Nuclear clustering and excitations in the EDF approach

J.-P. Ebran, E. Khan, T. Niksic, D. Vretenar, Nature 487(2012)341
PRC 87(2013)044307
PRC 89(2014)031303(R)
PRC 90(2014)054329

1) Unified microscopic approach

2) Clusters predictions

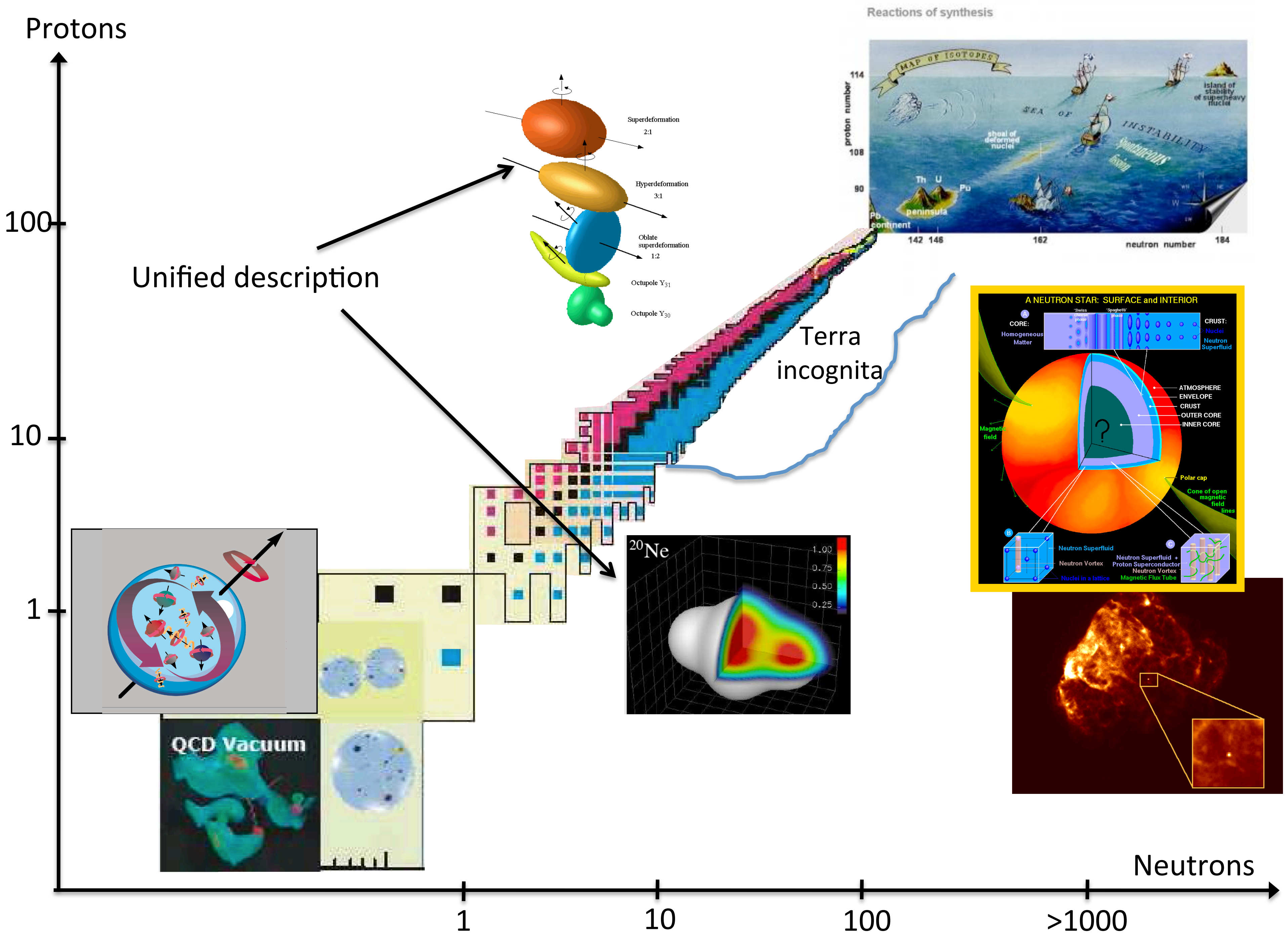
3) Excitations: towards comparison with exp.

4) Deeper understanding of cluster phenomenon ?

Elias Khan

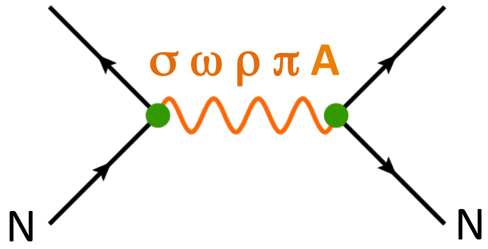
Comex5, 14-18 September 2015, Krakow

1) Unified microscopic approach



Protons
100
10
1
Neutrons
1
10
100
>1000
Unified description
Terra incognita
Reactions of synthesis
MAP OF ISOTOPES
SEA OF INSTABILITY
island of stability of superheavy nuclei
shoal of deformed nuclei
peninsula
Pb continent
Th
U
Pu
proton number
neutron number
114
108
90
142
146
162
184
Superdeformation 2:1
Hyperdeformation 3:1
Oblate superdeformation 1:2
Octupole Y_{31}
Octupole Y_{30}
^{20}Ne
1.00
0.75
0.50
0.25
A NEUTRON STAR: SURFACE and INTERIOR
CORE:
Homogeneous Matter
CRUST:
Neutron Superfluid
ATMOSPHERE
ENVELOPE
CRUST
OUTER CORE
INNER CORE
Polar cap
Cone of open magnetic field lines
Neutron Superfluid
Neutron Vortex
Neutron Superfluid + Proton Superconductor
Neutron Vortex
Magnetic Flux Tube
QCD Vacuum

Relativistic EDF in nuclei



$$\mathcal{L}_{int} = -g_\sigma(\rho_v)\bar{\psi}\sigma\psi - g_\omega(\rho_v)\bar{\psi}\gamma_\mu\omega^\mu\psi - g_\rho(\rho_v)\bar{\psi}\gamma_\mu\vec{\rho}^\mu.\vec{\tau}\psi - \frac{f_\pi(\rho_v)}{m_\pi}\bar{\psi}\gamma_5\gamma_\mu\partial^\mu\vec{\pi}.\vec{\tau}\psi - e\bar{\psi}\gamma_\mu A^\mu\psi$$

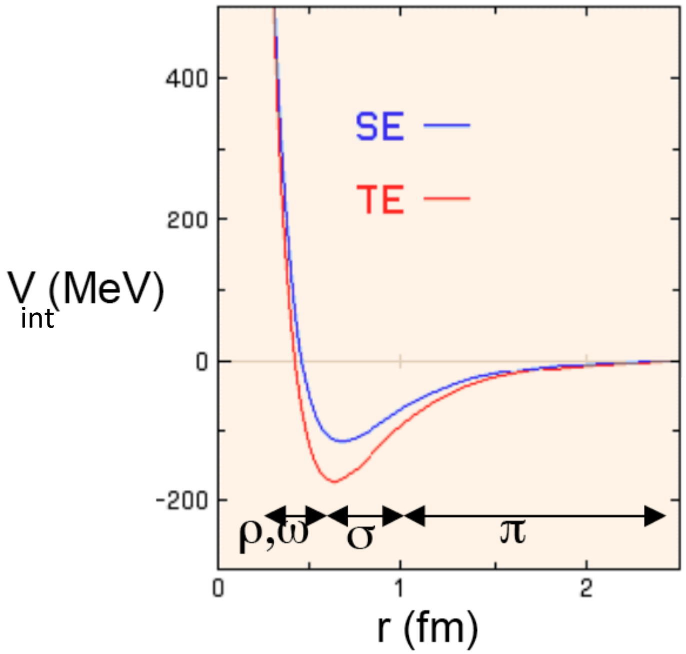
EDF $[\rho\ ;\ \sigma,\omega,\rho,\pi,A]$

