Rod-shaped nuclei at extreme spin and isospin

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• Introduction
• Theoretical Framework
• Results and Discussion
• Summary
Nuclear deformations provide us an excellent framework to investigate the fundamental properties of quantum many-body systems.

\[ R(\theta, \varphi) = R_0 \left[ 1 + \beta_{00} + \sum_{\lambda=1}^{\infty} \sum_{\mu=-\lambda}^{\lambda} \beta_{\lambda \mu}^* Y_{\lambda \mu}(\theta, \varphi) \right] \]

<table>
<thead>
<tr>
<th>(a) ( \beta_{\lambda \mu} = 0 )</th>
<th>(b) ( \beta_{20} &gt; 0 )</th>
<th>(c) ( \beta_{20} &lt; 0 )</th>
<th>(d) ( \beta_{40} &gt; 0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Graph" /></td>
<td><img src="image2" alt="Graph" /></td>
<td><img src="image3" alt="Graph" /></td>
<td><img src="image4" alt="Graph" /></td>
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<tr>
<td>(e) ( \beta_{22} \neq 0 )</td>
<td>(f) ( \beta_{30} \neq 0 )</td>
<td>(g) ( \beta_{32} \neq 0 )</td>
<td>(h) ( \beta_{20} \gg 0 )</td>
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<td><img src="image5" alt="Graph" /></td>
<td><img src="image6" alt="Graph" /></td>
<td><img src="image7" alt="Graph" /></td>
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Courtesy of Bing-Nan Lu (吕炳楠)
Evidence for the super- and hyper-deformation provide unique opportunity to study nuclear structure under extreme conditions.

Harmonic oscillator

Twin PRL 1986
There have been indications that even more exotic states above 1:3 might exist in light N = Z nuclei due to the a cluster structure.

Towards hyperdeformation

Cluster structure in light nuclei
■ Clustering in nuclei is an old story: John Archibald Wheeler, Molecular Viewpoints in Nuclear Structure, Physical Review 52 (1937) 1083

■ Lots of works have been done by, e.g. Ikeda, Horiuchi, Kanada-enyo, Freer, Itagaki, Khan, Maruhn, Schuck, Tohsaki, Zhou, Ichikawa, Funaki, Von Oertzen, ...

■ Linear-chain structure of three-$\alpha$ clusters was suggested about 60 years ago Morinaga, Phys. Rev. 101, 254 (1956) to explain the structure of the Hoyle state (the second $0^+$ state at 7.65 MeV in $^{12}$C) Hoyle, Astrophys J. Sup. 1, 121 (1954).

■ However, Hoyle state was later found to be a mixing of the linear-chain configuration and other three-$\alpha$ configurations, and recently reinterpreted as an $\alpha$-condensate-like state Fujiwara et al, PTP Sup. 68, 29 (1980). Tohsaki et al PRL 87, 192501 (2001). Suhara et al PRL 112, 062501 (2014).
Harmonic oscillator density

Freer RPP 2007

Green Function Monte Carlo

Wiringa PRC 2000

Because of ✓ antisymmetrization effects
✓ weak-coupling nature

it is difficult to stabilize the rod-shaped configuration in nuclear systems.
Long existing problem: how can we stabilize geometric cluster shapes for instance linear alpha chain?
Most of the linear chain structure have been predicted by the conventional cluster model with effective interactions determined from the binding energies and scattering phase shifts of the clusters.

Since the DFTs do not a priori assume the existence of $\alpha$ clusters, it is highly desirable to have investigations based on different approaches, such as density functional theories (DFTs).


- **ab initio lattice calculations of the low-energy even-parity states of 16O using chiral nuclear effective field theory.**
Studies have shown that the nucleons are prone to form cluster structure in the nuclear system with:

- high excitation energy and high spin with large deformation


- deep confining nuclear potential


- or expansion with low density

4$\alpha$-LCS in high-spin states of $^{16}$O from the cranking SHF calculation

How to stabilize linear chain configurations?

