|\, \hat{\vec{j}}\,(\vec{r})\,|\, 0 \rangle = \sum_{\nu\in[\omega_1,\omega_2]} D^*_{T\nu} \sum_{q=n,p} e^q_\beta\, \delta\vec{j}^{\,q}_\nu(\vec{r})$$

- bilinear combinations of ν
- for the energy interval $[\omega_1,\omega_2]$

$\hat{D}_1 = \frac{N}{A}\sum_i^Z (rY_1)_i - \frac{Z}{A}\sum_i^N (rY_1)_i$ - relevant for photoabsorption and (e,e')

e^q_β - effective charge

$\hat{D}_0 = \sum_i^A (r^3 Y_1)_i$ - relevant for isoscalar (α,α')

-The contributions of RPA states with a large D_T strength is enhanced

- There may be normalized weight

Nuclear current

$$\hat{\vec{j}}_{nuc}(\vec{r}) = \hat{\vec{j}}_{con}(\vec{r}) \; + \; \hat{\vec{j}}_{mag}(\vec{r}) = \frac{e\hbar}{m} \sum_{q=n,p} (\hat{\vec{j}}^{\,q}_{con}(\vec{r}) + \hat{\vec{j}}^{\,q}_{mag}(\vec{r}))$$

$$\hat{\vec{j}}^{\,q}_{con}(\vec{r}) = -ie^{q}_{eff} \sum_{k \ni q} \left(\delta(\vec{r} - \vec{r}_k) \vec{\nabla}_k - \overleftarrow{\nabla}_k \delta(\vec{r} - \vec{r}_k) \right)$$

← used in the present calculations

$$\hat{\vec{j}}^{\,q}_{mag}(\vec{r}) = \frac{g_s}{2} \sum_{k \ni q} \vec{\nabla}_k \times \hat{\vec{s}}_{qk} \delta(\vec{r} - \vec{r}_k)$$

T=0: $e^{n}_{eff} = e^{p}_{eff} = 1$

T=1: $e^{p}_{eff} = \frac{N}{A}, e^{p}_{eff} = -\frac{Z}{A}$

p: $e^{p}_{eff} = 1, \quad e^{n}_{eff} = 0$

n: $e^{p}_{eff} = 0, \quad e^{n}_{eff} = 1$

Center of mass corrections

$$\hat{O} = \sum_{k=1}^{A} o(\vec{r}_k) \quad \rightarrow \quad \hat{O} = \frac{1}{A}\sum_{k=1}^{A} z_k$$

$$\delta\langle\hat{O}\rangle = \int d\vec{r}\,\delta\rho(\vec{r})o(\vec{r})$$
$$= \int d\vec{r}\,\delta\vec{j}(\vec{r})\cdot\vec{\nabla}o(\vec{r}) = 0$$

translation invariance:
perturbation $\delta\rho$ **does not change**
z-coordinate of the c.m.

$$\vec{\nabla}o(\vec{r}) = \vec{\nabla}(rY_{10}) = \sqrt{3}\,\vec{Y}_{100}$$

$$\sum_{\nu}\langle 0|\hat{\vec{j}}(\vec{r})|\nu\rangle\langle 0|\hat{F}|\nu\rangle = \frac{1}{2mi}\rho_0(\vec{r})\vec{\nabla}f(\vec{r})$$

$$\sum_{\nu}\omega_{\nu}\langle 0|\hat{\rho}(\vec{r})|\nu\rangle\langle 0|\hat{F}|\nu\rangle = -\frac{1}{2m}\vec{\nabla}\cdot[\rho_0(\vec{r})\vec{\nabla}f(\vec{r})]$$

$$\delta\vec{j}_{\nu}(\vec{r}) = \langle 0|\hat{\vec{j}}(\vec{r})|\nu\rangle \propto \rho_0(\vec{r})\vec{\nabla}f(\vec{r}) \propto \rho_0(\vec{r})\vec{v}(\vec{r})$$

$$\delta\rho_{\nu}(\vec{r}) = \langle 0|\hat{\rho}(\vec{r})|\nu\rangle \propto \vec{\nabla}\cdot[\rho_0(\vec{r})\vec{\nabla}f(\vec{r})] \propto \vec{\nabla}\cdot[\rho_0(\vec{r})\vec{v}(\vec{r})]$$