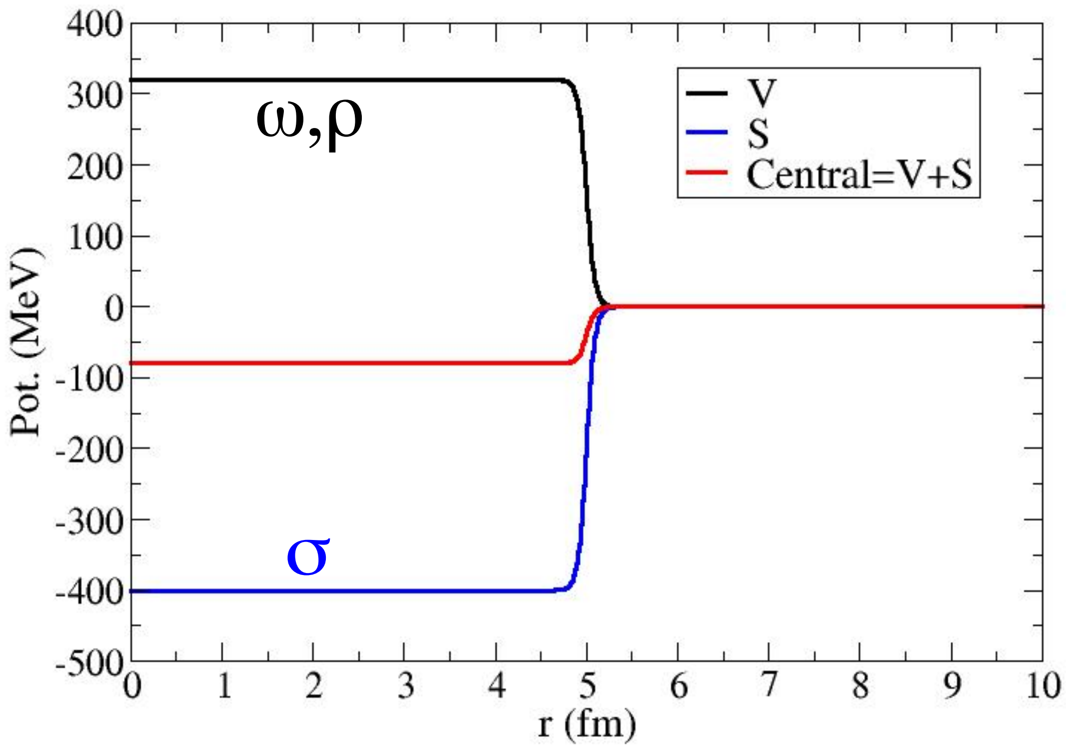
$$\left\{ \boldsymbol{p}\frac{1}{2\tilde{M}(r)}\boldsymbol{p} + W(r) + V_{ls}(r)\boldsymbol{l}.\boldsymbol{s} \right\} \varphi_i = \varepsilon_i^{NR}\varphi_i$$

$$W(r) = [V+S](r)$$

$$V_{ls}(r) = \frac{1}{2\tilde{M}^2(r)}\frac{1}{r}\frac{d}{dr}(V-S)$$

V and S potentials





EDF method & clusters

- EDF: many-body system mapped into the **one-body density** and its powers, gradient

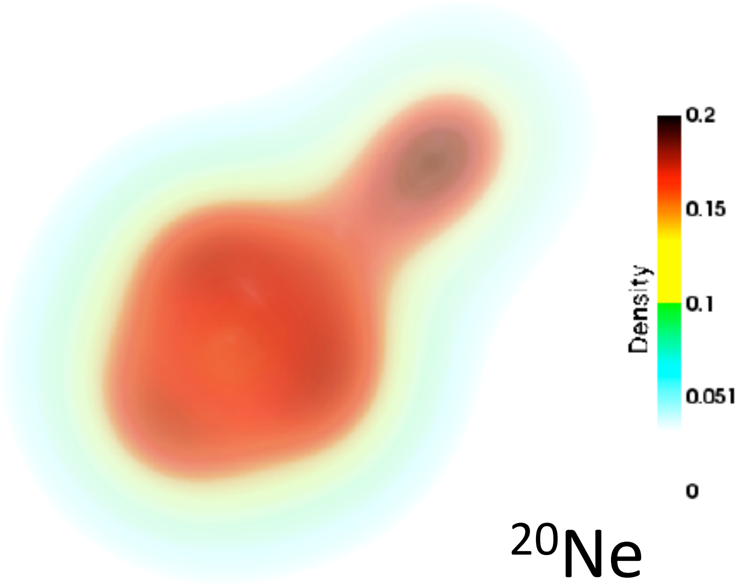
$$
\begin{aligned}
\rho_0(\mathbf{r}) &= \rho_0(\mathbf{r},\mathbf{r}) = \sum_{\sigma\tau} \rho(\mathbf{r}\sigma\tau;\mathbf{r}\sigma\tau) \\
\rho_1(\mathbf{r}) &= \rho_1(\mathbf{r},\mathbf{r}) = \sum_{\sigma\tau} \rho(\mathbf{r}\sigma\tau;\mathbf{r}\sigma\tau)\,\tau \\
\mathbf{s}_0(\mathbf{r}) &= \mathbf{s}_0(\mathbf{r},\mathbf{r}) = \sum_{\sigma\sigma'\tau} \rho(\mathbf{r}\sigma\tau;\mathbf{r}\sigma'\tau)\,\boldsymbol{\sigma}_{\sigma'\sigma} \\
\mathbf{s}_1(\mathbf{r}) &= \mathbf{s}_1(\mathbf{r},\mathbf{r}) = \sum_{\sigma\sigma'\tau} \rho(\mathbf{r}\sigma\tau;\mathbf{r}\sigma'\tau)\,\boldsymbol{\sigma}_{\sigma'\sigma}\,\tau
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{j}_T(\mathbf{r}) &= \tfrac{i}{2}(\nabla' - \nabla)\,\rho_T(\mathbf{r},\mathbf{r}')\big|_{\mathbf{r}=\mathbf{r}'} \\
\mathcal{J}_T(\mathbf{r}) &= \tfrac{i}{2}(\nabla' - \nabla)\otimes \mathbf{s}_T(\mathbf{r},\mathbf{r}')\big|_{\mathbf{r}=\mathbf{r}'} \\
\tau_T(\mathbf{r}) &= \nabla\cdot\nabla'\,\rho_T(\mathbf{r},\mathbf{r}')\big|_{\mathbf{r}=\mathbf{r}'} \\
\mathbf{T}_T(\mathbf{r}) &= \nabla\cdot\nabla'\,\mathbf{s}_T(\mathbf{r},\mathbf{r}')\big|_{\mathbf{r}=\mathbf{r}'}
\end{aligned}
$$

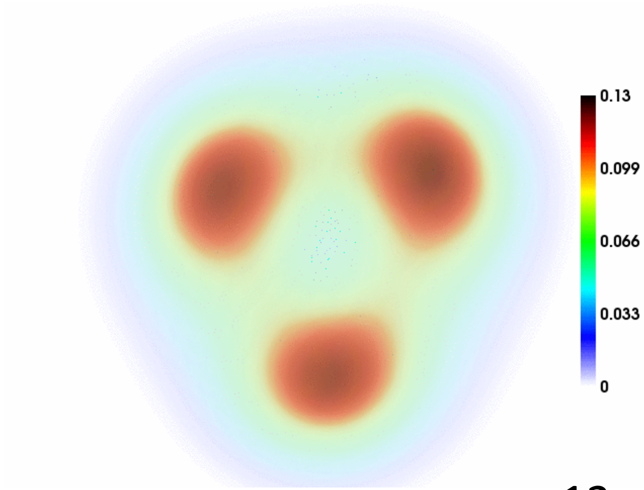
- **Most general** antisymmetrised product of nucleonic wavefunctions
- **Not any a priori assumption** on the nucleons' wave function
- **Correlations** beyond the mean-field effectively included by the EDF
- Results are obtained in the **intrinsic** frame of the nucleus
- Investigate nuclear structure on the **whole nuclear chart**
- **Relativistic**: the depth of the central potential is **consistently predicted**