Two important mechanisms

- Adding valence neutrons
  Itagaki, PRC2001

- Rotating the system
  Ichikawa, PRL2011

\[ \sigma \text{-orbital} \]

\[ \pi \text{-orbital} \]

Coherent effects exist?
It facilitates the stabilization?
Using the nuclear energy density functional, the conditions for single nucleon localization and formation of cluster structures in finite nuclei are examined.

A localized equilibrium density and the formation of cluster structures are visible in (a) DD-ME2 but not in (b) Skyrme SLy4

\[ ^{240}\text{Pu}: \text{3-dim. PES} (\beta_{20}, \beta_{22}, \beta_{30}) \]

- AS & RS for g.s. & isomer, the latter is stiffer
- Triaxial & octupole shape around the outer barrier
- Triaxial deformation crucial around barriers

Lu, Zhao, Zhou Phys. Rev. C 85, 011301(R)
Related review paper


孟杰, 郭建友, 李剑, 李志攀, 梁豪兆, 龙文辉, 牛一斐, 牛中明, 尧江明, 张颖, 赵鹏巍, 周善贵, 原子核物理中的协变密度泛函理论, 物理学进展, 第31卷04期 (2011) 199-336


Hyperdeformed Rod shaped $\alpha$-Linear Chain Structure
Cranking CDFT to investigate the stabilization of rod shape at extreme spin and isospin in a fully self-consistent and microscopic way.

By adding valence neutrons and rotating the system, the mechanism stabilizing the rod shape will be explored.

CDFT configuration mixing of PN-AM projected calculation will be carried out to find evidence for $4\alpha$ linear cluster structure.

Yao, Itagaki, Meng, Phys. Rev. C 90, 054307 (2014)
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The many-body problem is mapped onto an one-body problem without explicitly involving inter-nucleon interactions!

Kohn-Sham Density Functional Theory

For any interacting system, there exists a local single-particle potential \( h(r) \), such that the exact ground-state density of the interacting system can be reproduced by non-interacting particles moving in this local potential.

\[
E[\rho] \Rightarrow \hat{h} = \frac{\delta E}{\delta \rho} \Rightarrow \hat{h}\varphi_i = \varepsilon_i \varphi_i \Rightarrow \rho = \sum_{i=1}^{A} |\varphi_i|^2
\]

The practical usefulness of the Kohn-Sham scheme depends entirely on whether Accurate Energy Density Functional can be found!
• For nuclei, the energy density functional has been introduced by effective Hamiltonians

\[ E = \langle \Psi | H | \Psi \rangle \approx \langle \Phi | \hat{H}_{eff}(\hat{\rho}) | \Phi \rangle = E[\hat{\rho}] \]

• More degrees of freedom: spin, isospin, pairing, …

• Nuclei are self-bound systems;
  \( \rho(r) \) here denotes the intrinsic density.

• Density functional is probably not exact, but a very good approximation.

• The functional are adjusted to properties of nuclear matter and/or finite nuclei and (in future) to ab-initio results.
Why Covariant?

✓ Large spin-orbit splitting in nuclei
✓ Pseudo-spin Symmetry
✓ Success of Relativistic Brueckner
✓ Consistent treatment of time-odd fields
✓ Large fields $V \approx 350$ MeV, $S \approx -400$ MeV
✓ Relativistic saturation mechanism
✓ ... Liang, Meng, Zhou, Physics Reports 570 : 1-84 (2015)

Pseudospin symmetry

$P_{l \pm j} \Rightarrow n-1, \ l+2, \ j = l + 3/2$

pseudo-orbit : $\tilde{l} = l + 1$

pseudo-spin : $\tilde{s} = 1/2$

Hecht & Adler
NPA137(1969)129

Arima, Harvey & Shimizu
PLB 30(1969)517

Brockmann & Machleidt, PRC42, 1965 (1990)

