Roles of pairing interactions in the formation of low- and high-energy Gamow-Teller excitations

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Neptune driving Waves

Powerful Waves = strong interaction

Neptune and the waves, or "steeds," he rides.

_ Walter Crane, 1892_
### Vibration Modes in Nuclei (Schematic)

<table>
<thead>
<tr>
<th></th>
<th>Electric Mode ($\Delta S=0$)</th>
<th>Magnetic Mode ($\Delta S=1$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>IS ($\Delta T=0$)</td>
<td>IV ($\Delta T=0$)</td>
</tr>
<tr>
<td>L=0</td>
<td>![Diagram L=0 IS]</td>
<td>![Diagram L=0 IV]</td>
</tr>
<tr>
<td></td>
<td>![Diagram L=0 IS]</td>
<td>![Diagram L=0 IV]</td>
</tr>
<tr>
<td>L=1</td>
<td>![Diagram L=1 IS]</td>
<td>![Diagram L=1 IV]</td>
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<td></td>
<td>![Diagram L=1 IS]</td>
<td>![Diagram L=1 IV]</td>
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<tr>
<td>L=2</td>
<td>![Diagram L=2 IS]</td>
<td>![Diagram L=2 IV]</td>
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<tr>
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<td>![Diagram L=2 IS]</td>
<td>![Diagram L=2 IV]</td>
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<tr>
<td>L=3</td>
<td>![Diagram L=3 IS]</td>
<td>![Diagram L=3 IV]</td>
</tr>
<tr>
<td></td>
<td>![Diagram L=3 IS]</td>
<td>![Diagram L=3 IV]</td>
</tr>
</tbody>
</table>

**Gamow-Teller mode ($\sigma\tau$)**

Isovector & Spin excitation
Gamow-Teller transitions

Mediated by $\sigma \tau$ operator

$\Delta S = -1, 0, +1$ and $\Delta T = -1, 0, +1$

($\Delta L = 0$, no change in radial w.f.)

$\Rightarrow$ no change in spatial w.f.

Accordingly, transitions among $j_>$ and $j_<$ configurations

$\begin{align*}
  j_> &\rightarrow j_>, \\
  j_< &\rightarrow j_<, \\
  j_> &\leftrightarrow j_<
\end{align*}$

example $f_{7/2} \rightarrow f_{7/2}, f_{5/2} \rightarrow f_{5/2}, f_{7/2} \leftrightarrow f_{5/2}$

Note that Spin and Isospin are unique quantum numbers in atomic nuclei!

$\Rightarrow$ GT transitions are sensitive to Nuclear Structure!

$\Rightarrow$ GT transitions in each nucleus are UNIQUE!
IS & IV pairing and “Residual Interactions”

However, $J^\pi$ values of even-even nuclei are $J^\pi=0^+$. We notice the importance of the spin-spin coupling.

In general, interactions that are not included in a model are called “residual interactions”

ex. “deuteron model”

Isovector $T=1$

$J^\pi=0^+$ unbound

Isoscalar $T=0$

$J^\pi=1^+$ bound=deuteron

ex. $1s_{1/2}$

(pairing interaction)

$^4\text{He} = \alpha$

$J^\pi=0^+$

ex. “deuteron model”

Isovector $T=1$

$J^\pi=0^+$ unbound

Isoscalar $T=0$

$J^\pi=1^+$ bound=deuteron
**Basic common understanding of $\beta$-decay and Charge-Exchange reaction**

$\beta$ decays:
- Absolute $B$(GT) values,
  - but usually the study is limited to low-lying states
- $(p,n)$, $(^3\text{He},t)$ reaction at $0^\circ$:
  - Relative $B$(GT) values, but Highly Excited States

**Both are important for the study of GT transitions!**
**β-decay & Nuclear Reaction**

*β-decay GT tra. rate = \( \frac{1}{t_{1/2}} = \frac{\lambda^2}{\hbar} B(\text{GT}) \)

\( B(\text{GT}) \) : reduced GT transition strength
\( \propto (\text{matrix element})^2 = |<f|\sigma\tau|i>|^2 \)