2) Predictions

Quadrupole + octupole deformations



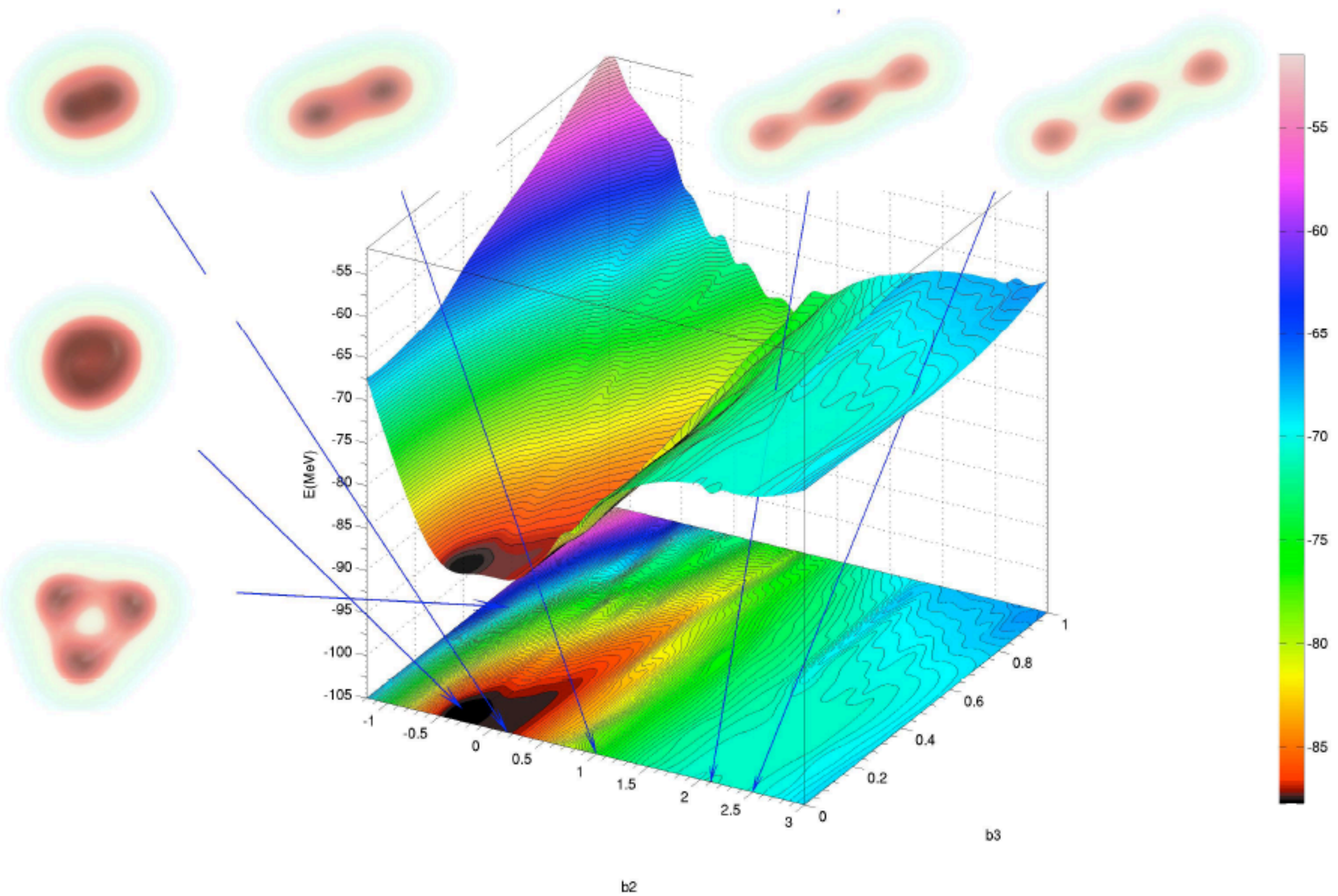
^{20}Ne



^{12}C

Constrained RHB (DDME2)
β_2, β_3, parity proj.

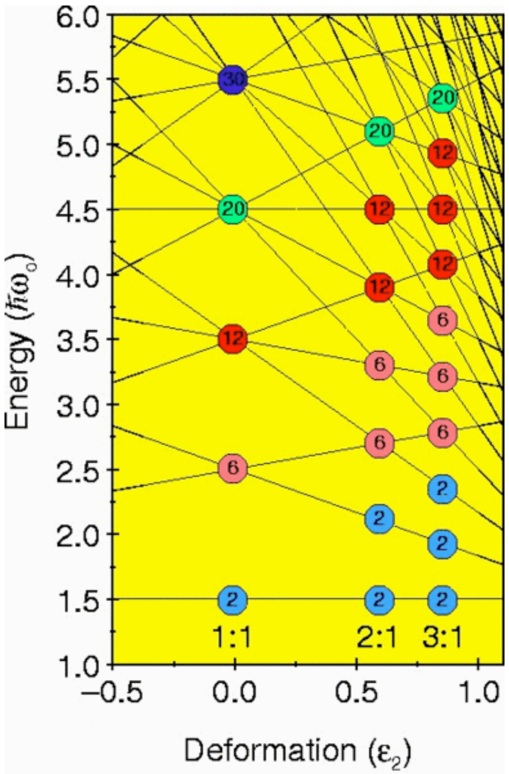
Parity-projected quadrupole/octupole results



$^{12}\mathrm{C}\ (K^{\pi} = 0^{+})\ \mathrm{PAV}$

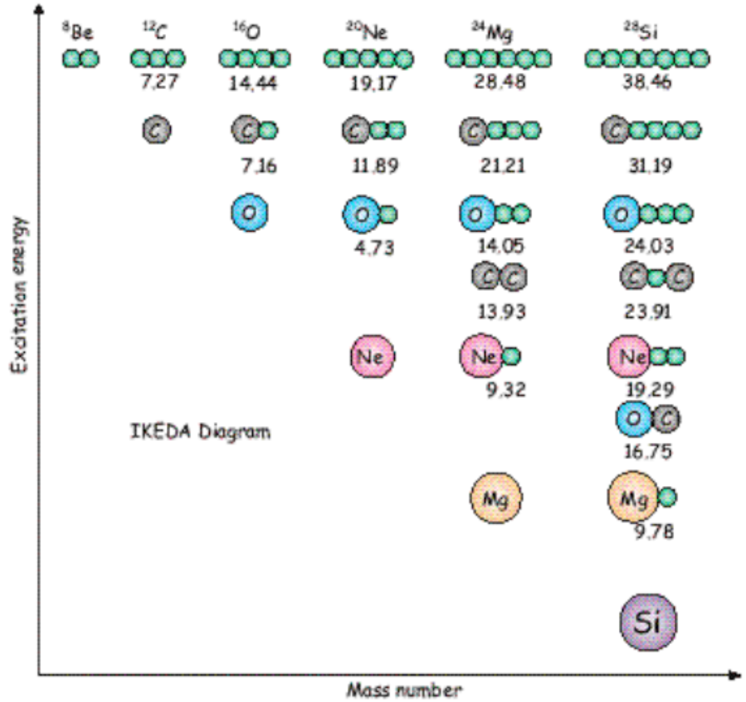
Towards a global picture

deformation and degeneracy



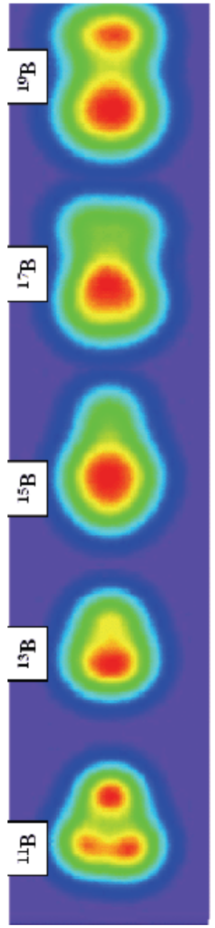
Von Oertzen, Freer, Kanada-En'yo,
Phys. Rep. 432(2006)43

excitations



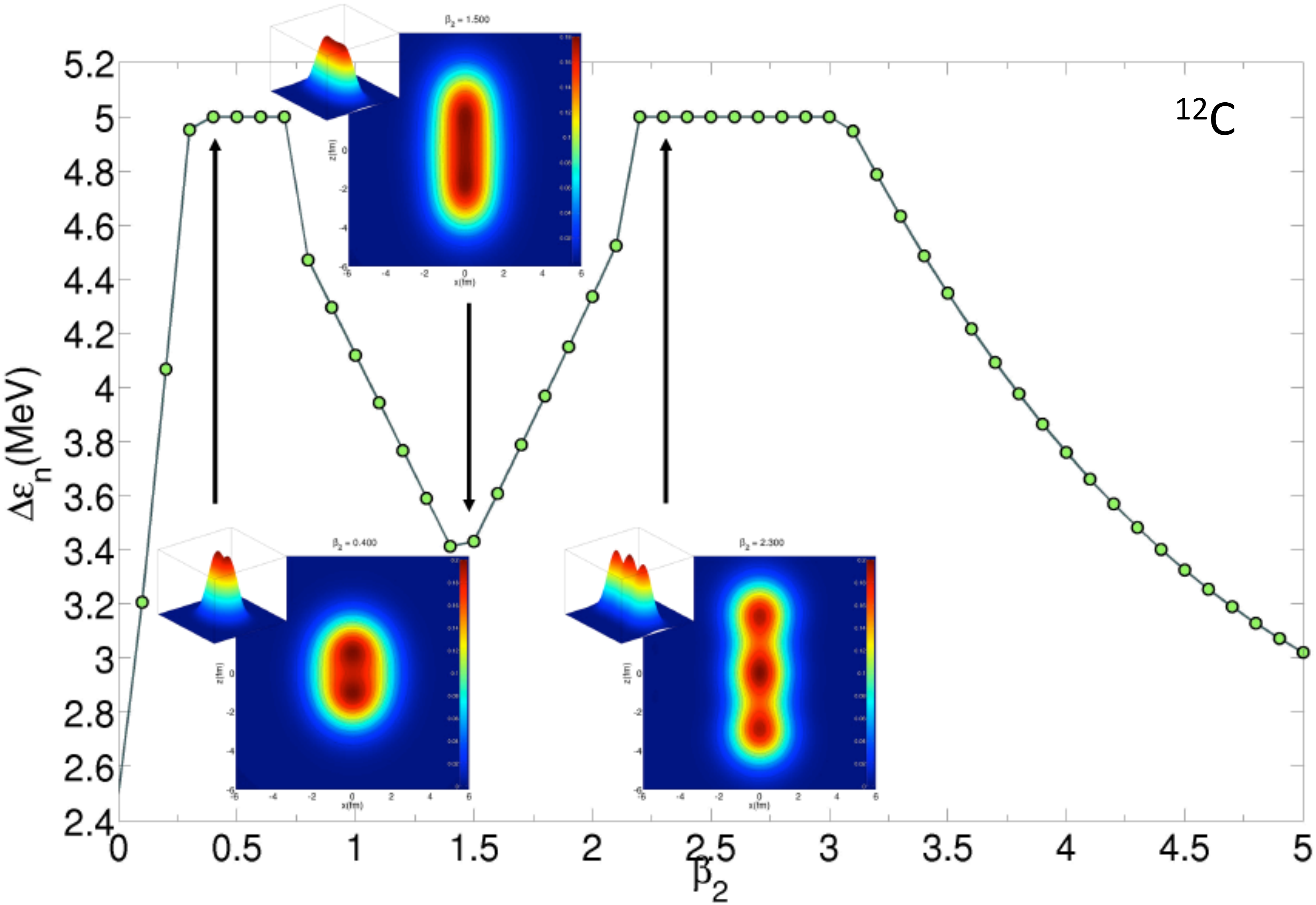
Ikeda, Tagikawa, Horiuchi,
Prog. Theor. Phys. 464(1968)464