Covariant Density Functional Theory

Elementary building blocks

\[(\bar{\psi} \mathcal{O}_\tau \Gamma \psi) \quad \mathcal{O}_\tau \in \{1, \tau_i\} \quad \Gamma \in \{1, \gamma_\mu, \gamma_5, \gamma_5 \gamma_\mu, \sigma_{\mu\nu}\}\]

Densities and currents

**Isoscalar-scalar**

\[\rho_S(r) = \sum_k \bar{\psi}_k(r) \psi_k(r)\]

**Isoscalar-vector**

\[j_\mu(r) = \sum_k \bar{\psi}_k(r) \gamma_\mu \psi_k(r)\]

**Isovector-scalar**

\[\bar{\rho}_S(r) = \sum_k \bar{\psi}_k(r) \bar{\gamma}_\mu \psi_k(r)\]

**Isovector-vector**

\[\bar{j}_\mu(r) = \sum_k \bar{\psi}_k(r) \bar{\gamma}_\mu \psi_k(r)\]

Energy Density Functional

\[E_{\text{kin}} = \sum_k \hbar^2 k^2 \int \bar{\psi}_k (-\gamma \nabla + m) \psi_k dr\]

\[E_{2nd} = \frac{1}{2} \int (\alpha_S \rho_S^2 + \alpha_V \rho_V^2 + \alpha_{tV} \rho_{tV}^2) dr\]

\[E_{\text{hot}} = \frac{1}{12} \int (4\beta_S \rho_S^3 + 3\gamma_S \rho_S^4 + 3\gamma_V \rho_V^4) dr\]

\[E_{\text{der}} = \frac{1}{2} \int (\delta_S \rho_s \Delta \rho_S + \delta_V \rho_V \Delta \rho_V + \delta_{tV} \rho_{tV} \Delta \rho_{tV}) dr\]

\[E_{\text{em}} = \frac{e}{2} \int j_\mu A^\mu dr\]
Cranking Covariant Density Functional Theory

Transform to the frame rotating with a uniform velocity

\[
x^\alpha = \begin{pmatrix} t \\ x \end{pmatrix} \rightarrow \tilde{x}^\mu = \begin{pmatrix} \tilde{t} \\ \tilde{x} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & R_x(t) \end{pmatrix} \begin{pmatrix} t \\ x \end{pmatrix}
\]

Rotating Density Functional

Kohn-Sham/Dirac Equation:

Dirac equation for single nucleon

\[
\begin{pmatrix}
  m + S + V - \Omega \cdot J & \sigma (p - V) \\
  \sigma (p - V) & -m - S + V - \Omega \cdot J \\
\end{pmatrix}
\begin{pmatrix}
  \mathcal{J} \\
  \mathcal{K} \\
\end{pmatrix} = \epsilon
\begin{pmatrix}
  \mathcal{J} \\
  \mathcal{K} \\
\end{pmatrix}
\]

\[
V(r) = \alpha_\uparrow \rho_\uparrow + \gamma_\uparrow \rho_\uparrow^i + \delta_\uparrow \Delta \rho_\uparrow + \tau_\uparrow \alpha_\uparrow \rho_\uparrow + \tau_\uparrow \delta_\uparrow \Delta \rho_\uparrow + e^{\frac{1 - \tau_\uparrow}{2}} A \\
V(r) = \alpha_\downarrow \rho_\downarrow + \gamma_\downarrow \rho_\downarrow^i + \delta_\downarrow \Delta \rho_\downarrow + \tau_\downarrow \alpha_\downarrow \rho_\downarrow + \tau_\downarrow \delta_\downarrow \Delta \rho_\downarrow + e^{\frac{1 - \tau_\downarrow}{2}} A \\
S(r) = \rho_\uparrow + \beta_\uparrow \rho_\uparrow^i + \gamma_\uparrow \rho_\uparrow + \delta_\uparrow \Delta \rho_\uparrow \\
\]

\[V(r)\] vector potential time-like
\[V(r)\] vector potential space-like \[S(r)\] scalar potential
Observables