*Nuclear (CE) reaction rate (cross-section)
= reaction mechanism
\( \otimes \) operator
\( \times \) structure
\( = (\text{matrix element})^2 \)

*At intermediate energies (100 < \( E_{\text{in}} < 500 \text{ MeV} \))
\( \rightarrow \frac{d\sigma}{d\omega}(q=0) : \text{proportional to } B(\text{GT}) \)
Simulation of $\beta$-decay spectrum

$^{50}$Cr$(^{3}$He,t)$^{50}$Mn

$E=140$ MeV/nucleon

$\theta=0^\circ$

$Q_{EC}$ = 8.152 MeV

$^{50}$Cr$(^{3}$He,t)$^{50}$Mn

$E=140$ MeV/nucleon

$\theta=0^\circ$

$Q_{EC}$ = 8.152 MeV

$\beta$-decay: $^{50}$Fe $\rightarrow$ $^{50}$Mn

*expected spectrum assuming isospin symmetry

$Q_{EC}$ = 8.152 MeV

$(p, n)$ spectra for Fe and Ni Isotopes

- $^{54}$Fe$(p,n)^{54}$Co
  - $E_p = 160$ MeV
- $^{58}$Ni$(p,n)^{58}$Cu
- $T = 1$

Rapaport & Sugarbaker
Comparison of $(p, n)$ and $(^3\text{He}, t)$ $0^\circ$ spectra

$^{58}\text{Ni}(p, n)^{58}\text{Cu}$
$E_p = 160$ MeV

$^{58}\text{Ni}(^3\text{He}, t)^{58}\text{Cu}$
$E = 140$ MeV/u

J. Rapaport et al.
NPA (‘83)

Y. Fujita et al.,
EPJ A 13 (’02) 411.

H. Fujita et al.,
PRC 75 (’07) 034310

Excitation Energy (MeV)
Grand Raiden Spectrometer

Large Angle Spectrometer

$(^3\text{He}, \text{t})$ reaction

$^3\text{He}$ beam

140 MeV/u
Dispersion Matching Techniques were applied!

RCNP, Osaka Univ.

Grand Raiden

WS Beam Line

Dispersio

Dispersion Matching Techniques were applied!

Ring Cyclotron

ΔE=30 keV

ΔE=150 keV
T=1 Isospin Symmetry

\[ ^{42}_{20}\text{Ca}_{22} \quad T_z = +1 \]
\[ ^{42}_{21}\text{Sc}_{21} \quad T_z = 0 \]
\[ ^{42}_{22}\text{Ti}_{20} \quad T_z = -1 \]
T=1 symmetry: Structures & Transitions

\[
\begin{array}{c}
\text{(e, e')} \\
\text{(p, p')} \\
\text{\sigma\tau} \\
0^+ \\
g.s.
\end{array}
\quad
\begin{array}{c}
1^+ \\
\text{\sigma\tau} \\
(3\text{He},t) \\
0^+, \text{IAS} \\
g.s.
\end{array}
\quad
\begin{array}{c}
1^+ \\
\beta^+ \text{ decay} \\
0^+ \\
g.s.
\end{array}
\quad
\begin{array}{c}
T=2 \\
T=1 \\
T=0 \\
T=1
\end{array}
\]

\[
\begin{array}{c}
42^{\text{Ca}} \\
T_Z = +1
\end{array}
\quad
\begin{array}{c}
42^{\text{Sc}} \\
T_Z = +0
\end{array}
\quad
\begin{array}{c}
42^{\text{Ti}} \\
T_Z = -1
\end{array}
\]
\[ \text{\( \beta \)-decay GT tra. rate} = \frac{1}{t_{1/2}} = f \frac{\lambda^2}{K} B(\text{GT}) \]

*Study of Weak Response of Nuclei by means of Strong Interaction!*

*\( B(\text{GT}) \): reduced GT transition strength \( \propto (\text{matrix element})^2 \)

A simple reaction mechanism should be achieved!

