Monopole E0 resonance in deformed nuclei

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Motivation

Giant Monopole Resonance (GMR) centroid E_{GMR} is connected with finite – nucleus incompressibility K_A by (see e.g. J.Blaizot, Phys. Rep. 64, 171 (1980))

$$E_{GMR} = \sqrt{\frac{\hbar^2 K_A}{m \langle r^2 \rangle}}$$

The incompressibility (together with the nucleus mass and radius) belongs to the bulk properties used for the determination of the energy functional (n-n effective interaction) parameters

GMR is the subject of intensive investigation from 60-s up to now

From the point of view of theory the position of E_{GMR} is usually obtained by means of moments of energy weighted E0 strength functions

$$E_{GMR} = m_1 / m_0 \quad \Big| \quad m_k = \int dE \; S_k(E0;E) \quad \Big| \quad S_k(E0;E) = \sum_{\nu} E^k \, |\langle \nu | \hat{M}^{(IS)}_{\lambda=\mu=0}(el) | 0 \rangle|^2 \, \delta(E - E_\nu)$$

where $\hat{M}^{(IS)}_{\lambda=\mu=0}(el) = \sum_{i=1}^{A} (r^2 Y_{00})_i$ is the isoscalar E0 transition operator

Motivation

Using this approach a lot of papers analyzing centroids of GMR appeared:

- some of the latest:

P. Avogadro, C.A.Bertulani, PRC 88, 044319 (2013)
P.Veselý, J.Toivanen, B.C.Carlsson, J.Dobaczewski, M.Michel, A.Pastore, PRC 86, 024303 (2012)
L.Cao, H.Sagawa, G.Coló, PRC 86, 054313 (2012)
P.Avogadro, T.Nakatsukasa, PRC 87, 014331 (2013)
K.Yoshida, T.Nakatsukasa, PRC88, 034309 (2013)

Analyses performed in these papers (based on the GMR centroids calculated in terms of the RPA) showed that the energy – density - functional (EDF) approaches with the incompressibilities $K_{nm} \approx 230$ MeV give the good agreement with the experimentally determined centroids in ^{208}Pb and ^{144}Sm. However, the experimental data on Sn (see T.Li, U.Garg, et al., PRL 99, 162503 (2007)) and Cd (see D.Patel, et al., Phys.Lett. B 718, 447 (2012)) cannot be reproduced equally well with the same functionals in the comparison with Pb-Sm data.

In papers P.Avogadro, et al., PRC88, 044319 (2013) and P.Veselý, , et al., PRC 86, 024303 (2012) the modification of the pairing interaction was used for the explanation of the problem of the simultaneous reproduction of Sn-Cd and Pb-Sm data.

$$V_{pair}(\vec{r},\vec{r}\,') = V_0\,\delta(\vec{r}-\vec{r}\,') \quad \rightarrow \quad V_{pair}(\vec{r},\vec{r}\,') = V_0\left[1-\eta\left(\frac{\rho(\vec{r})}{\rho_0}\right)^{\gamma}\right]\delta(\vec{r}-\vec{r}\,')$$

$\eta = 0$ - volume pairing

$\eta = 1$ - surface pairing

Motivation

However, these attempts of the solving of the problem of the simultaneous reproduction of Sn-Cd and Pb-Sm data by the new type of the pairing have not helped.

In the paper K.Yoshida, T.Nakatsukasa, PRC88, 034309 (2013) microscopical fully self-consistent Skyrme QRPA analyses of the shape evolution of giant resonances of different types (ISGMR including):

double-peak structure of the GMR in deformed nuclei is caused by the mixing of E0 and E2 modes (the higher peak is a primal ISGMR and the lower peak is induced by the E2-E0 mixing from ISGQR)

in spite of the fact that in this paper the calculated energy distribution of GMR is shown only the comparison of calculated positions (centroids) and widths of the GMR with corresponding experimental values was performed – relatively good agreement for Sm isotopes was obtained

So, in spite of the fact that the experimental energy distributions of the ISGMR are available for $^{144, 154}$Sm the comparison with experimental values was done only for positions (centroids) and widths of the ISGMR (theoretical positions and widths were determined by the fitting of one- (for spherical nuclei) or two- (for deformed nuclei) Lorentzians to the calculated values of the isoscalar E0 excitation probability for individual RPA solutions)

Motivation

There are two main groups in the world providing the data on E0 resonance, namely:

Texas A&M University (TAMU):

D.H.Youngblood, et al., PRC69, 034315 (2004) - ^{116}Sn, ^{208}Pb, ^{144}Sm, ^{154}Sm

D.H.Youngblood, et al., PRC69, 054312 (2004) - ^{90}Zr

D.H.Youngblood, et al., PRC88, 021301(R) (2004) - ^{92}Zr, ^{92}Mo, ^{90}Zr, ^{96}Mo, ^{96}Mo, ^{98}Mo, ^{100}Mo

Research Center for Nuclear Physics (RCNP) at Osaka University

M.Uchida, et al., PRC69, 051301 (2004) - ^{90}Zr, ^{116}Sn, ^{208}Pb

M.Itoh, et al., PRC68, 064602 (2003) - ^{144}Sm, ^{148}Sm, ^{150}Sm, ^{152}Sm, ^{154}Sm

T.Li, et al., PRC99, 162503 (2007) - $^{112\text{-}124}$Sn

All these papers give not only GMR centroids but also shapes of the GMR and both experimental groups used (α,α') reaction for the determination of E0 strength functions. However, in the case when both groups measured E0 strength function for the same nucleus (^{90}Zr, ^{144,154}Sm, ^{208}Pb) one can see substantial differences in the E0 strength functions between both groups (mentioned already in P.Avogadro, et al., PRC88, 044319 (2013)).

Motivation

- In spite of the fact that the experimental shapes of E0 strength functions are available for many spherical and also for several deformed nuclei all papers with theoretical analyses have compared only GMR centroids determined by simple expr. $E_{GMR} = m_1 / m_0$ or widths (determined by the fitting of Lorentzian to calculated values of the excitation probabilities of individual RPA solutions)

- The deeper theoretical analyses of the GMRs were done in the paper K. Yoshida, T.Nakatsukasa, PRC 88, 034309 (2013) with the Skyrme QRPA approach for SkM*, SLy4 and SkP Skyrme interactions (for Sm isotopes) but the comparison with experimental data was done only for positions (centroids) and widths of GMR

⇒ We analyze the shape and position of the GMR from the point of view of the comparison of the experimental values of the ISGMR energy distribution with the calculated values with different Skyrme parametizations for a broad ensemble of Sm, Pb, Sn, Mo isotopes (not only position and width). E0 strength is also determined for some superheavy nuclei.

Deformation effect (double peak structure of the GMR) is illustratively demonstrated in terms of the Separable RPA (SRPA) approach

Energy distribution of the ISGMR in spherical and deformed nuclei is analyzed from the point of view of different Skyrme parametrizations (with different incompressibility modulus)

Theoretical background - SRPA

In this contribution two theoretical approaches are used:

1. separable RPA (sRPA) - 1 code
2. standard RPA (fRPA) - 2 codes < coupled scheme (spherical nuclei); m- scheme (deformed nuclei)

Separable RPA

SRPA = modification of the RPA based on the Skyrme energy functional for axially deformed nuclei using multi-dimensional response approach

V.O.Nesterenko, J.Kvasil, P.-G.Reinhard, PRC66, 044307 (2002) - formulation of SRPA

V.O.Nesterenko, W.Kleinig, J.Kvasil, P.Veselý, P.-G.Reinhard, PRC74, 064306 (2006) - GDR

P.Veselý, J.Kvasil, V.O.Neterenko, W.Kleinig, P.-G.Reinhard, V.Yu.Ponomarev, PRC80, 0313012(R) (2009) - M1 giant resonance

V.O.Nesterenko, J.Kvasil, P.Veselý, W.Kleinig, P.-G.Reinhard, V.Yu.Ponomarev, J. Phys. G37, 064034 (2010) - M1 giant resonance

J.Kvasil, V.O.Nesterenko, W.Kleinig, P.-G.Reinhard, P.Veselý, PRC84, 034303 (2011) - toroidal and compression E1 modes

A.Repko, P.-G.Reinhard, V.O.Nesterenko, J.Kvasil, PRC87, 024305 (2013) - toroidal nature of low-lying E1 modes

J.Kvasil, V.O.Nesterenko, W.Kleinig, D.Božík, P.-G.Reinhard, N.Lo Iudice , Eur. Phys. J. A49, 119 (2013) - toroidal, compression E1 modes

Theoretical background - sRPA

The sRPA starts with the Skyrme energy functional (see Appendix B for details):

$$\mathcal{E}(\rho, \tau, \vec{J}, \vec{j}, \vec{s}, \vec{T}) = \mathcal{E}_{kin} + \mathcal{E}_{Sk} + \mathcal{E}_{pair} + \mathcal{E}_{coul}$$