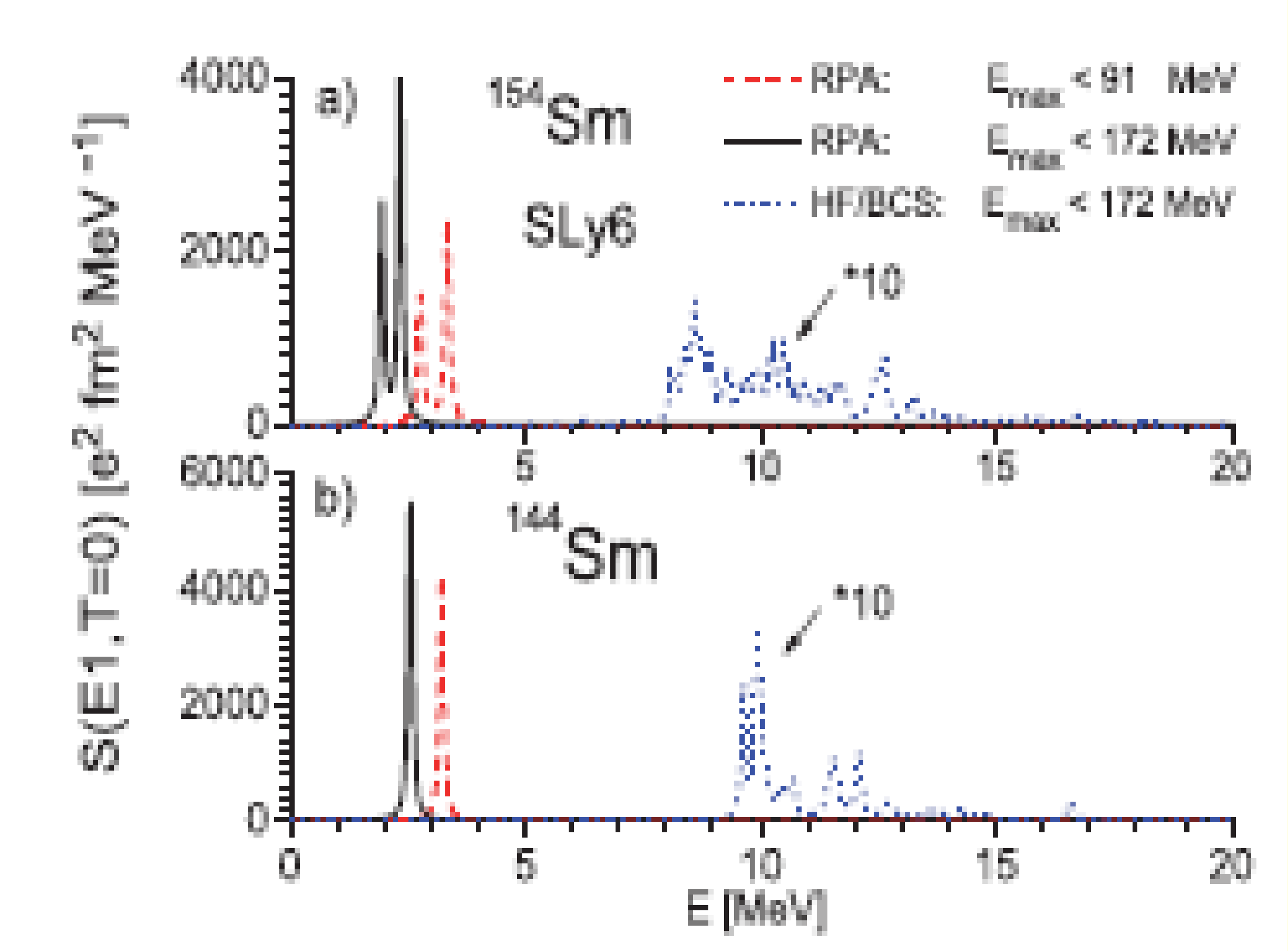
$$\vec{v}_{vor} = r^2\vec{Y}_{12\mu} + \eta\vec{Y}_{10\mu}$$
$$\vec{v}_{tor} = \frac{\sqrt{2}}{5}r^2\vec{Y}_{12\mu} + (r^2-\eta)\vec{Y}_{10\mu}$$
$$\vec{v}_{com} = \frac{\sqrt{2}}{5}r^2\vec{Y}_{12\mu} - (r^2-\eta)\vec{Y}_{10\mu}$$