\( \Rightarrow \) we have to go to high incoming energy
**GT transitions in each nucleus are UNIQUE!**

- *pf-shell nuclei* -
$^{42}\text{Ca}(^{3}\text{He},t)^{42}\text{Sc}$ in 2 scales

80% of the total $B(\text{GT})$ strength is concentrated in the excitation of the 0.611 MeV state.

$B(\text{GT}) = 2.2$
(from mirror $\beta$ decay)

$^{42}\text{Ca}(^{3}\text{He},t)^{42}\text{Sc}$
$E = 140$ MeV/nucleon
$\theta = 0^\circ$
$S_p = 4.272$
GT strengths in $A=42-58$

$^{58}\text{Ni}(^{3}\text{He},t)^{58}\text{Cu}$

GT-GR
GT states in $A=42-54 \ T_z=0$ nuclei

Y. Fujita et al.  
PRL 2014  
PRC 2015

T. Adachi et al.  
PRC 2006

Y. Fujita et al.  
PRL 2005

T. Adachi et al.  
PRC 2012

Peak heights are proportional to $B(GT)$ values
GT-strength: Cumulative Sum

(a) Experiment

(b) SM calculation

M. Homma et al.
SM Configurations of GT transitions

Target nuclei: $N = Z + 2$ ($T_z = +1$)
Final nuclei: $N = Z$ ($T_z = 0$)
$rp$-process Path
(T=1 system)

f-shell nuclei!
→ transition among f$_{7/2}$ & f$_{5/2}$ shells!

** $\Delta E (f_{5/2} - f_{7/2}) \approx 5 - 6$ MeV
Role of Residual Int. (repulsive)

Single particle-hole strength distribution

Graphical solution of the RPA dispersive eigen-equation

positive = repulsive

p-h configuration + IV excitation = repulsive

Collective excitation formed by the repulsive residual interaction

1p-1h strength

collective strength (GR)
Role of Residual Int. (repulsive)

Collective excitation formed by the repulsive residual interaction

\[ ^{54}\text{Fe}(^{3}\text{He},t)^{54}\text{Co} \]

1p-1h strength

Collective strength (GR)
\[ ^{42}\text{Ca} (^{3}\text{He},t)^{42}\text{Sc} \text{ in 2 scales} \]

\[ B(\text{GT}) = 2.2 \]

(from mirror $\beta$ decay)

\[ ^{42}\text{Ca} (^{3}\text{He},t)^{42}\text{Sc} \]

\[ E=140 \text{ MeV/nucleon} \]

\[ \theta=0^\circ \]

\[ S_p = 4.272 \]
QRPA calculations

using Skyrme int.
(with IV pairing corr.)

Calculation by
P. Sarrigren,
CSIC, Madrid
SM Configurations of GT transitions

particle-hole configuration
+ IV-type int.
= REPULSIVE
SM Configurations of GT transitions

\[ \pi-p \rightarrow \nu-p \] configurations sensitive to IS pairing int. 
\[ \rightarrow \text{attractive} \]

(spin-triplet, IS int. is stronger than spin-singlet, IV int.)

particle-hole configurations
+ IV-type excitation \((\sigma \tau)\)
\[ \rightarrow \text{repulsive} \]

by Engel, Bertsch, Macchiavelli
SM Configurations of GT transitions

particle-particle int. (attractive) \rightarrow particle-hole int. (repulsive)
(IS p-n int. is attractive)

Isoscalar interaction can play Important roles!
GT strength Calculations:
HFB+QRPA + pairing int.

Bai, Sagawa, Colo et al., PL B 719 (2013) 116

The density dependent contact pairing interactions are adopted for both $T = 1$ and $T = 0$ channels,

\[ V_{T=1}(\mathbf{r}_1, \mathbf{r}_2) = V_0 \frac{1 - P_\sigma}{2} \left( 1 - \frac{\rho(\mathbf{r})}{\rho_0} \right) \delta(\mathbf{r}_1 - \mathbf{r}_2), \tag{1} \]

\[ V_{T=0}(\mathbf{r}_1, \mathbf{r}_2) = f V_0 \frac{1 + P_\sigma}{2} \left( 1 - \frac{\rho(\mathbf{r})}{\rho_0} \right) \delta(\mathbf{r}_1 - \mathbf{r}_2), \tag{2} \]

Results (using Skyrme int. SGII)
at $f = 0$: there is little strength in the lower energy part,
at $f = 1.0 \sim 1.7$: coherent low-energy strength develops!
QRPA-cal. GT-strength (with IS-int.)