Basic idea of the sRPA: nucleus is excited by external s.p. fields: $(\hat{Q}_k, \hat{P}_k), k = 1, \ldots, K$

$$\hat{Q}_k^+ = \hat{Q}_k \quad ; \quad T\hat{Q}_k T^{-1} = \hat{Q}_k \quad ; \quad [\hat{H}, \hat{Q}_k] = -i\hat{P}_k$$

$$\hat{P}_k^+ = \hat{P}_k \quad ; \quad T\hat{P}_k T^{-1} = -\hat{P}_k \quad ; \quad [\hat{H}, \hat{P}_k] = -i\hat{Q}_k$$

The optimal set of generators $(\hat{Q}_k, \hat{P}_k)$ was discussed in:

E1 T=1 modes

V.O.Nesterenko, W.Kleinig, J.Kvasil, P.Vesely, P.-G.Reinhard, PRC 74, 064306 (2006)
P.Vesely, J.Kvasil, V.O.Nesterenko, W.Kleinig, P.-G.Reinhard, V.Yu.Ponomarev, PRC. 80, 031302(R) (2009)
J.Kvasil, V.O.Nesterenko, W.Kleinig, P.-G.Reinhard, P.Vesely, PRC 84, 034303 (2011)

M1 modes

M1 modes

Using linear response theory corresponding Hamiltonian is: $\hat{H} = \hat{h}_{HFB} + \hat{V}_{res}$

HFB mean field

separable residual interaction self-consistently obtained from mean field

$$\hat{h}_{HFB} = \int d^3r \sum_{d+} \frac{\partial \mathcal{E}}{\partial J_{d+}(\vec{r})} \hat{J}_{d+}$$

$$\hat{V}_{res} = -\frac{1}{2} \sum_{k,k'=1}^{K} \left(\kappa_{kk'} \hat{X}_k \hat{X}_{k'} + \eta_{kk'} \hat{Y}_k \hat{Y}_{k'} \right)$$

where $\kappa_{kk'}, \eta_{kk'}, \hat{X}_k, \hat{Y}_k$ are given by the 2-d derivatives of the functional $\mathcal{E}$ with respect to densities and currents → no free paramerters except those of the Skyrme functional

Theoretical background – full RPA

Total Hamiltonian consists from the BCS (HFB) mean field and residual interactions:

$$\hat{H} = \hat{h}_{BCS} + \frac{1}{2}\sum_{d\,d'}\int d^3r \int d^3r' \frac{\delta^2\mathcal{E}}{\delta J_d(\vec{r})\,\delta J_{d'}(\vec{r}\,')} : \hat{J}_d(\vec{r})\hat{J}_{d'}(\vec{r}\,') :$$

with the standard full RPA equation

$$\begin{pmatrix} A & B \\ B^* & A^* \end{pmatrix}\begin{pmatrix} c^{(\nu-)} \\ c^{(\nu+)} \end{pmatrix} = \begin{pmatrix} E_\nu & 0 \\ 0 & -E_\nu \end{pmatrix}\begin{pmatrix} c^{(\nu-)} \\ c^{(\nu+)} \end{pmatrix}$$

with

$$A_{ijkl} = \sum_{dd'} \frac{(-1)^{l_k+l_l}}{2\lambda+1}\int_0^\infty \frac{\delta^2\mathcal{E}}{\delta J_d \delta J_{d'}} J^{\lambda}_{d;\,ij}(r)\, J^{\lambda *}_{d';\,kl}(r)\, r^2 dr \;+\; \delta_{ij,kl}\,\varepsilon_{ij}$$

$$B_{ijkl} = \sum_{dd'} \gamma_T^{(J_d)} \frac{(-1)^{l_k+l_l}}{2\lambda+1}\int_0^\infty \frac{\delta^2\mathcal{E}}{\delta J_d \delta J_{d'}} J^{\lambda}_{d;\,ij}(r)\, J^{\lambda *}_{d';\,kl}(r)\, r^2 dr$$

The solving of the sRPA or fRPA equations gives the forward and backward amplitudes $c_{ij}^{(\nu-)}$ and $c_{ij}^{(\nu+)}$ of the phonon creation operator

$$Q_\nu^+(\lambda\mu) = \sum_{i>j}\left(c_{i\,j}^{(\nu-)}\,\alpha_i^+\alpha_j^+ - c_{i\,j}^{(\nu+)}\,\alpha_{\bar{j}}\alpha_{\bar{i}} \right)$$

with corresponding phonon energy E_ν

Theoretical background

Knowing the structure and energies of one-phonon states one can determine the strength function of given transition operator (in our case the monopole electric operator

$$\hat{M}^{(is)}_{\lambda=\mu=0}(el) = \sum_i (r^2 Y_{00})_i$$

with the corresponding energy weighted strength function:

$$S_k(E0; E) = \sum_\nu (E_\nu)^k \ |\langle \nu | \hat{M}^{(is)}_{\lambda=\mu=0}(el) | RPA \rangle|^2 \ \delta(E - E_\nu)$$

$$\longrightarrow \quad S_k(E0; E) = \sum_\nu (E_\nu)^k \ |\langle \nu | \hat{M}^{(is)}_{\lambda=\mu=0}(el) | RPA \rangle|^2 \ \xi_\Delta(E - E_\nu)$$

to simulate escape width and coupling to complex configurations

Where $\xi_\Delta(E - E_\nu)$ is the Lorentz weight function

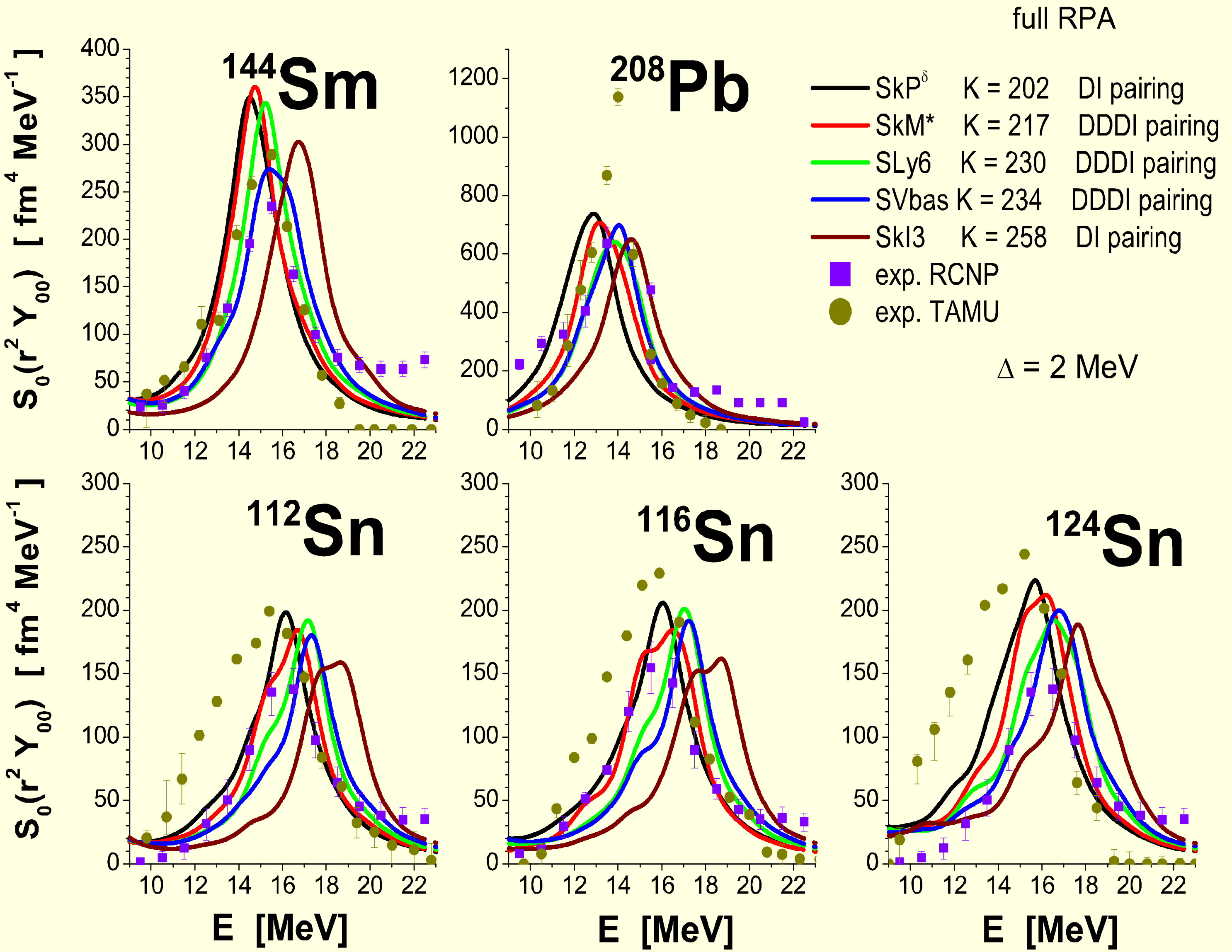
$$\xi_\Delta(E - E_\nu) = \frac{1}{2\pi} \frac{\Delta}{(E - E_\nu)^2 + \frac{\Delta^2}{4}}$$