$\Rightarrow$

$$\eta_{vor} = 0$$
$$\eta_{tor} = \eta_{com} = \langle r^2\rangle_0$$
$$\eta'_{com} = \frac{5}{3}\langle r^2\rangle_0$$

Exclusion of spurious admixtures:

J. Kvasil, V.O. Nesterenko, W. Kleinig, D. Bozik, P.-G. Reinhard, and N. Lo Iudice, EPJA, **49**, 119 (2013)

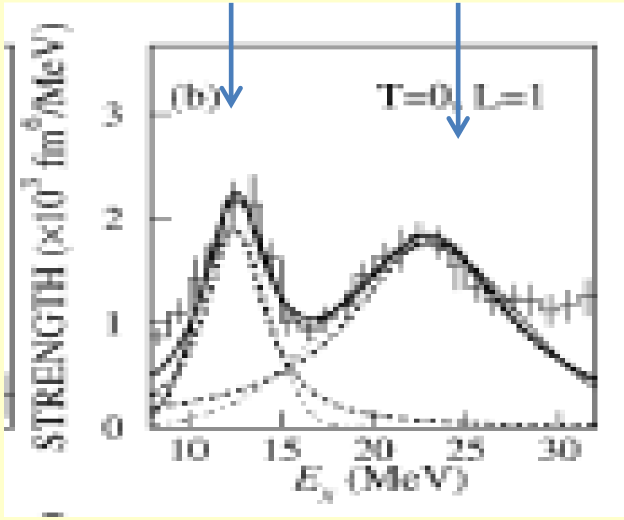


TR: experimental status

Experiment: (α, α')

M.Uchida et al, PLB 557, 12 (2003),
PRC 69, 051301(R) (2004)

LE (toroidal) HE (compression)



Looks reasonable since the theory predicts only TR to form the low-energy part of ISGDR.

Anyway is it possible to propose a reaction where TR:
- could be observed alone or
- could demonstrate a particular fingerprint?

The reaction should be:
- IS (to suppress the effect of the dominant E1(T=1) modes)
- transversal but not polluted by magnetic form-factors
- sensitive to nuclear interior

(e,e'), - both IS/IV, strong magnetic forf-mactor
(α, α') - peripheral, not sensitive to nuclear interior,
(p,p') - both IS/IV

} not good

Reactions with polarized beams/targets?

So far (α, α') is the best option where TR can be excited:
- not directly but through the coupling with CR or PDR
- through peripheral part of TR

TR and CR consitute low- and high-energy ISGDR branches

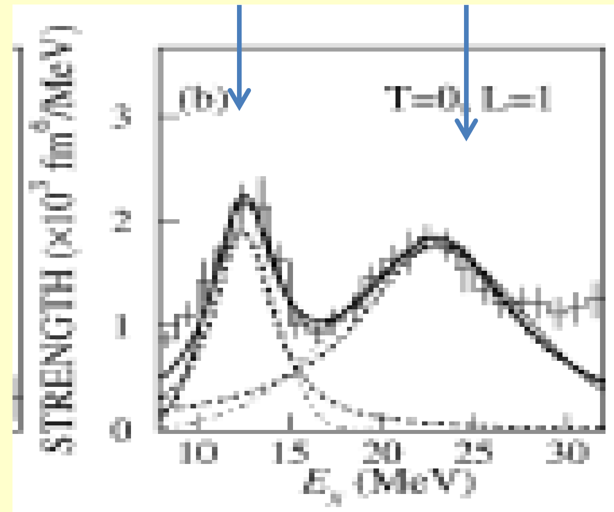
Experiment: (α, α')

