Partial Symmetries in Nuclear Models

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Hamiltonian symmetry

The symmetry of the Hamiltonian $\hat{H}$:

$$\hat{g}\hat{H}\hat{g}^{-1} = \hat{H}.$$

for all $g \in \mathrm{G}$.

- Degeneracy of the energy spectrum
- Selection rules
- Wigner-Eckart theorem

Wigner-Eckart theorem

Clebsch-Gordan series and coefficients:

$$\Delta^{\Gamma_1} \times \Delta^{\Gamma_2} \sim \bigotimes_{\Gamma} n^{\Gamma}_{\Gamma_1 \Gamma_2} \Delta^{\Gamma}$$

$$\Psi^{\Gamma,\alpha}_{c} = \sum_{a=1}^{\dim(\Gamma_1)} \sum_{b=1}^{\dim(\Gamma_2)} (\Gamma_1 a \Gamma_2 b | \Gamma c; \alpha) \phi^{\Gamma_1}_{a} \xi^{\Gamma_2}_{b}$$

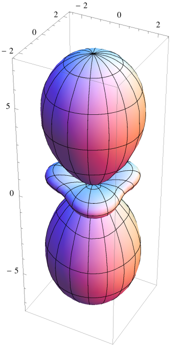
Irreducible tensor for a group G

$$\hat{g} Q^{\Gamma}_{a} \hat{g}^{-1} = \sum_{k=1}^{\dim(\Gamma)} \Delta^{\Gamma}_{ka}(g) Q^{\Gamma}_{k}$$

Wigner-Eckart theorem:

$$\langle \phi^{\Gamma_1}_{a} | Q^{\Gamma}_{k} | \xi^{\Gamma_2}_{b} \rangle = \sum_{\alpha}^{n^{\Gamma}_{\Gamma_1 \Gamma_2}} (\Gamma_1 a \Gamma_2 b | \Gamma l; \alpha)^{\star} \langle \phi^{\Gamma_1} || Q^{\Gamma} || \xi^{\Gamma_2} \rangle_{\alpha}$$

Two symmetries in one body

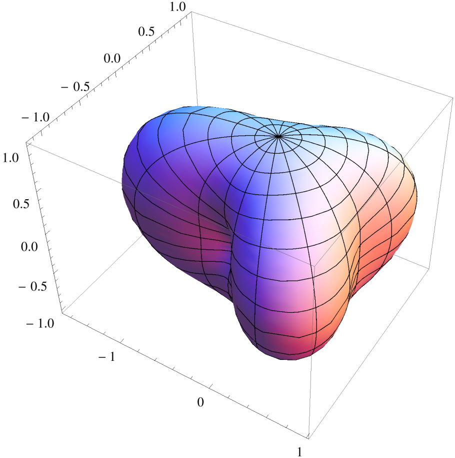


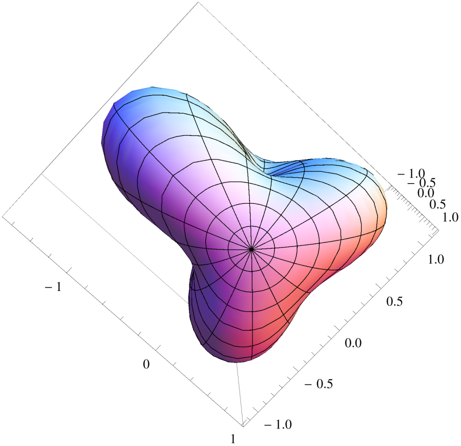
'Nuclear' surface Nr 1.: $\alpha_{20} = 10,\ \alpha_{33} = 0.5, \overline{\mathrm{SO}(2)}, \overline{\mathrm{C}}_3$

Nuclear surface:

$$R(\{\alpha\}; \theta, \phi) = R_0 \left(1 + \sum_{\lambda\mu} \alpha^{\star}_{\lambda\mu} Y_{\lambda\mu}(\theta, \phi)\right)$$

2 in one,





'Nuclear' surface Nr 2.: $\alpha_{22} = 0.3, \ \alpha_{33} = 0.5, \overline{\mathrm{C}}_2, \overline{\mathrm{C}}_3$

Partial–symmetries, non-orthogonal decomposition

The schematic quadrupole+octupole model Hamiltonian:

$$\hat{\mathcal{H}} = \hat{\mathcal{H}}_{vib} + \hat{\mathcal{H}}_{rot}$$

where

$$\hat{\mathcal{H}}_{vib} = \hat{\mathcal{H}}_{vib;2}(\alpha_2) + \hat{\mathcal{H}}_{vib;3}(\alpha_3)$$
$$\hat{\mathcal{H}}_{rot} = \hat{\mathcal{H}}_{rot}(\Omega)$$