Binding energy

\[ E_{\text{tot}} = \sum_{k=1}^{A} \epsilon_k - \int d^3r \left\{ \frac{1}{2} \alpha_s \rho_s^2 + \frac{1}{2} \alpha_v j_{\nu}^\mu (j_{\nu})_\mu \right. \\
+ \frac{1}{2} \alpha_{TV} j_{TV}^\mu (j_{TV})_\mu + \frac{2}{3} \beta_s \rho_s^3 + \frac{3}{4} \gamma_s \rho_s^4 \\
+ \frac{3}{4} \gamma_v (j_{\nu}^\mu (j_{\nu})_\mu)^2 + \frac{1}{2} \delta_s \rho_s \Delta \rho_s + \frac{1}{2} \delta_v (j_{\nu})_\mu \Delta j_{\nu}^\mu \\
+ \frac{1}{2} \delta_{TV} j_{TV}^\mu \Delta (j_{TV})_\mu + \frac{1}{2} \epsilon (0) A_0 \right\} + \sum_{k=1}^{A} \langle k| \Omega | j | k \rangle \\
+ E_{\text{c.m.}}. \]

Angular momentum

\[ J = \sqrt{\langle \hat{J}_x \rangle^2 + \langle \hat{J}_z \rangle^2} \equiv \sqrt{I(I+1)}. \]

Quadrupole moments and magnetic moments

\[ Q_{20} = \sqrt{\frac{5}{16\pi}} \langle 3z^2 - r^2 \rangle, \]
\[ Q_{22} = \sqrt{\frac{15}{32\pi}} \langle x^2 - y^2 \rangle, \]
\[ \mu = \sum_{i=1}^{A} \int d^3r \left[ \frac{mc^2}{\hbar c} q \psi_i^\dagger (r) r \times \alpha \psi_i (r) + \kappa \psi_i^\dagger (r) \beta \Sigma \psi_i (r) \right]. \]
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Angular momentum

- C-12, C-13, C-14: constant moments of inertia (MOI); like a rotor
- C-15, C-16, C-17, C-18: abrupt increase of MOI; some changes in structure
- C-19; C-20: constant moments of inertia; much larger

Rod shape are obtained in all isotopes by tracing the corresponding rod-shaped configuration.

Proton density distribution

Larger deformation
Clearer clustering
Stable against particle-hole deexcitations?

Single-proton energy: configurations stabilized against particle-hole deexcitations.

For C-15:

- Low spin: deexcitations easily happen
- High spin: More stable against deexcitations

Spin effects?
Valence neutron density distribution

π-orbital  σ-orbital  σ-orbital  σ-orbital

C-15: valence neutrons
Low spin: \( \pi \)-orbital; proton unstable
High spin: \( \sigma \)-orbital; proton stable

C-20: valence neutrons
Low spin: \( \sigma \)-orbital; proton stable
High spin: \( \sigma \)-orbital; proton stable

Isospin effects?

\( h\hbar \nu \) 0.0 MeV 3.0 MeV

Single-neutron energy

Isospin effects?

Spin and Isospin Coherent Effects
Related experiment is highly demanded!

Beyond RMF calculation with GCM for low-spin states

1. Generate a large set of highly correlated RMF+BCS wave functions with triaxial deformation (β,γ) by minimizing

\[
E[\rho, \kappa] = \frac{\langle \Phi(q) | \hat{H} | \Phi(q) \rangle}{\langle \Phi(q) | \Phi(q) \rangle} - \sum_{\mu=0,2} \frac{1}{2} C_\mu (q_{2\mu} - Q_{2\mu})^2
\]