Dependence of E0 strength function in the spherical nuclei (heavier 208Pb,144Sm; lighter 112, 116, 124Sn) on Skyrme parametrizations with different K

parametrizations with K~230 MeV fits the experimental values for heavier nuclei (Sm, Pb)

experimental values for lighter Sn isotopes require parametrizat-ions with the lower values of K (K~200 MeV like SkP$^{\delta}$)

differences in RCNP and TAMU data



GMR in the isotopes $^{144-154}$Sm

comparison of SRPA results with TAMU and RCNP exp. data

TAMU data renormalized for absolute units:

$$S_0(r^2Y_{00};\ fm^4 MeV^{-1}) = S_1(r^2Y_{00};\ fraction\ EWSR\ MeV^{-1}) \times EWSR_{cl} \times E^{-1}$$

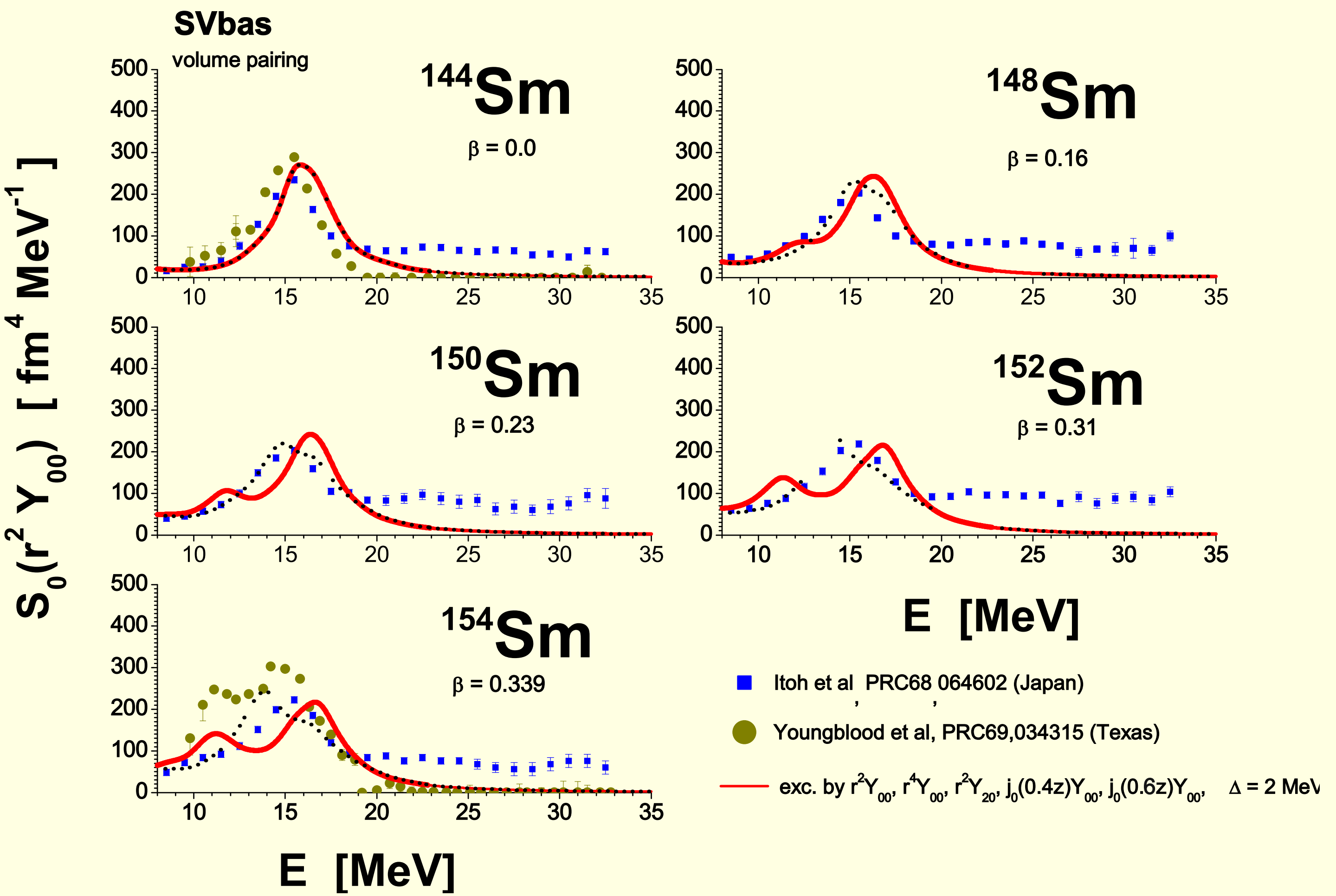
$$EWSR_{cl} = m_1(r^2Y_{00}) = \frac{\hbar^2}{2\pi m} A \langle r^2 \rangle$$

different shapes of exp. GMR for TAMU and RCNP data: much bigger deformation effect in TAMU data

SRPA calculation: nucleus excited by 5 external fields:

$$\sum r^2 Y_{00},\ \sum r^4 Y_{00},\ \sum r^2 Y_{20},\ \sum j_0(0.4z) Y_{00},\ \sum j_0(0.6z) Y_{00}$$

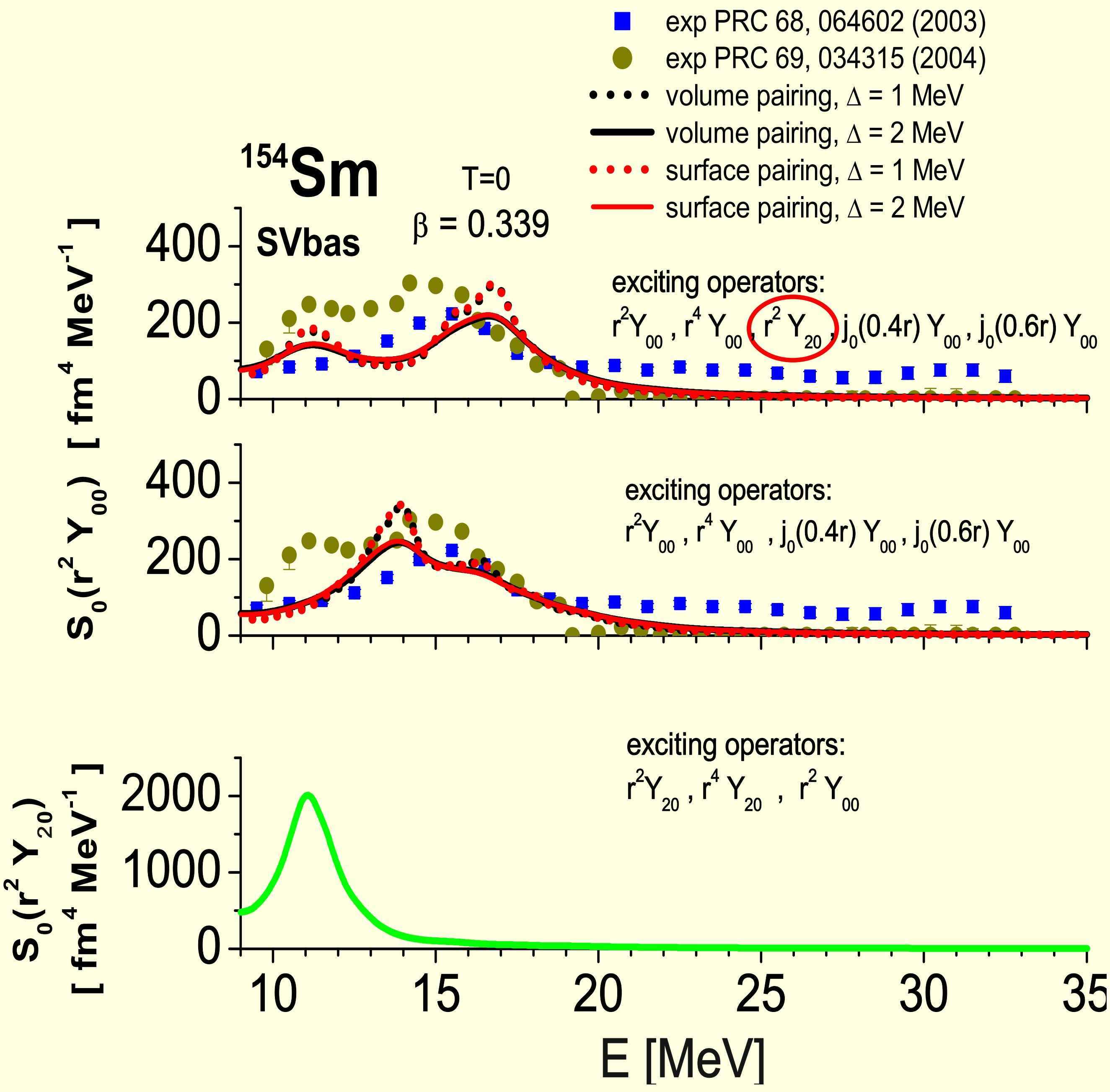
	$EWSR_{cl}$ fm^4 MeV	$EWSR_{RPA}$ fm^4 MeV
^{144}Sm	23158	24306
^{148}Sm	24335	24650
^{150}Sm	25128	25232
^{152}Sm	26782	26990
^{154}Sm	27162	27458