If the Hamiltonian is related to the above nuclear shape Nr. 1:

$$\mathrm{Sym}(\hat{\mathcal{H}}_{vib;2}) = \overline{\mathrm{SO}(2)}_{vib} \quad \mathrm{Sym}(\hat{\mathcal{H}}_{vib;3}) = \overline{\mathrm{C}}_{3;vib} \quad \mathrm{Sym}(\hat{\mathcal{H}}_{rot}) = \overline{\mathrm{G}}_{rot}$$

Partial–symmetries, non-orthogonal decomposition

No vib-rot coupling terms ⇒ the eigenfunctions:

$$\Psi^{JM}_{\nu_2\nu_3\nu_{rot}}(\alpha,\Omega) = \phi^{J}_{2;\kappa_2\Gamma_2 a}(\alpha_2)\phi^{J}_{3;\kappa_3\Gamma_3 b}(\alpha_3)R^{JM}_{\kappa_{rot}\Gamma_{rot} c}(\Omega),$$

$\nu_2 = (\kappa_2\Gamma_2 a)$, $\nu_3 = (\kappa_3\Gamma_3 b)$, $\nu_{rot} = (\kappa_{rot}\Gamma_{rot} c)$

In the simplest approximation the electric transition operator:

$$Q_{\lambda\mu} = A_\lambda \alpha_{\lambda\mu}$$

The reduced (in respect to $J_3 = M$) matrix elements:

$$\langle \Psi^{J'}_{\nu'} || Q^{lab}_{\lambda} || \Psi^{J}_{\nu} \rangle = \sum_{\mu} \langle \phi^{J'}_{\sigma'} | Q_{\lambda\mu} | \phi^{J}_{\sigma} \rangle \langle R^{J'}_{\nu'_{rot}} || D^{\lambda\star}_{\cdot\mu} || R^{J}_{\nu_{rot}} \rangle,$$

$\nu = (\nu_2\nu_3\nu_{rot})$, $\sigma = (\nu_2, \nu_3)$

Partial–symmetries, non-orthogonal decomposition

The schematic quadrupole+octupole model.

The quadrupole transitions:

$$\langle \Psi^{J'}_{\nu'} || Q^{lab}_{2} || \Psi^{J}_{\nu} \rangle = \delta_{\nu'_3 \nu_3} \sum_{\mu} \langle \phi^{J'}_{\nu'_2} | \alpha_{2\mu} | \phi^{J}_{\nu_2} \rangle \langle R^{J'}_{\nu'_{rot}} || D^{\lambda\star}_{\cdot\mu} || R^{J}_{\nu_{rot}} \rangle$$

The selection rules for axial symmetry $\overline{\mathrm{SO}(2)}_{vib} + \overline{\mathrm{G}}_{rot}$

The octupole transitions:

$$\langle \Psi^{J'}_{\nu'} || Q^{lab}_{2} || \Psi^{J}_{\nu} \rangle = \delta_{\nu'_2 \nu_2} \sum_{\mu} \langle \phi^{J'}_{\nu'_3} | \alpha_{3\mu} | \phi^{J}_{\nu_3} \rangle \langle R^{J'}_{\nu'_{rot}} || D^{\lambda\star}_{\cdot\mu} || R^{J}_{\nu_{rot}} \rangle$$

The selection rules for the symmetry $\overline{\mathrm{C}}_{3;vib} + \overline{\mathrm{G}}_{rot}$.

Partial–symmetries, orthogonal decomposition

Spectral theorem

Assume the discrete spectrum of $\hat{\mathcal{H}}$, then:

$$\hat{\mathcal{H}} = \sum_{\nu} \epsilon_{\nu} P_{\nu}$$

Notation:

A) The operator A has the symmetry G:

$$\mathrm{G} = \mathrm{Sym}(A)$$

B) Collection of the projectors P_{ν} having the same symmetry G:

$$\mathcal{O}_{\mathrm{G}} = \{P_{\nu} : \mathrm{Sym}(P_{\nu}) = \mathrm{G}\}$$

Partial–symmetries, orthogonal decomposition

The partial Hamiltonians:

$$\hat{\mathcal{H}}_{\mathrm{G}} = \sum_{P_\nu \in \mathcal{O}_{\mathrm{G}}} \epsilon_\nu P_\nu .$$

$\hat{\mathcal{H}}_{\mathrm{G}}$ has the symmetry G.