neutron excess

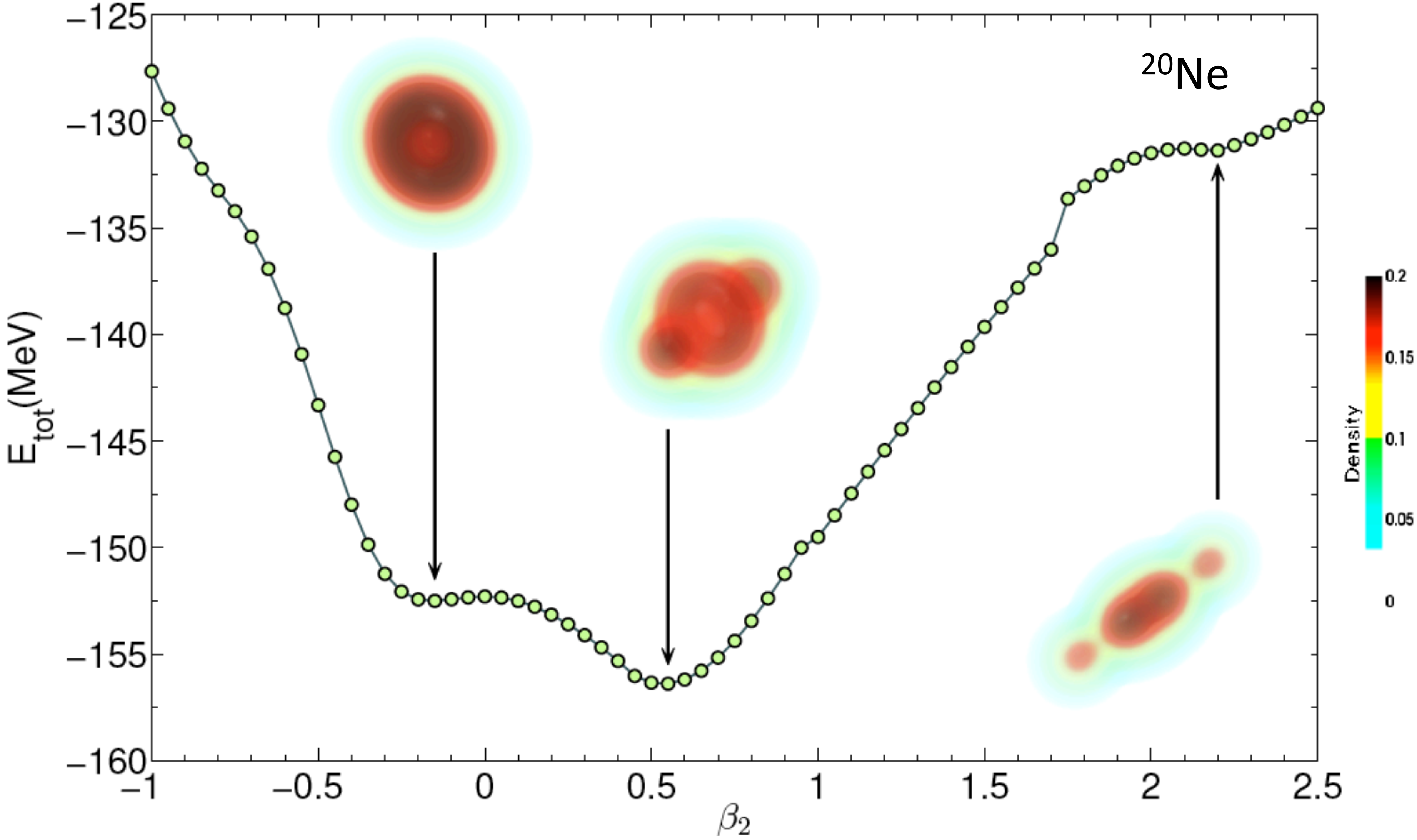


Kanada-En'yo, Horiuchi,
PRC 52(1995)647

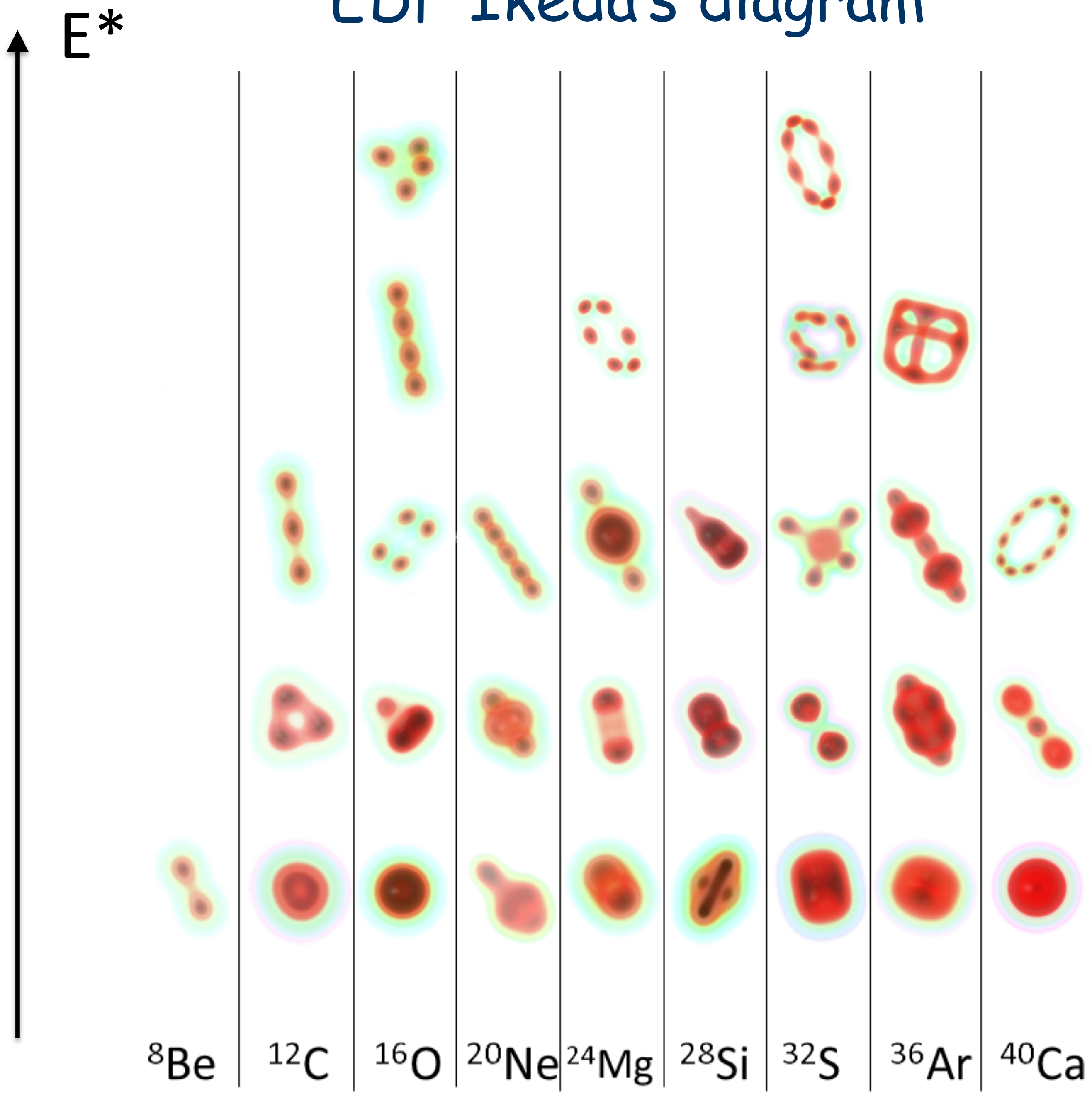
Effect of the deg. raising



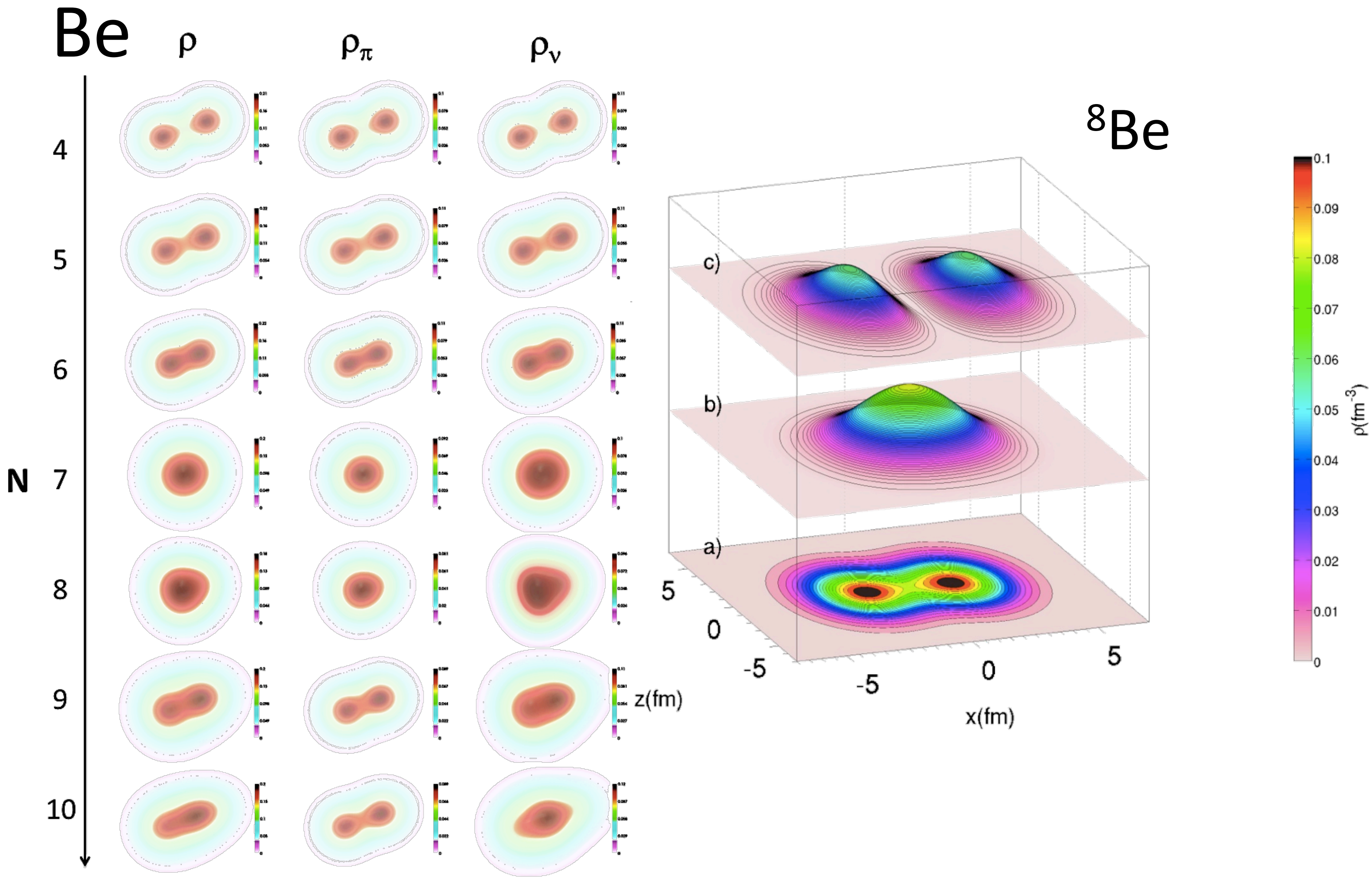