GMR in ^{154}Sm – coupling of E0 and E2 modes

involving the E2 field $\sum(r^2Y_{20})$ among the external exciting fields in the SRPA calculation of GMR improves the agreement with the TAMU GMR data

the volume and surface pairing give practically the same results

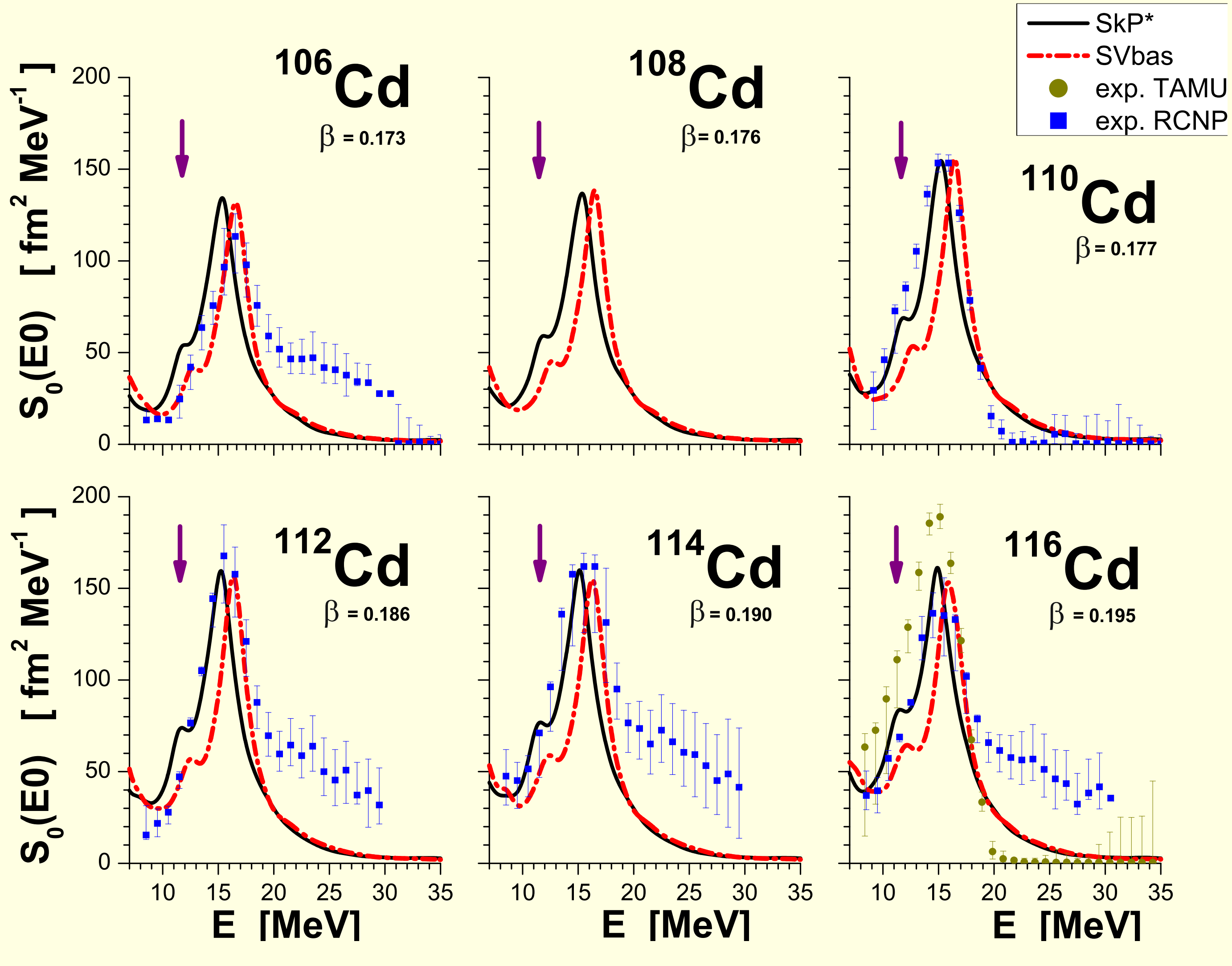


GMR in isotopes $^{106\text{-}116}$Cd

Comparison of the sRPA (vithout and with E0-E2 mixing) with the fRPA results and with experimental data

small discrepancies between sRPA and fRPA

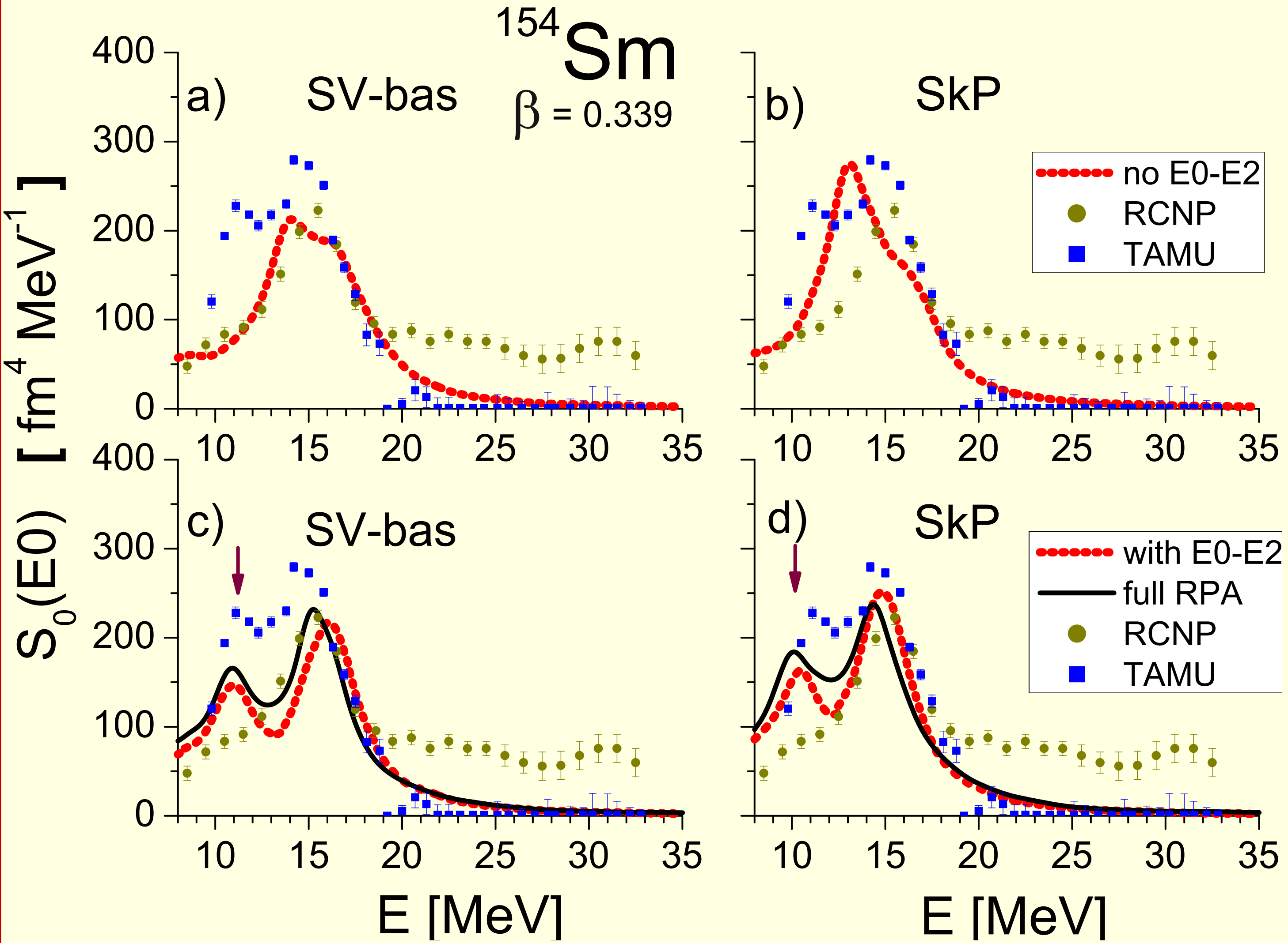
in $^{110\text{-}116}$Cd the fRPA agrees better with the experimental data (at least in the positions of maxima)



Comparison of the GMR in ^{154}Sm calculated with SV-bas and SkP$^{\delta}$ parametrizations in the framework of sRPA (without and with E0-E2 mixing) and fRPA with corresponding experimental data