LE (toroidal) HE (compression)

^{208}Pb

D.Y. Youngblood et al, 1977
H.P. Morsch et al, 1980
G.S. Adams et al, 1986
B.A. Devis et al, 1997
H.L. Clark et al, 2001
D.Y. Youngblood et al, 2004

M.Uchida et al, PLB 557, 12 (2003), PRC 69, 051301(R) (2004)



There are also the ISGDR data in

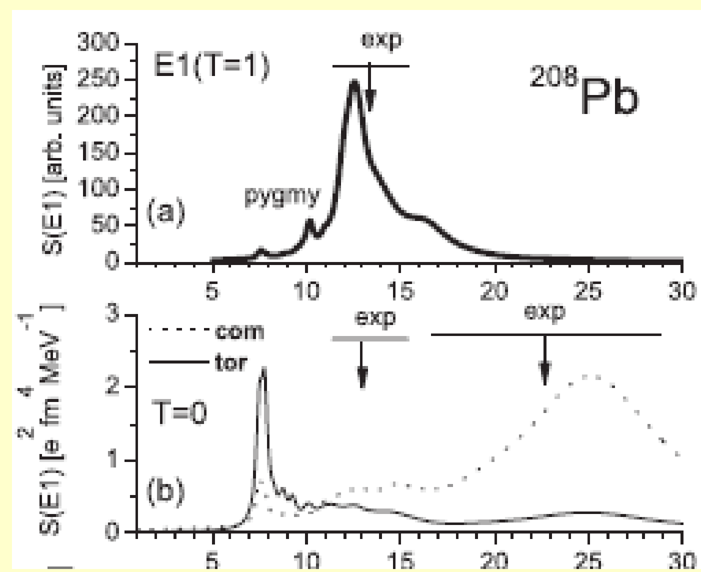
^{56}Fe, $^{58,60}Ni$, ^{90}Zr, ^{116}Sn, ^{144}Sm, ...

Preliminary results on TR (F. Guerelly) $(^{16}O, ^{16}O')$ in ^{27}Al

Theory:

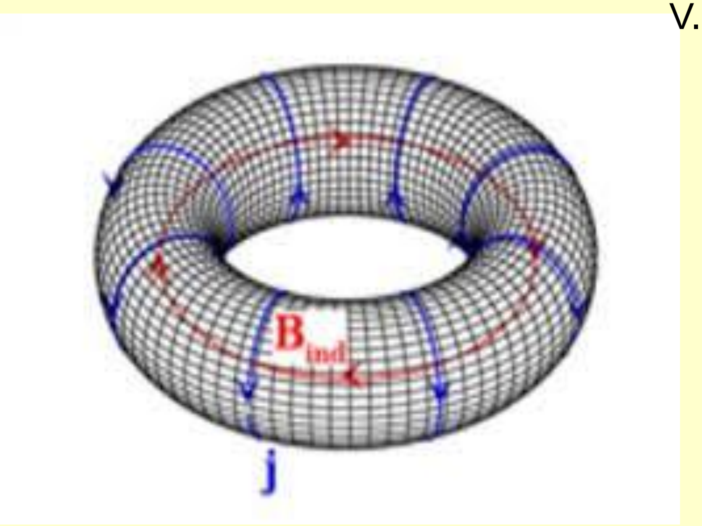
A. Repko, P.-G. Reinhard, V.O.N. and J. Kvasil, PRC 87, 024305 (2013).