Effect of deformation & excitation



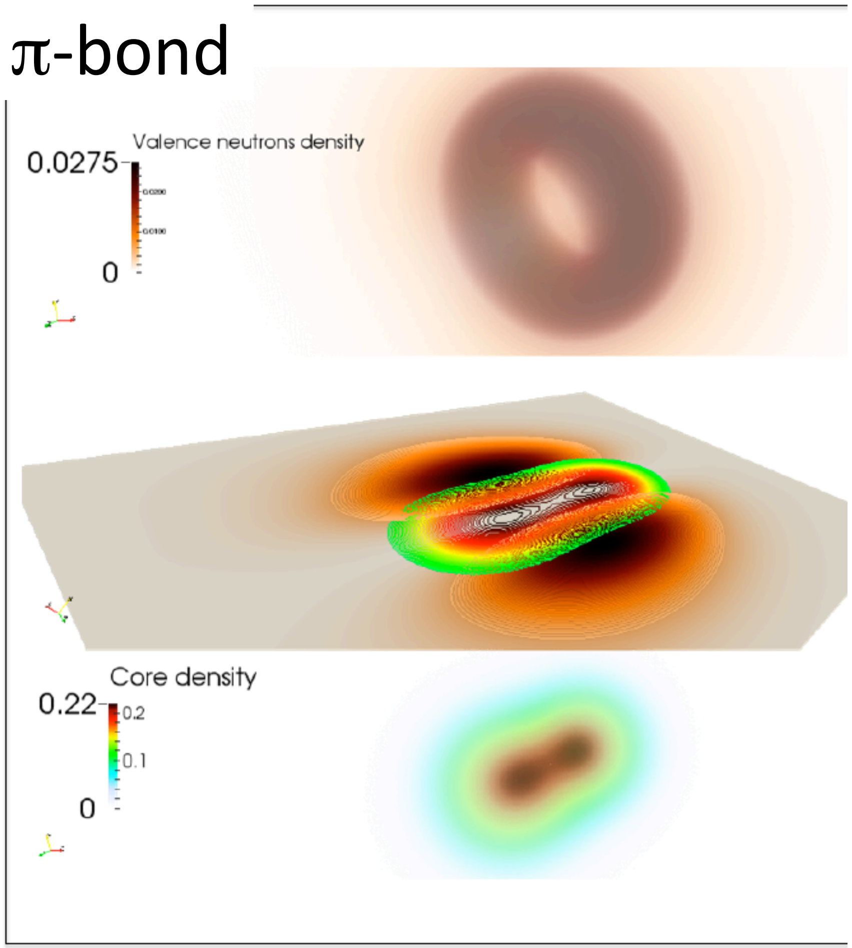
EDF Ikeda's diagram



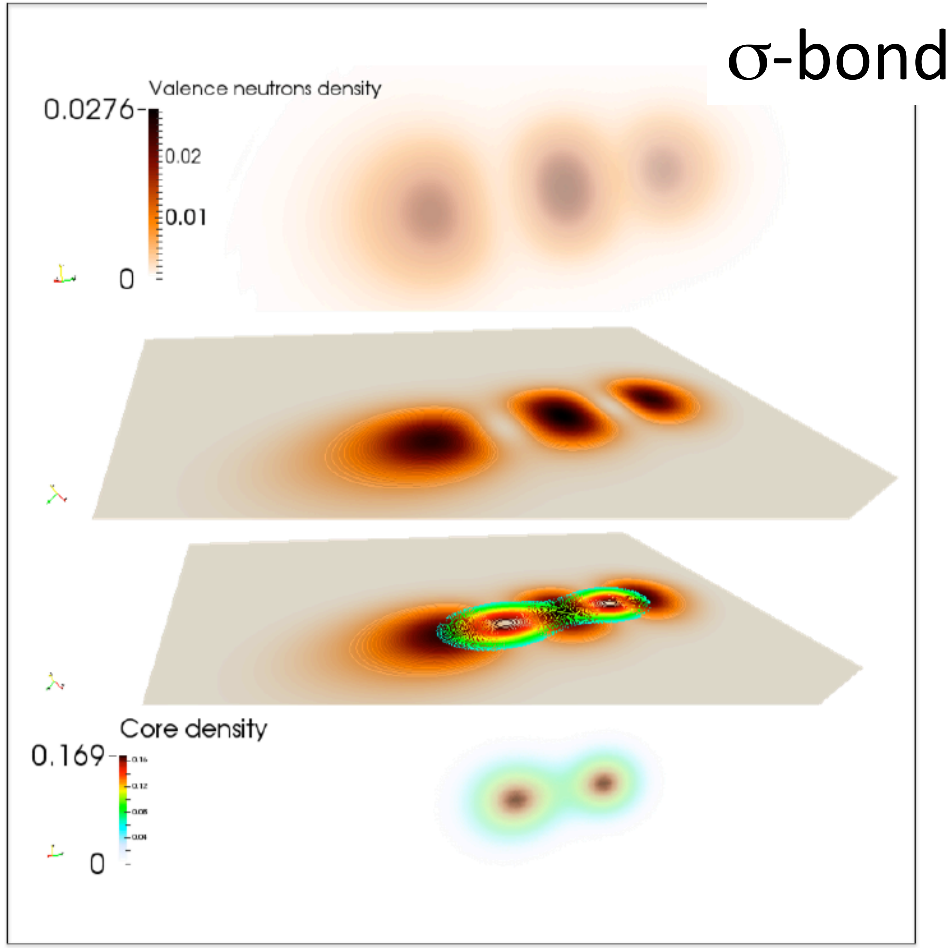
Isotopic dependence



n valence molecular bond



^{10}Be g.s.



^{10}Be exc.