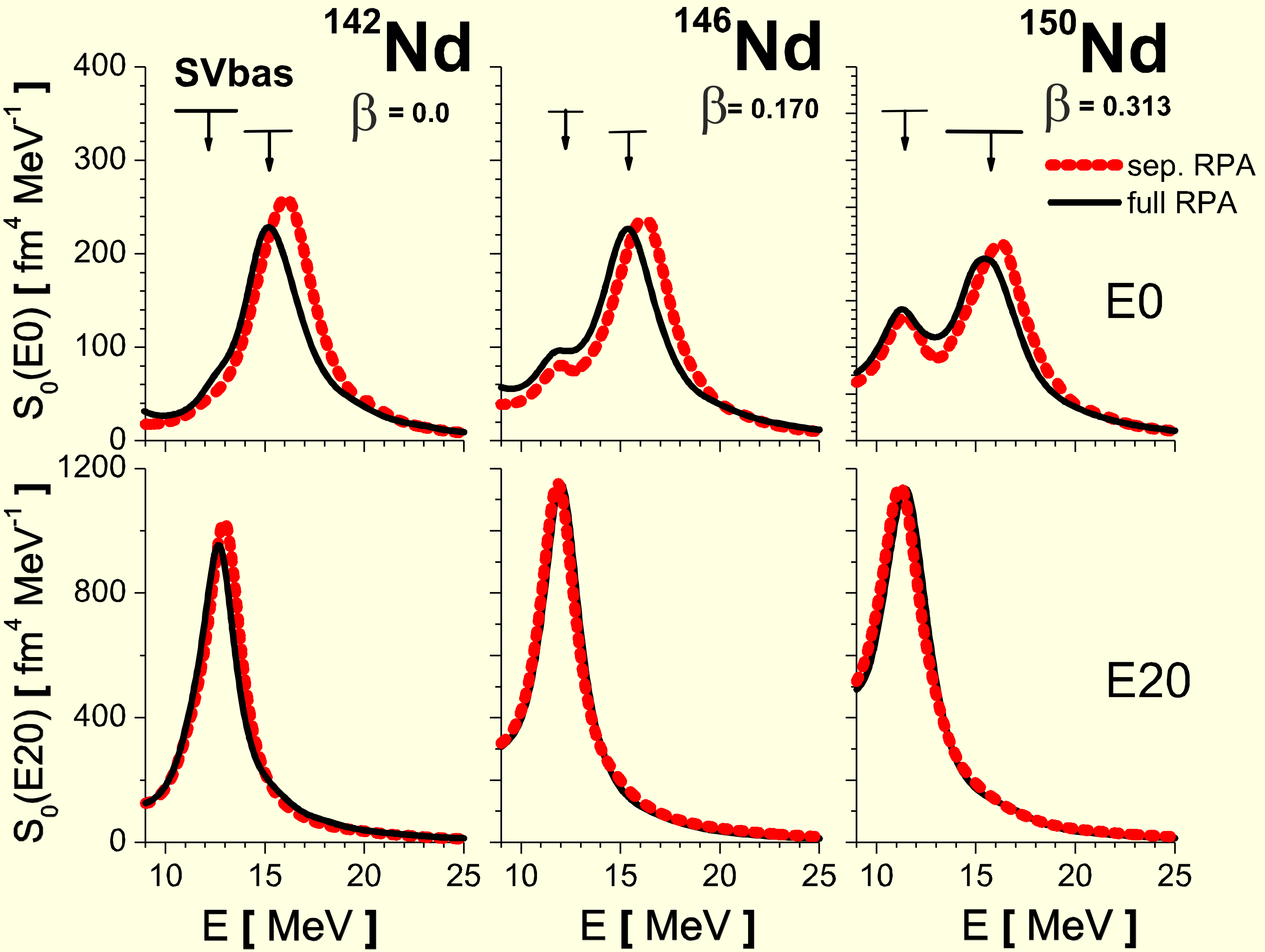
small discrepancies between sRPA and fRPA

the SV-bas Skyrme interaction gives better agreement with experiment



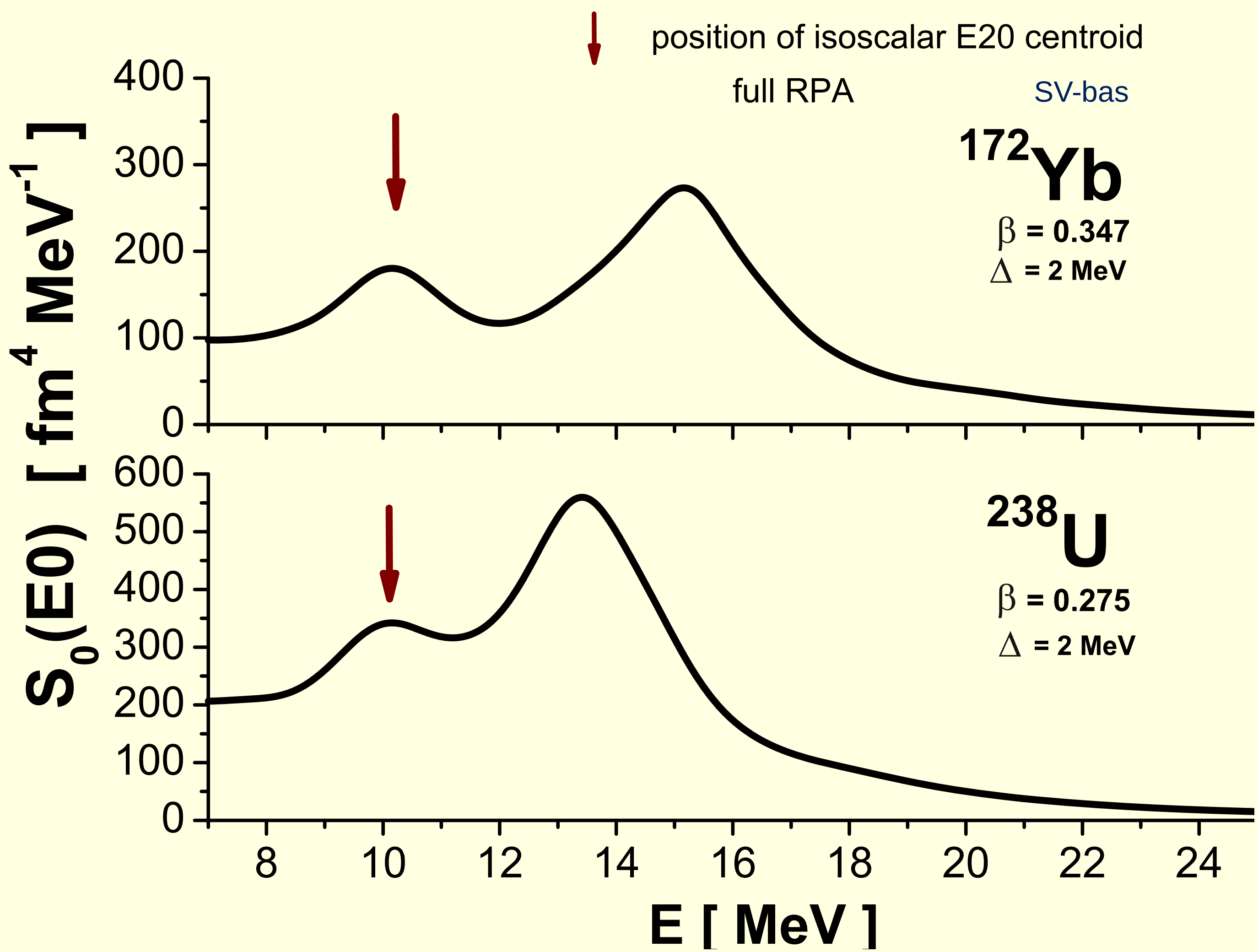
E0 and E2 strength functions in Nd isotopes
(the comparison with experimental GMR centroids)

in the deformed case the position of the 1-st GMR maximum agrees with the position of the maximum of the E2 K=0 strength