Skyrme RPA, SLy6

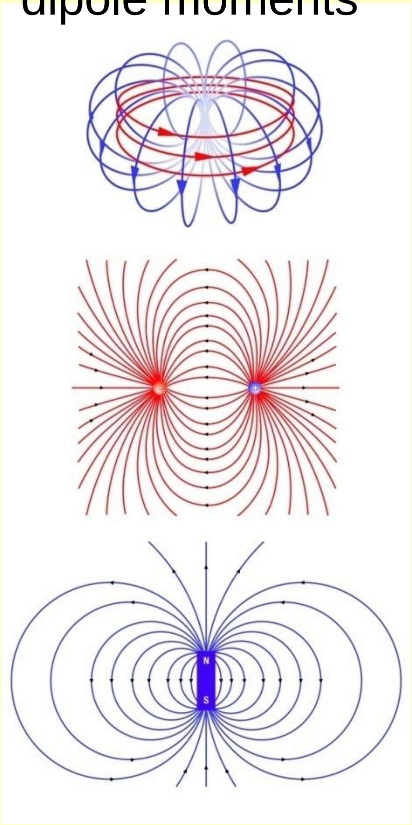


Toroidal moment

Ya. B. Zel'dovich, Zh. Eksp. Teor. Fiz. 33, 1531 (1957)
V.M. Dubovik and L.A. Tosunyan, Part. Nucl., 14, 1193 (1983)



dipole moments



anapole

electric

magnetic

- No electric and magnetic moments but the toroidal (anapole) moment

$$\vec{T} = \frac{1}{10c}\int d\vec{r}\ [(\vec{j}\cdot\vec{r})\vec{r} - 2r^2\vec{j}]$$

$$T = \frac{\pi}{2c} jR_0^2 bn$$

Speculations with toroidal stuff:

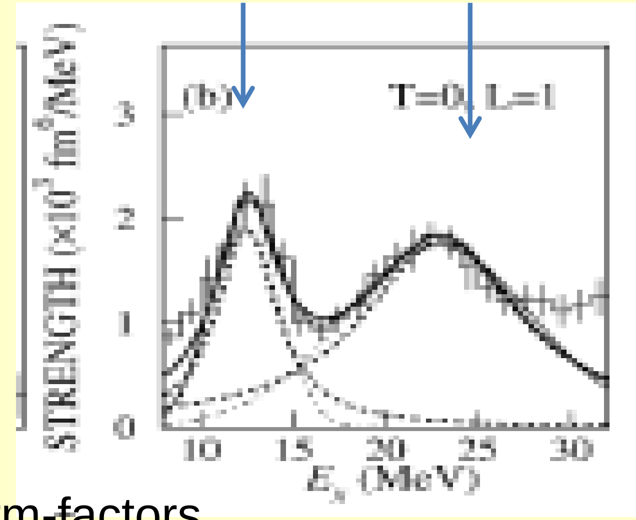
- Robert Scherrer and Chiu Man Ho (2013): attepmt to explain dark matter by existence of Maiorana fermions with the anapole moment

TR: experimental perspectives

Experiment: (α,α')

M.Uchida et al, PLB 557, 12 (2003),
PRC 69, 051301(R) (2004)

LE (toroidal)   HE (compression)



Looks reasonable since the theory predicts only TR to form the low-energy part of ISGDR.

The reaction should be:

- IS (to suppress the dominant E1(T=1) modes)
- (partly) transversal but not polluted by magnetic form-factors

(e,e'), - both IS/IV, strong magnetic form-mactor
(p,p') - both IS/IV

} not good

Peripheral IS reactions (α,α') and $(^{16}O,^{16}O')$ seem to be the best options:

TR can be excited though its peripheral part (together with IS PDR and CR).

TR: experimental perspectives -2

The response depends on the probe:

(α,α') $r^3Y_{1\mu}$ → TR

Photoabsorp. $rY_{1\mu}$ → PDR

Always small angles but different E_α

Different conditions of (α,α') -experiments:

- PDR, Endres, E_α = 136 MeV
- TR+CR , Uchida E_α = 400 MeV
- TR+CR, Youngblood E_α = 240 MeV

It would be interesting to observe:

1) Deformation splitting (sequence of K-branches) in TR/PDR energy region by using $(\alpha,\alpha'\gamma)$.

2) Comparison of photoabsorption and (α,α') data in nuclei with N=Z and N>Z. For example:

			^{40}Ca	^{48}Ca
(α,α')	IS,	$r^3Y_{1\mu}$	TR	TR, PDR
Photoabsorp.	IS/IV,	$rY_{1\mu}$	--	PDR

To compare TR, PDR and GDR (α,α') formfactors. The TR formfactors should have maxima at higher transfer mom. (talk of P.G. Reinhard)