Orthogonal decomposition of $\hat{\mathcal{H}}$ into the partial Hamiltonians:

$$\hat{\mathcal{H}} = \sum_{\mathrm{G}} \hat{\mathcal{H}}_{\mathrm{G}}$$

$\mathrm{G} \neq \mathrm{G}' \Rightarrow$

$$\hat{\mathcal{H}}_{\mathrm{G}} \hat{\mathcal{H}}_{\mathrm{G}'} = 0 \qquad (*)$$

Eigenproblem

To solve the eigen-equation for $\hat{\mathcal{H}} = \sum_{\mathrm{G}} \hat{\mathcal{H}}_{\mathrm{G}}$ it is sufficient to solve the eigenproblems for all partial Hamiltonians:

$$\hat{\mathcal{H}}_{\mathrm{G}} |\mathrm{G}; \mu \Gamma a\rangle = \epsilon^{\mathrm{G}}_{\mu\Gamma} |\mathrm{G}; \mu \Gamma a\rangle.$$

By definition, for $\mathrm{G}' \neq \mathrm{G}$

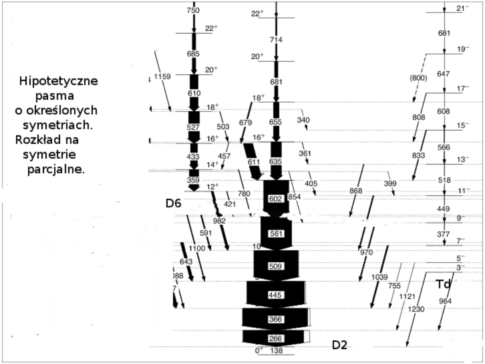
$$\hat{\mathcal{H}}_{\mathrm{G}'} |\mathrm{G}; \mu \Gamma a\rangle = 0.$$

Here: μ labels the equivalent i.r. of the group G. We get

$$\hat{\mathcal{H}} |\mathrm{G}; \mu \Gamma a\rangle = \epsilon^{\mathrm{G}}_{\mu\Gamma} |\mathrm{G}; \mu \Gamma a\rangle.$$

and reversely.

Band with symmetries



The energy band $\overline{\mathrm{D}}_6$ is generated by $\hat{\mathcal{H}}_{D_6}$, the ground state band $\overline{\mathrm{D}}_2$ by $\hat{\mathcal{H}}_{D_2}$, and the tetrahedral band $\overline{\mathrm{T}}_d$ by $\hat{\mathcal{H}}_{T_d}$.

Illustrative example 1/2

Let the set of subhamiltonians be defined as:

$$H_1, H_2, \ldots, H_N, H', \quad H_k H_{k'} = 0,\ H_k H' = 0,\ \text{for } k \neq k',$$

where H_k generate the kth excitation mode,
H' represents a coupling among these modes.

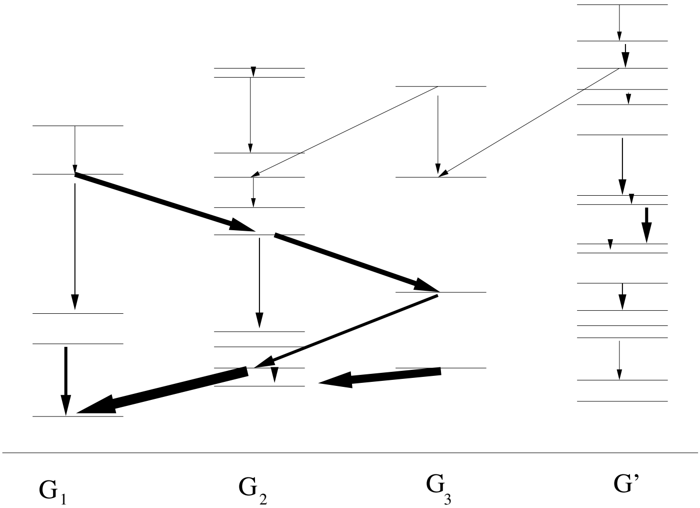
$$\mathrm{Sym}(H_k) = \overline{\mathrm{G}}_k, \mathrm{Sym}(H') = \overline{\mathrm{G'}}.$$

Notation: $H_{\overline{\mathrm{G}}_k} \equiv H_k,\ H_{\overline{\mathrm{G'}}} \equiv H',\ k = 1, 2, \ldots, N.$

The Hamiltonian

$$H = \sum_{k=1}^{N} H_{\overline{\mathrm{G}}_k} + H_{\overline{\mathrm{G'}}} \quad \mathrm{Sym}(H) = \bigcap_k \overline{\mathrm{G}}_k \cap \overline{\mathrm{G'}}.$$

Illustrative example 2/2



Energy bands groupped by symmetries, selection rules

Problems

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