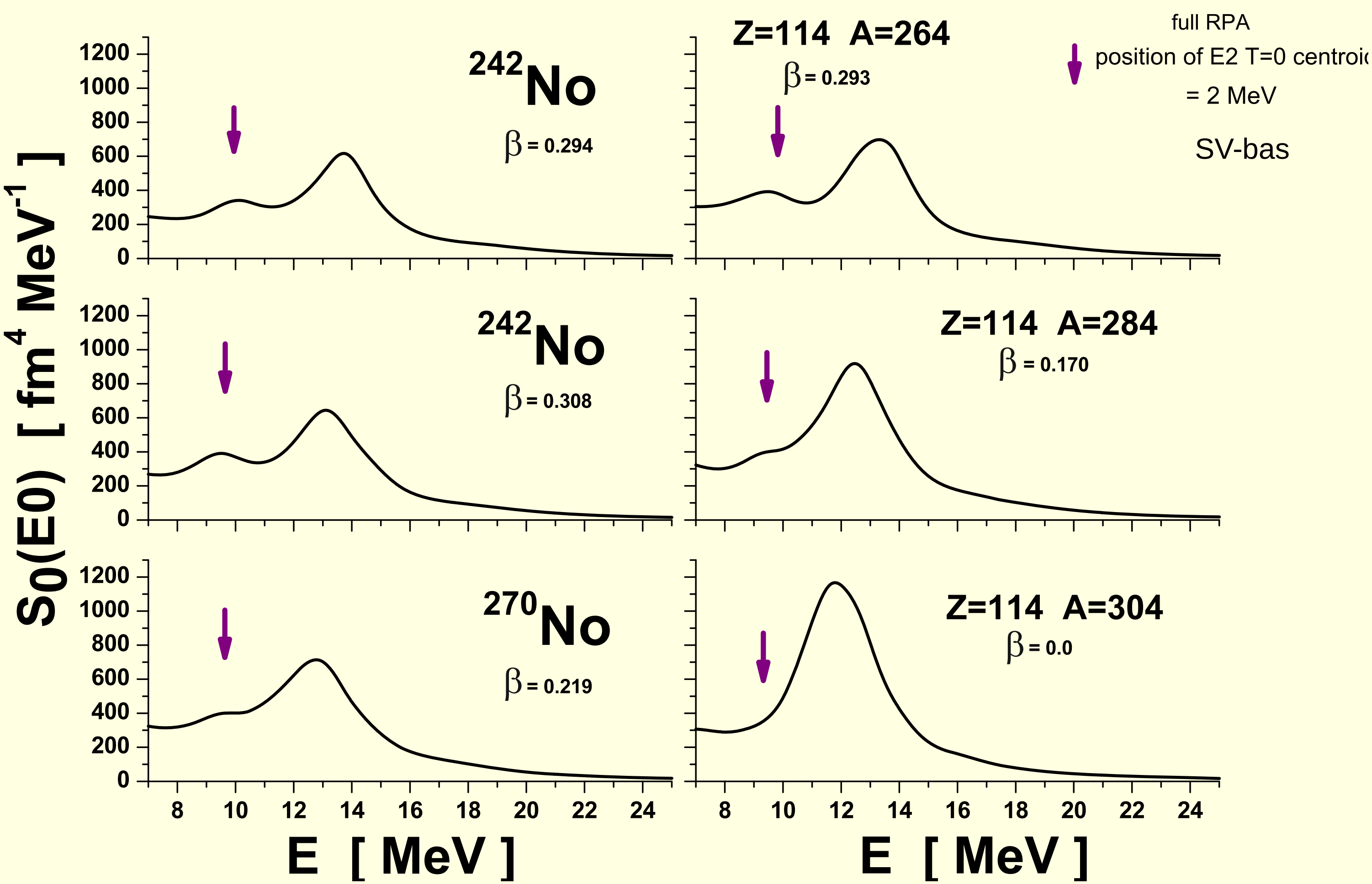
GMR in ^{172}Yb and ^{238}U

the position of the 1-st GMR maximum agrees with the position of the maximum of the E2 K=0 strength



GMR in superheavy nuclei

the position of the 1-st GMR maximum agrees with the position of the maximum of the E2 K=0 strength



Conclusion

Positions of the GMR depends on the incompressibility of the nuclear matter. The better agreement of calculated and exp. GMR values was obtained for parametrizations with K ~ 230 MeV for heavier nuclei (^{144}Sm, ^{208}Pb) while with K ~ 200 MeV for lighter nuclei (^{112,116,124}Sn)

Significant deformation effect observed in the TAMU GMR data is in the agreement with the SRPA and fRPA results (double-peak structure of the GMR in deformed nuclei)

Double-peak structure of the GMR for deformed nuclei is caused by the coupling of E2 and E0 modes for nonzero deformation (original idea – U.Garg, et al., PRC29, 93 (1984))

There are discrepancies between TAMU and RCNP experimental data from the point of view of the shape of GMR

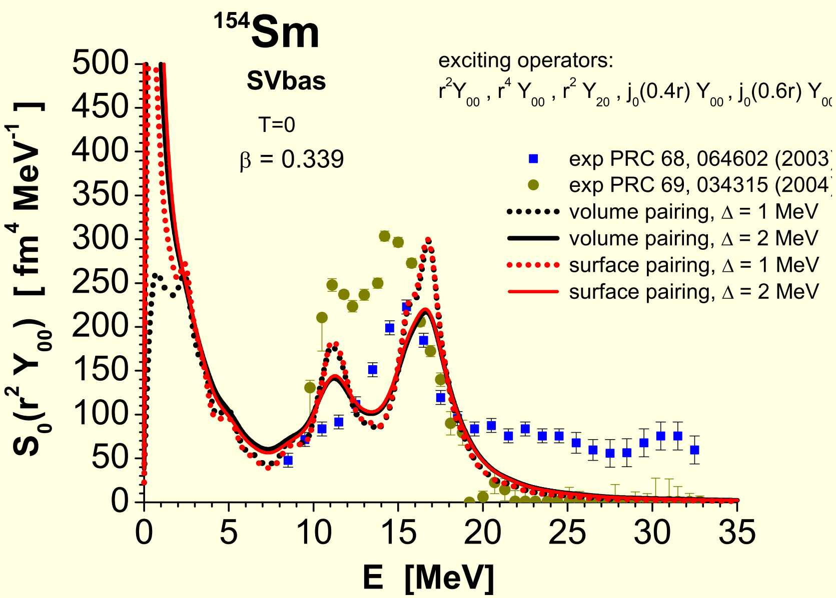
Volume and surface pairing give the similar results (confirmation of previous results)

Theoretical analyses and comparison with exp. values of the GMR cannot be restricted only on resonance centroids (the shape of E0 strength function is important)

Thank you for your attention

Appendix A: Spurious mode connected with the number of particles

In the energy interval E > 8 MeV the influence of the spurious mode is very small



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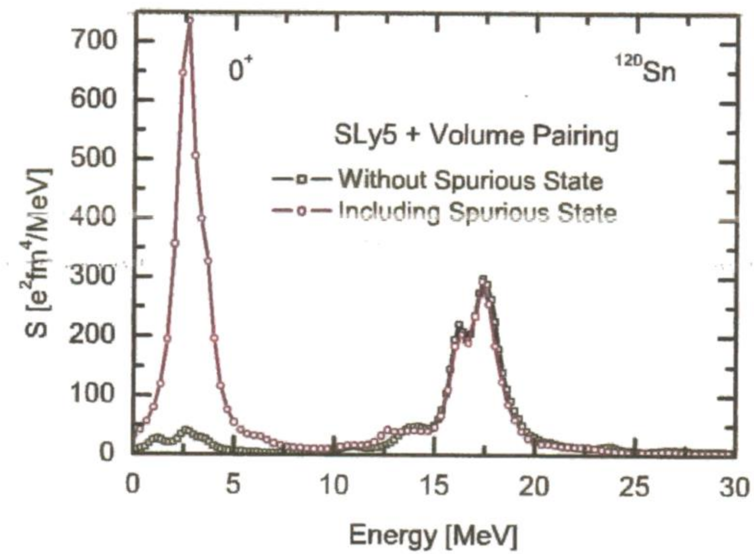

FIG. 2. (Color online) The isoscalar 0^+ strength function (averaged with Lorentzians having 1 MeV width) including spurious state (line with circles) or not (line with squares) in ^{120}Sn using the Skyrme force SLy5 and the volume pairing force.

Appendix b: Brief formulation of the SRPA approach

(more details)

Starting energy functional:

$$E\left(J^{\alpha}_{\tau}(t)\right) = <\Psi(t)|\hat{H}|\Psi(t)> = \int \mathcal{H}(\vec{r},t)\, d^3r$$

where $|\Psi(t)>$ is the time-dependent Slater determinant or time-dependent quasiparticle vacuum. Time-dependent densities and currents are: $J^{\alpha}_{\tau}(\vec{r},t) = <\Psi(t)|\hat{J}^{\alpha}_{\tau}(\vec{r})|\Psi(t)>$

Time-dependent Slater determinant $|\Psi(t)>$ can be related to the equilibrium Slater determinant $|>$ by (see E.Lipparini, S.Stringari, Nucl.Phys. A371, 430 (1981))

$$|\Psi(t)> = \prod_{\tau=n,p}\prod_{k=1}^{K} \exp\left[-i(q_{\tau k}(t) - <q_{\tau k}>)\hat{Q}_{\tau k}\right]\exp\left[-ip_{\tau k}(t)\hat{P}_{\tau k}\right]|>$$

with T-even generators $\hat{Q}_{\tau k}$ and T-odd generators $\hat{P}_{\tau k}$:

$$\hat{Q}^{+}_{\tau k} = \hat{Q}_{\tau k} \quad ; \quad T\hat{Q}_{\tau k}T^{-1} = \hat{Q}_{\tau k} \quad ; \quad [\hat{H}, \hat{Q}_{\tau k}] = -i\hat{P}_{\tau k}$$

$$\hat{P}^{+}_{\tau k} = \hat{P}_{\tau k} \quad ; \quad T\hat{P}_{\tau k}T^{-1} = -\hat{P}_{\tau k} \quad ; \quad [\hat{H}, \hat{P}_{\tau k}] = -i\hat{Q}_{\tau k}$$