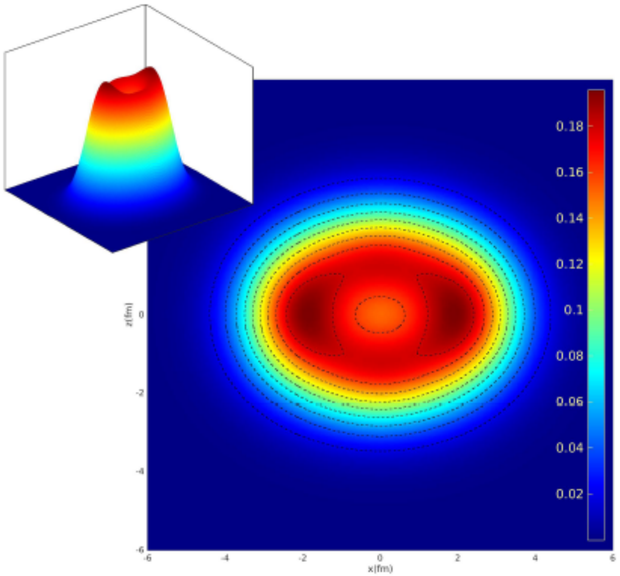
Beyond pairing: Quarteting

R. Lasseri, N. Sandulescu

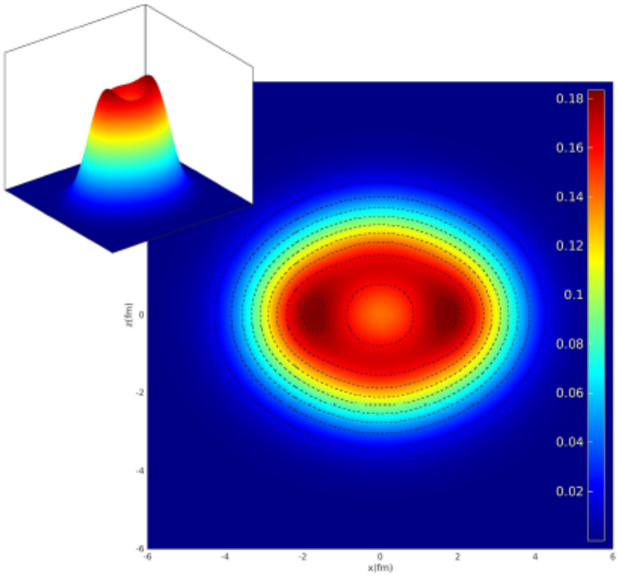
$$\hat{H} = \sum_i \epsilon_i (N_i^\nu + N_i^\pi) + \sum_{i,j} V_{ij} \sum_{\tau=0,\pm 1} P^\dagger_{i,\tau} P_{j,\tau}$$

(see PRL115(2015)112501, Sept. 9)

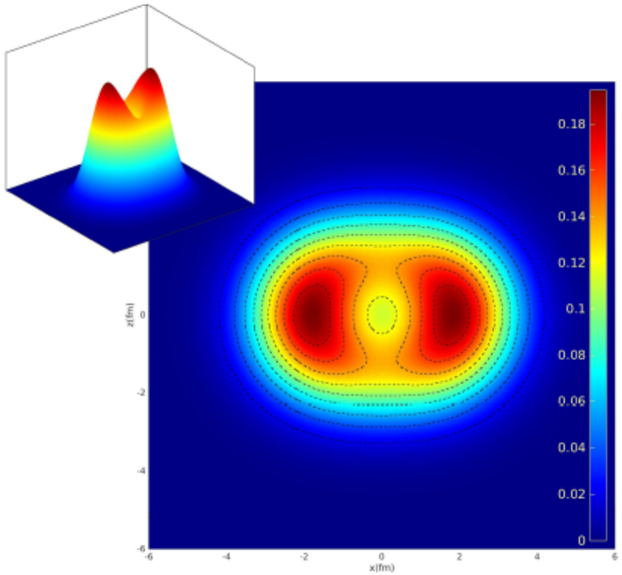
^{24}Mg



RMF



+Pairing



+Quarteting

3) Excitations:
towards comparison with experiment

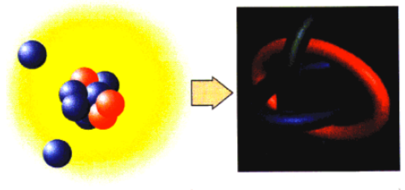
Excitations modes as clustering signature

- Relativistic + deformation: RQRPAz
- Vibration + rotations: collective Bohr Hamiltonian
- Correlations: IBM mapping

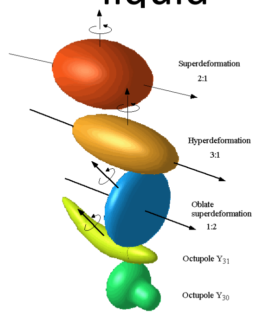
4) Deeper understanding of the cluster phenomenon