\[
E_{DF}[\rho_i, \nabla \rho_i, j_i^\mu, \nabla j_i^\mu] = \text{Tr}[(\alpha \cdot p + \beta m)\rho_V] + \int dr \left( \frac{\alpha_S}{2} \rho_S^2 + \frac{\beta_S}{3} \rho_S^3 + \frac{\gamma_S}{4} \rho_S^4 + \frac{\delta_S}{2} \rho_S \Delta \rho_S \right) + \frac{\alpha_V}{2} j_\mu j^\mu + \frac{\gamma_V}{4} (j_\mu j^\mu)^2 + \frac{\delta_V}{2} j_\mu \Delta j^\mu + \frac{\alpha_{TV}}{2} j_\mu (j_{TV})_\mu + \frac{\delta_{TV}}{2} j_\mu \Delta (j_{TV})_\mu + \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - F^{0\mu} \partial_0 A_\mu + e \frac{1 - \tau_3}{2} j_\mu A^\mu,
\]

Mean-field states |q(β,γ)⟩ are Slater determinants of single-(quasi)particle states from the RMF+BCS calculation with constraints on the mass quadrupole moments

\[
Q_{20} = \sqrt{\frac{5}{16\pi}} \langle 2z^2 - x^2 - y^2 \rangle \quad \text{and} \quad Q_{22} = \sqrt{\frac{15}{32\pi}} \langle x^2 - y^2 \rangle,
\]

where the deformation parameters β,γ are related to the quadrupole moments by

\[
\beta = \frac{4\pi}{3AR^2} Q_{20}, \quad \gamma = \tan^{-1} \left( \sqrt{2} \frac{Q_{22}}{Q_{20}} \right),
\]

respectively, with \( R = 1.2A^{1/3} \) and \( A \).
Beyond RMF calculation with GCM for
low-spin states

- The wave function of nuclear low-spin state is given by the superposition of a set of both particle-number and angular-momentum projected (PNAMP) quadrupole deformed mean-field states in the framework of GCM.

\[ |JNZ;\alpha\rangle = \sum_{q,K} f^{JNZK}_\alpha(q) \hat{P}^J_{MK} \hat{P}^N \hat{P}^Z |q(\beta, \gamma)\rangle. \]

Minimization of nuclear total energy with respect to the coefficient \( f \) leads to the Hill-Wheeler-Griffin (HWG) equation:

(\text{Restricted to be axially deformed, and } K=0)

\[ \sum_{\beta'} [\mathcal{H}^J(\beta, \beta') - E^J_{\alpha} \mathcal{N}^J(\beta, \beta')] f^{JNZ}_\alpha(\beta') = 0, \]
• Linear-Chain-Structure (LCS) in the low-spin GCM states with moment of inertia around 0.11 MeV is found.
• 4-alpha clusters stay in z-axis and nucleons occupy the states in a nonlocal way.
• Spin and orbital angular momenta of all nucleons are parallel in the LCS states.
• Fully microscopic GCM calculation has reproduced the excitation energies and B(E2) values rather well for the rotational band built on the second 0^+ state.
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The relativistic density functional theories have been proved to be very successful in describing a large variety of nuclear phenomena from finite nuclei to nuclear matter, from stable to extreme unstable nuclei, from spherical shape to nuclei with novel shapes, from nuclear ground-state to excited-state properties, etc.
Novel shape, rod-shaped C isotopes, known to be difficult to stabilize for a long time, has been studied.

The advantages of cranking CDFT include (i) the cluster structure is investigated without assuming the existence of clusters a priori, (ii) the nuclear currents are treated self-consistently, (iii) the density functional is universal, and (iv) a microscopic picture can be provided in terms of intrinsic shapes and single-particle shells self-consistently.

Two mechanisms to stabilize the rod shape: rotation (high spin) and adding neutrons (high Isospin), coherently work in C isotopes.

Coherent Effects: Rotation makes the valence sigma neutron-orbital lower, and thus 1) lower the sigma proton orbitals 2) enhances the prolate deformation of protons.

Outlook: bend motion? valence proton? …
In collaboration with

Naoyuki Itagaki, Jiangming Yao, Pengwei Zhao

Thank you for your attention!