$q_{\tau k}(t)$ and $p_{\tau k}(t)$ are T-even and T-odd periodically time- dependent deformations, respectively:

$$q_{\tau k}(t) = <\Psi(t)|\hat{Q}_{\tau k}|\Psi(t)>$$

$$<q_{\tau k}> = <|\hat{Q}_{\tau k}|>$$

$$p_{\tau k}(t) = <\Psi(t)|\hat{P}_{\tau k}|\Psi(t)>$$

$$<p_{\tau k}> = <|\hat{P}_{\tau k}|> = 0$$

The equilibrium Slater determinant (HF ground state) is given by the HF equation which gives also the HF mean field:

$$\hat{h}_0(\vec{r}) = \sum_{\alpha,\tau} \frac{\partial E}{\partial J_\tau^\alpha(\vec{r})} \hat{J}_\tau^\alpha(\vec{r}) \qquad \hat{h}_0 = \int \hat{h}_0(\vec{r})\, d^3 r$$

In the small amplitude limit (up to the linear order in the deformations $q_{\tau k}(t)$ and $p_{\tau k}(t)$ the time- dependent Slater determinant is:

$$|\Psi(t)> \approx |> + |\delta\Psi(t)>$$

$$|\delta\Psi(t)> = -i \sum_{k\tau} \left[(q_{\tau k}(t) - <q_{\tau k}>)\, \hat{P}_{\tau k} + p_{\tau k}(t)\, \hat{Q}_{\tau k} \right] |>$$

Therefore for the time-dependence of densities:

$$J^{\alpha}_{\tau}(\vec{r},t) \approx J^{\alpha}_{\tau}(\vec{r}) + \delta J^{\alpha}_{\tau}(\vec{r},t)$$

with

$$\delta J^{\alpha}_{\tau}(\vec{r},t) = <\Psi(t)|\hat{J}^{\alpha}_{\tau}(\vec{r})|\Psi(t)> - <|\hat{J}^{\alpha}_{\tau}(\vec{r})|> =$$

$$= -i\sum_{\tau k}\left\{(q_{\tau k}(t) - <q_{\tau k}>) <|\left[\hat{P}_{\tau k}, \hat{J}^{\alpha}_{\tau}(\vec{r})\right]|> + p_{\tau k}(t) <|\left[\hat{Q}_{\tau k}, \hat{J}^{\alpha}_{\tau}(\vec{r})\right]|>\right\}$$

and similarly for time-dependent (vibrating) s.p. mean field:

$$\hat{h}(\vec{r},t) \approx \hat{h}_0(\vec{r}) + \hat{h}_{res}(\vec{r},t)$$

where $\hat{h}_0(\vec{r})$ is the static HF mean field and time- dependent vibrating part is

$$\hat{h}_{res}(\vec{r},t) = \sum_{\alpha'\tau'}\left[\frac{\partial \hat{h}_0}{\partial J^{\alpha'}_{\tau'}(\vec{r})}\right] \delta J^{\alpha'}_{\tau'}(\vec{r},t) =$$

$$= \int d^3r' \sum_{\alpha\tau}\sum_{\alpha'\tau'}\left[\frac{\partial^2 E}{\partial J^{\alpha}_{\tau}(\vec{r})\,\partial J^{\alpha'}_{\tau'}(\vec{r}')}\right] \delta J^{\alpha'}_{\tau'}(\vec{r},t)\ \hat{J}^{\alpha}_{\tau}(\vec{r}') =$$

$$= \sum_{\tau k}\left\{(q_{\tau k}(t) - <q_{\tau k}>)\ \hat{X}_{\tau k}(\vec{r})\ +\ p_{\tau k}(t)\,\hat{Y}_{\tau k}(\vec{r})\right\}$$

$$\hat{h}_{res}(t) = \int d^3r\ \hat{h}_{res}(\vec{r},t)$$

where

$$< |[\hat{A}_+ , \hat{B}_+]| > = < |[\hat{A}_- , \hat{B}_-]| > = 0$$

$$< |[\hat{A}_+ , \hat{B}_-]| > \neq 0 \qquad \text{where} \qquad T\,\hat{A}_\pm\, T^{-1} = \pm \hat{A}_\pm$$

$$\hat{X}_{\tau k}(\vec{r}) = \sum_{\tau'} \hat{X}^{\tau'}_{\tau k}(\vec{r}) =$$

$$= i \int d^3 r' \sum_{\tau' \alpha_+ \alpha'_+} \left[\frac{\partial^2 E}{\partial J^{\alpha'_+}_{\tau'}(\vec{r}\,')\, \partial J^{\alpha_+}_{\tau}(\vec{r})} \right] < |[\hat{P}_{\tau k}, \hat{J}^{\alpha'_+}_{\tau}(\vec{r}\,')]| > \hat{J}^{\alpha_+}_{\tau'}(\vec{r})$$

where α_+ enumerates T- even densities

α_- enumerates T- odd densities

$k = 1, \cdots, K \qquad \tau = n, p$

$$\hat{Y}_{\tau k}(\vec{r}) = \sum_{\tau'} \hat{Y}^{\tau'}_{\tau k}(\vec{r}) =$$

$$= i \int d^3 r' \sum_{\tau' \alpha_- \alpha'_-} \left[\frac{\partial^2 E}{\partial J^{\alpha'_-}_{\tau'}(\vec{r}\,')\, \partial J^{\alpha_-}_{\tau}(\vec{r})} \right] < |[\hat{Q}_{\tau k}, \hat{J}^{\alpha'_-}_{\tau}(\vec{r}\,')]| > \hat{J}^{\alpha_-}_{\tau'}(\vec{r})$$

with $\hat{X}_{\tau k} \equiv \int \hat{X}_{\tau k}(\vec{r})\; d^3 r \qquad \hat{Y}_{\tau k} \equiv \int \hat{Y}_{\tau k}(\vec{r})\; d^3 r$

and $T\,\hat{X}_{\tau k}\, T^{-1} = \hat{X}_{\tau k} \qquad T\,\hat{Y}_{\tau k}\, T^{-1} = -\hat{Y}_{\tau k}$

Similarly as for $\hat{J}_{\tau}^{\alpha}$ we can write for the time-dependent variations of $\hat{X}_{\tau k}$ and $\hat{Y}_{\tau k}$

$$< \delta \hat{X}_{\tau k}(t) > = < \Psi(t) | \hat{X}_{\tau k} | \Psi(t) > - < | \hat{X}_{\tau k} | > =$$

$$= \sum_{\tau' k'} (q_{\tau' k'}(t) - < q_{\tau' k'} >) \, \kappa^{-1}_{\tau k, \tau' k'}$$

$$< \delta \hat{Y}_{\tau k}(t) > = < \Psi(t) | \hat{Y}_{\tau k} | \Psi(t) > - < | \hat{Y}_{\tau k} | > =$$

$$= \sum_{\tau' k'} p_{\tau' k'}(t) \, \eta^{-1}_{\tau k, \tau' k'}$$

where we introduced inverse strength constants $\kappa^{-1}_{\tau k, \tau' k'}$ and $\eta^{-1}_{\tau k, \tau' k'}$

$$\kappa^{-1}_{\tau k, \tau' k'} = \kappa^{-1}_{\tau' k', \tau k} \equiv i < | [\hat{P}_{\tau' k'} , \hat{X}_{\tau k}] | > =$$

$$= \int d^3 r \int d^3 r' \sum_{\alpha_+ \alpha'_+} < | [\hat{P}_{\tau k} , \hat{J}_{\tau}^{\alpha_+}(\vec{r})] | > [\frac{\partial^2 E}{\partial J_{\tau}^{\alpha_+}(\vec{r}\,') \, \partial J_{\tau'}^{\alpha'_+}(\vec{r})}] < | [\hat{P}_{\tau' k'} , \hat{J}_{\tau'}^{\alpha'_+}(\vec{r}\,')] | >$$

$$\eta^{-1}_{\tau k, \tau' k'} = \eta^{-1}_{\tau' k', \tau k} \equiv i < | [\hat{Q}_{\tau' k'} , \hat{P}_{\tau k}] | > =$$

$$= \int d^3 r \int d^3 r' \sum_{\alpha_- \alpha'_-} < | [\hat{Q}_{\tau k} , \hat{J}_{\tau}^{\alpha_-}(\vec{r})] | > [\frac{\partial^2 E}{\partial J_{\tau}^{\alpha_-}(\vec{r}\,') \, \partial J_{\tau'}^{\alpha'_-}(\vec{r})}] < | [\hat{Q}_{\tau' k'} , \hat{J}_{\tau'}^{\alpha'_-}(\vec{r}\,')] | >$$