Nuclear states

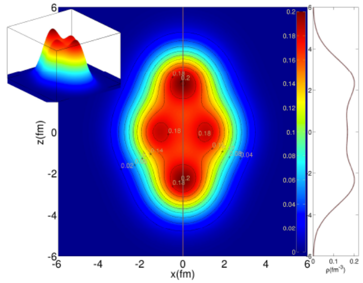
Halo



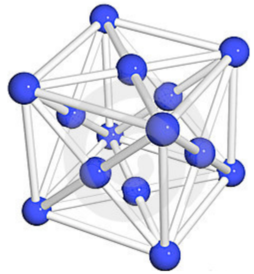
Quantum liquid

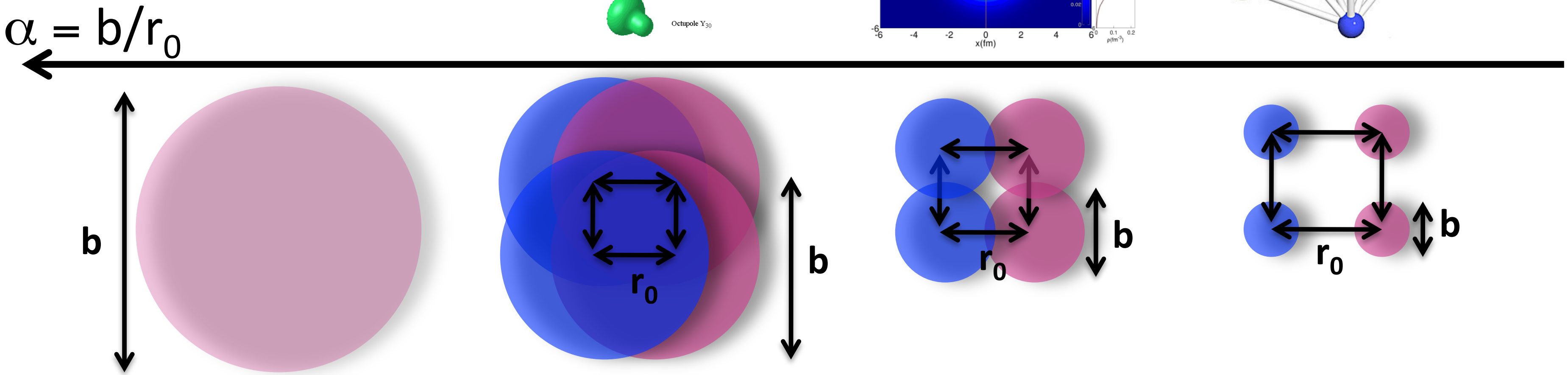


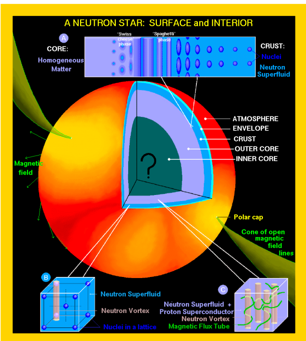
Cluster

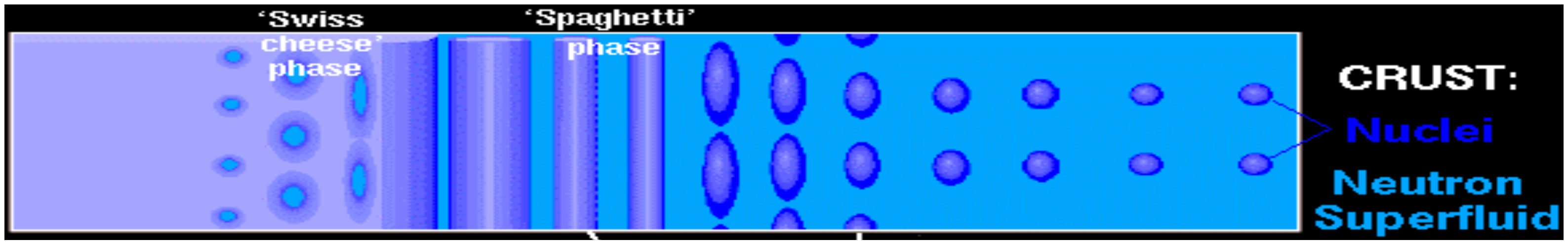


Crystal

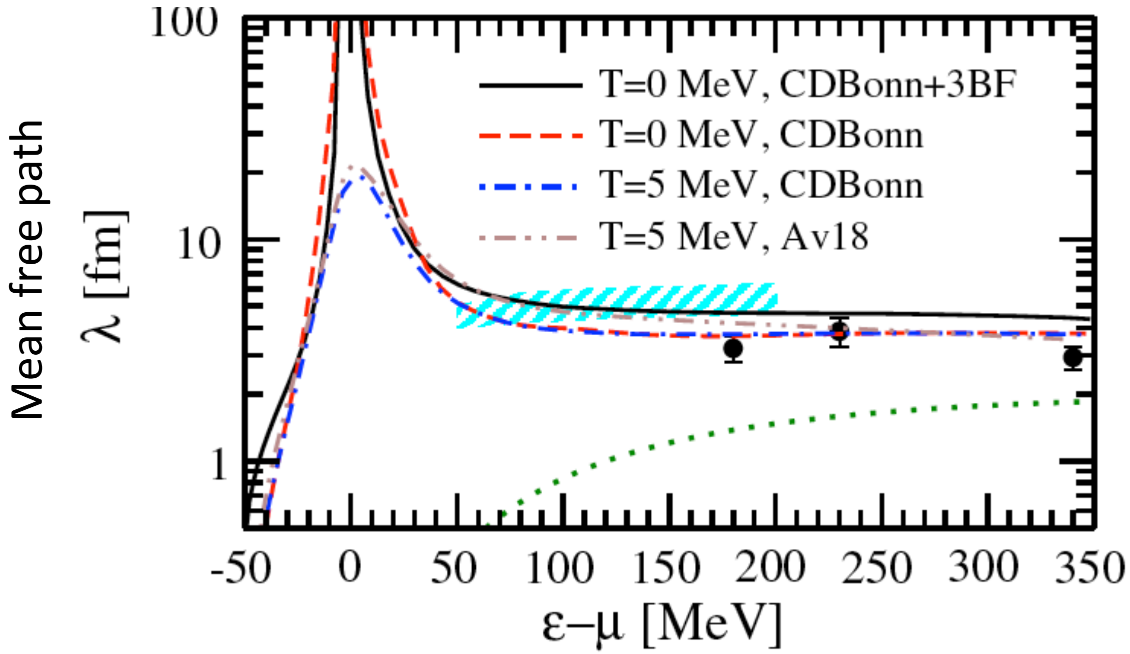




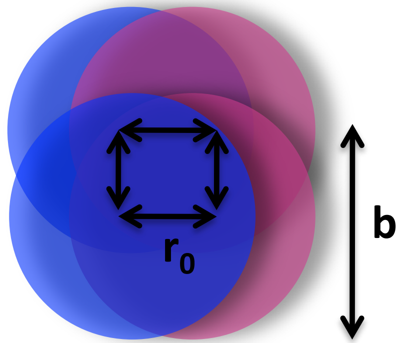




Nuclei: a quantum liquid feature



A. Rios & V. Soma PRL108(2012)012501



B. Mottelson ⇒ the concept of independent particle motion is based on the fact that the orbits of individual nucleons are delocalized and reflect the shape and radial dependence of the effective potential over the entire nucleus!

The quantality and the localisation parameter

Mottelson: **quantality** =
Quantal kinetic energy/potential energy

$$\Lambda \hat{=} \frac{\hbar^2}{m r_0^2 V_0'}$$

B. Mottelson, Proc. Les Houches school (1996)

Localisation parameter =
Localisation/internucleon distance

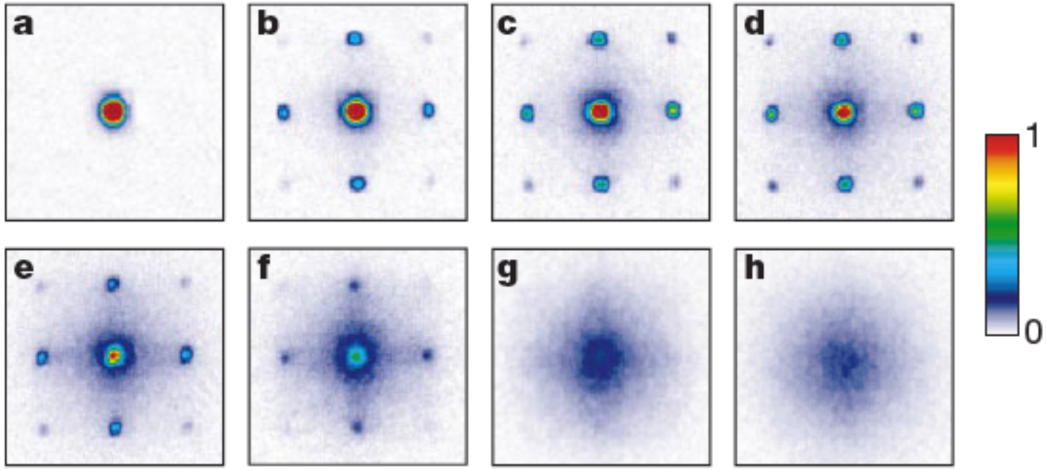
$$\alpha \hat{=} \frac{b}{r_0} = \frac{\sqrt{\hbar} A^{1/6}}{(2 m V_0 r_0^2)^{1/4}}$$

Quantality does not take into account finite size effects at work for clusterisation

The depth of the potential

- **Ultracold atoms** : optical trap of variable depth V_0

M. Greiner at al., Nature 415 (2002) 39



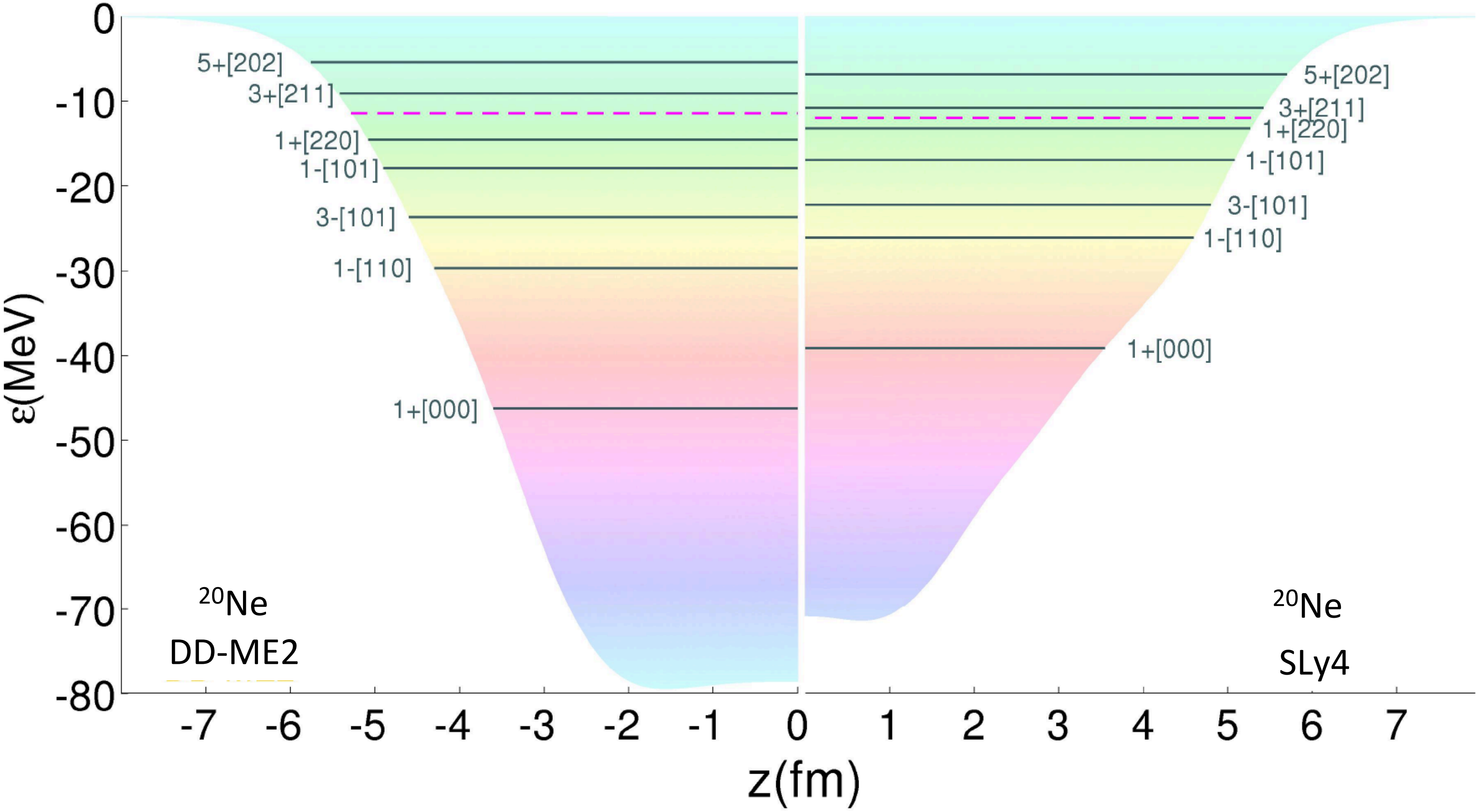
- **Nuclei** : depth of the potential **consistently** determined (relativisitic)

$$\left\{ p \frac{1}{2\tilde{M}(r)} p + W(r) + V_{ls}(r) l.s \right\} \varphi_i = \varepsilon_i \varphi_i$$