For the determination of the vibrating shifts $q_{\tau k}(t) - < q_{\tau k} >$ and $p_{\tau k}(t)$:

$$q^{\nu}_{\tau k}(t) - < q_{\tau k} > = \bar{q}^{\nu}_{\tau k} \cos(\omega t) = \frac{1}{2} \bar{q}^{\nu}_{\tau k} (e^{i\omega t} + e^{-i\omega t})$$

$$p^{\nu}_{\tau k}(t) = \bar{p}^{\nu}_{\tau k} \sin(\omega t) = \frac{1}{2} \bar{p}^{\nu}_{\tau k} (e^{i\omega t} - e^{-i\omega t})$$

the TDHF or TDHFB approach can be used starting from the Thouless theorem for the the vibrating Slater determinant:

$$|\Psi(t)>_{\nu} = \exp\Big(\sum_{\varpi = ij, \bar{i}\bar{j}, i\bar{j}} c^{(\nu)}_{\varpi}(t) b^{+}_{\varpi} \Big) |> \approx \Big(1 + \sum_{\varpi = ij, \bar{i}\bar{j}, i\bar{j}} c^{(\nu)}_{\varpi}(t) b^{+}_{\varpi} \Big) |>$$

$$c^{(\nu)}_{\varpi}(t) = c^{(\nu)+}_{\varpi} e^{i\omega t} + c^{(\nu)-}_{\varpi} e^{-i\omega t}$$

where $b^{+}_{ij}, b^{+}_{\bar{i}\bar{j}}, b^{+}_{i\bar{j}}$ are two-quasiparticle quasi-boson operators:

$$b^{+}_{ij} = \alpha^{+}_{i} \alpha^{+}_{j} \qquad b^{+}_{\bar{i}\bar{j}} = \alpha^{+}_{\bar{i}} \alpha^{+}_{\bar{j}} \qquad b^{+}_{i\bar{j}} = \alpha^{+}_{i} \alpha^{+}_{\bar{j}}$$

with

$$< | [b_{ij}, b^{+}_{i'j'}] | > = \delta_{ii'} \delta_{jj'} - \delta_{ij'} \delta_{ji'} \qquad < | [b_{\bar{i}\bar{j}}, b^{+}_{\bar{i}'\bar{j}'}] | > = \delta_{ii'} \delta_{jj'} - \delta_{ij'} \delta_{ji'}$$

$$< | [b_{i\bar{j}}, b^{+}_{i'\bar{j}'}] | > = \delta_{ii'} \delta_{jj'}$$

Using

$$i\frac{d}{dt}|\Psi(t)>_\nu = [\,\hat{h}_0 + \hat{h}_{res}(t)\,]|\Psi(t)>_\nu$$

we can express $c_\varpi^\pm = c_\varpi^\pm(\bar{q}_{\tau k}, \bar{p}_{\tau k})$ to obtain alternative expressions for $_\nu\!<\delta X_{\tau k}(t)>_\nu$ and $_\nu\!<\delta Y_{\tau k}(t)>_\nu$ through $c_\varpi^\pm(\bar{q}_{\tau k}, \bar{p}_{\tau k})$. By comparison with previous ones we finallly have a system of equations for unknown amplitudes $\bar{q}_{\tau k}^\nu$, $\bar{p}_{\tau k}^\nu$

$$\sum_{\tau' k'}\left\{ \bar{q}_{\tau' k'}^\nu \left[F_{\tau' k',\tau k}^{(XX)} - \kappa_{\tau' k',\tau k}^{-1}\right] + \bar{p}_{\tau' k'}^\nu F_{\tau' k',\tau k}^{(XY)} \right\} = 0$$

$$\sum_{\tau' k'}\left\{ \bar{q}_{\tau' k'}^\nu F_{\tau' k',\tau k}^{(YX)} + \bar{p}_{\tau' k'}^\nu \left[F_{\tau' k',\tau k}^{(YY)} - \eta_{\tau' k',\tau k}^{-1}\right] \right\} = 0$$

$$k, k' = 1, \cdots, K \qquad \tau, \tau' = n, p$$

where we introduced following matrices:

$$F^{(AA)}_{\tau' k', \tau k} = \sum_{\bar{\tau}} \sum_{\substack{\varpi \in \bar{\tau} \\ \varpi = ij, i\bar{j}, \bar{i}\bar{j}}} \frac{\varepsilon_{\varpi} < \varpi | \hat{A}^{\bar{\tau}}_{\tau' k'} | > < \varpi | \overset{\frown}{A}^{\bar{\tau}}_{\tau k} | >}{\varepsilon^2_{\varpi} - \omega^2_{\nu}} \qquad \hat{A} = \hat{X} \text{ or } \hat{Y}$$

$$F^{(AB)}_{\tau' k', \tau k} = F^{(BA)}_{\tau k, \tau|' k'} = \sum_{\bar{\tau}} \sum_{\substack{\varpi \in \bar{\tau} \\ \varpi = ij, i\bar{j}, \bar{i}\bar{j}}} \frac{\omega_{\nu} < \varpi | \hat{A}^{\bar{\tau}}_{\tau' k'} | > < \varpi | \overset{\frown}{A}^{\bar{\tau}}_{\tau k} | >}{\varepsilon^2_{\varpi} - \omega^2_{\nu}}$$

where ε_{ϖ} are two-quasiparticle energies:

$$\varepsilon_{\varpi} = \begin{cases} E_i + E_j & \text{for } \varpi = ij \\ E_i + E_{\bar{j}} & \text{for } \varpi = i\bar{j} \\ E_{\bar{i}} + E_{\bar{j}} & \text{for } \varpi = \bar{i}\bar{j} \end{cases}$$

- the matrix of the eq. system for $\bar{q}^{\nu}_{\tau k}$ and $\bar{p}^{\nu}_{\tau k}$ is symmetric and real
- this eq. system has nontrivial solution only if the determinat of its matrix is zero, $F(\omega_\nu) = \det F(\omega_\nu) = 0$ - dispersion equation for ω_ν

It can be shown that the eq. system for $\bar{q}^{\nu}_{\tau k}$ and $\bar{p}^{\nu}_{\tau k}$ is the same as that one obtained from the standard RPA equations:

$$[\hat{H}_{RPA},\ \hat{O}^{+}_{\nu}] = \omega_{\nu}\ \hat{O}^{+}_{\nu} \qquad [\hat{H}_{RPA},\ \hat{O}_{\nu}] = -\omega_{\nu}\ \hat{O}_{\nu}$$

$$[\hat{O}_{\nu}, \hat{O}^{+}_{\nu'}] = \delta_{\nu\nu'}$$

with the RPA Hamiltonian:

$$\hat{H}_{RPA} = \hat{h}_0 + \hat{V}^{(sep)}_{res}$$

where $\hat{h}_0$ is the HF average field and $\hat{V}^{(sep)}_{res}$ is the residual interaction (see p. 3):

$$\hat{V}^{(sep)}_{res} = -\frac{1}{2}\sum_{\tau k}\sum_{\tau' k'}\left\{\kappa_{\tau k,\tau' k'}\ \hat{X}^{(1)}_{\tau k}\ \hat{X}^{(1)}_{\tau k} + \eta_{\tau k,\tau' k'}\ \hat{Y}^{(1)}_{\tau k}\ \hat{Y}^{(1)}_{\tau k}\right\}$$

$$k, k' = 1, \cdots, K$$

$\hat{X}^{(1)}_{\tau k}, \hat{Y}^{(1)}_{\tau k} \longrightarrow$ p-h (two-qp) part of corresponding operator where Q_ν^+ is the phonon creation operator:

$$Q_\nu^+ = \sum_{\substack{\tau=n,p;\varpi\in\tau \\ \varpi=ij,i\bar{j},\bar{i}\bar{j}}} \left\{ \psi_\varpi^{(\nu,\tau)} b_\varpi^+ - \phi_\varpi^{(\nu,\tau)} b_\varpi \right\}$$

with two-quasiparticle amplitudes:

$$\psi_\varpi^{(\nu,\tau)} = 4\,\xi_\varpi \frac{\sum_{\tau'k'} \bar{q}_{\tau'k'}^{(\nu)} < \varpi \,|\, \widehat{X}_{\tau'k'}^{\tau} \,|> - i \sum_{\tau'k'} \bar{p}_{\tau'k'}^{(\nu)} < \varpi \,|\, \widehat{Y}_{\tau'k'}^{\tau} \,|>}{\varepsilon_\varpi - \omega_\nu}$$

$$\phi_\varpi^{(\nu,\tau)} = 4\,\xi_\varpi \frac{\sum_{\tau'k'} \bar{q}_{\tau'k'}^{(\nu)} < \varpi \,|\, \widehat{X}_{\tau'k'}^{\tau} \,|>^* + i \sum_{\tau'k'} \bar{p}_{\tau'k'}^{(\nu)} < \varpi \,|\, \widehat{Y}_{\tau'k'}^{\tau} \,|>^*}{\varepsilon_\varpi + \omega_\nu}$$

where

$$\xi_\varpi = \begin{cases} 1/4 & \text{for } \varpi = ij \\ 1/4 & \text{for } \varpi = i\bar{j} \\ 1/2 & \text{for } \varpi = \bar{i}\bar{j} \end{cases}$$

Advantages of the SRPA:

- instead of the construction and diagonalization of huge matrices it is sufficient to solve the system of eqs. for $\bar{q}^{\nu}_{\tau k}$ and $\bar{p}^{\nu}_{\tau k}$ with matrix of the dimension 4K (K is the number of (Q_k, P_k) modes)

- in the opposite to the standard separable interaction RPA the SRPA method gives the receipt for the determination of the strength constant $\kappa_{\tau k, \tau' k'}$ and $\eta_{\tau k, \tau' k'}$.

- only a few correctly chosen exciting modes (Q_k, P_k) are sufficient for the description of each giant resonance of given type and multipolarity