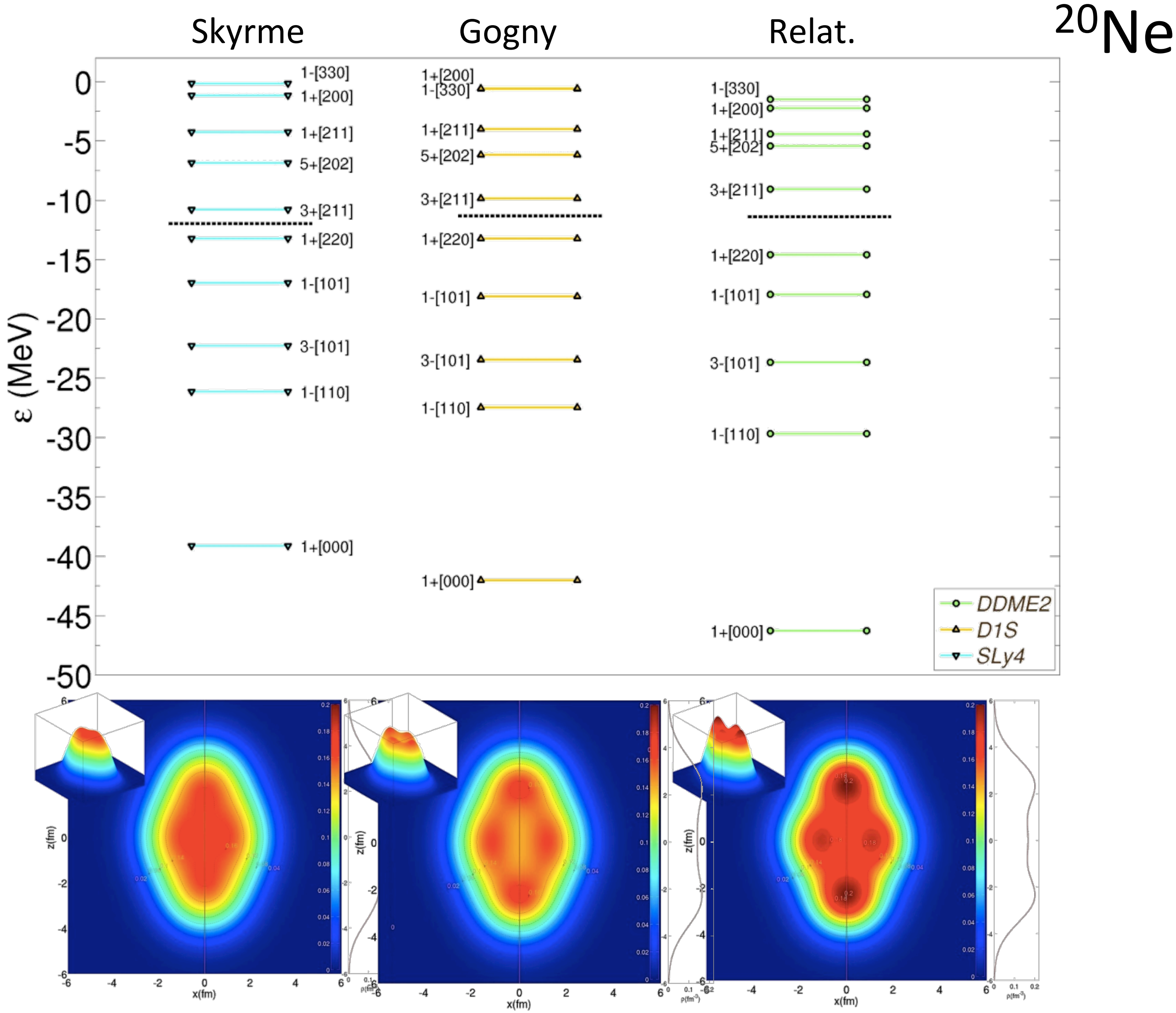
S≈ - 400 MeV
V≈ 320 MeV → $V_0 \approx 80$ MeV

$$W(r) = [V + S](r)$$
$$V_{ls}(r) = \frac{1}{2\tilde{M}^2(r)} \frac{1}{r} \frac{d}{dr}(V - S)$$

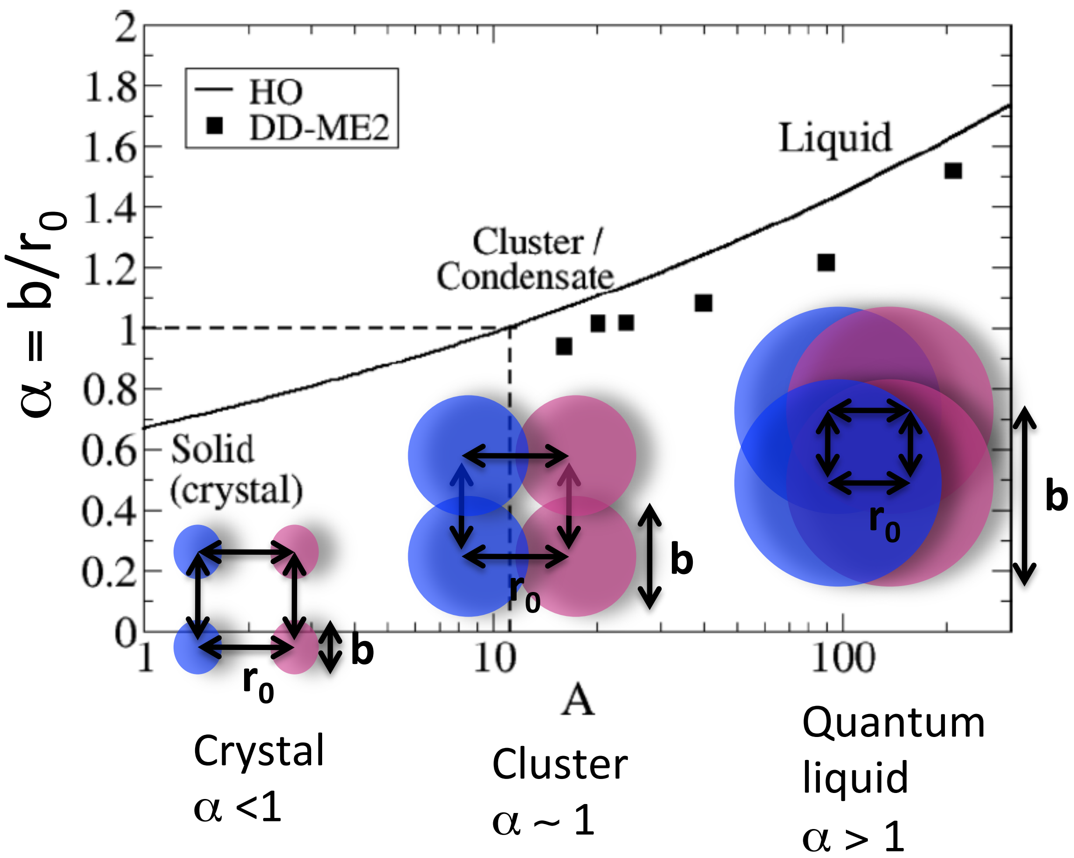
A way to vary the depth of the potential



Deeper potential leads to localisation



From a nuclear crystal to a nuclear liquid

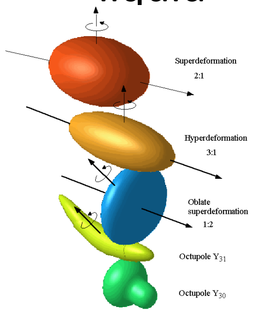


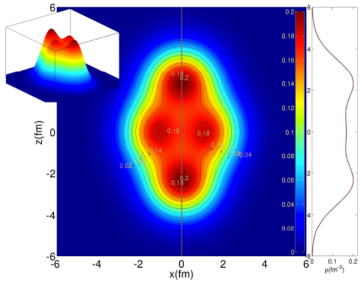
In finite nuclei: localisation α

$$\alpha \hat{=} \frac{b}{r_0} = \frac{\sqrt{\hbar} A^{1/6}}{(2mV_0 r_0^2)^{1/4}}$$

States of matter

Quantum liquid	Cluster	Solid	Liquid	Gas

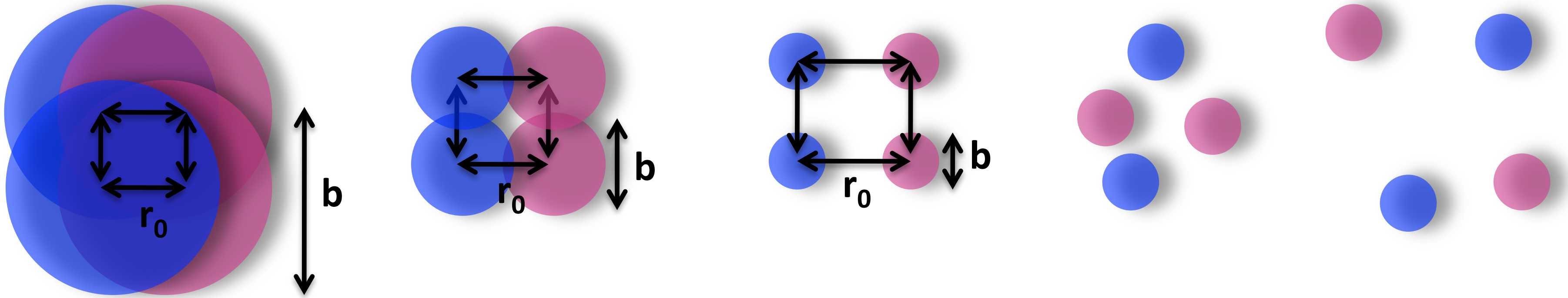




MACRO

$\alpha = b/r_0; \Lambda ; \lambda$

ρ; A ⟷ T



MICRO

Delocalised dense system	Molecule	Crystal	Epot<Ekin	Epot<<Ekin

Quantum ⟷ Classical

Summary

- Rel. EDF provides unified description of nuclear states: liquid drop, cluster and halo

→ Clusters = hybrid states between quantum liquid and crystal

→ Role of localisation, deformation, excitation, n excess

→ Exotic shapes, phase transition

→ Key role of saturation

→ Specific mode of excitation

→ Comparison with Exp. excitation spectra