42Ca → 42Sc (Q-value)

by Bai Sagawa Colo
Role of Residual Int. (attractive)

Single particle-hole strength distribution

Graphical solution of the RPA dispersive eigen-equation

negative=attractive

Collective excitation formed by the attractive IS residual interaction
Role of Residual Int. (attractive)

Collective excitation formed by the attractive IS residual interaction

$^{42}\text{Ca}(^{3}\text{He},t)^{42}\text{Sc}$

$E = 140 \text{ MeV/nucleon}$

$\theta = 0^\circ$

Collective strength (GR)
QRPA cal. including IS int.

C.L. Bai, H. Sagawa, G. Colo

| f | Bnp  | neutron | proton | \((X_{upvn}+Y_{unvp}) \langle p | GT | n \rangle\) |
|---|------|---------|--------|---------------------------------------------|
| 0 | 1.34 | 1f7/2   | 1f7/2  | 0.427                                       |
|   |      |         |        | 1.3689                                      |
| 0.5| 2.051| 1f7/2   | 1f7/2  | 0.432                                       |
|    |      |         |        | 1.384                                       |
| 1  | 4.75 | 1f7/2   | 1f7/2  | 0.053                                       |
|    |      |         |        | 0.2158                                      |
|    |      | 1f5/2   | 1f7/2  | 0.053                                       |
|    |      |         |        | 0.474                                       |
|    |      | 1f7/2   | 1f7/2  | 0.129                                       |
|    |      |         |        | 1.059                                       |
|    |      | 1f7/2   | 1f7/2  | 0.33                                        |
Shell Model Cal.: Transition Matrix Elements

TABLE VI. Results of the \( pf \)-shell SM calculation using the GXPF1J interaction. The matrix elements \( M(GT) \) of GT transitions exciting individual \( J^\pi = 1^+ \) GT states in \( ^{42}\text{Sc} \) from the g.s. of \( ^{42}\text{Ca} \) are shown for each configuration. The results are shown for all excited GT states predicted in the region up to 9.82 MeV. The notation \( f7 \to f7 \), for example, stands for the transition with the \( \nu f_{7/2} \to \pi f_{7/2} \) type and \( p3 \to p3 \) the \( \nu p_{3/2} \to \pi p_{3/2} \). The summed value of the matrix elements is denoted by \( \Sigma M(GT) \) and its squared value is the \( B(GT) \), where the \( B(GT) \) values do not include the quenching factor of the SM calculation.

<table>
<thead>
<tr>
<th>States in ( ^{42}\text{Sc} )</th>
<th>Configurations</th>
<th>Transition strengths</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E_x ) (MeV)</td>
<td>( T )</td>
<td>( f7 \to f7 )</td>
</tr>
<tr>
<td>0.33</td>
<td>0</td>
<td>1.383</td>
</tr>
<tr>
<td>4.41</td>
<td>0</td>
<td>0.719</td>
</tr>
<tr>
<td>7.41</td>
<td>0</td>
<td>0.193</td>
</tr>
<tr>
<td>8.62</td>
<td>0</td>
<td>-0.151</td>
</tr>
<tr>
<td>9.82</td>
<td>1</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Matrix Elements are in-phase!
$^{42}\text{Ca}(^{3}\text{He},t)^{42}\text{Sc}$ in 2 scales

Low-energy collective GT excitation!
(collectivity is from IS p-n int. !)

$B(\text{GT}) = 2.2$

PRC 91, 064316 (2015).

Low Energy Super GT state
Super-allowed GT transitions in $\beta$ decay

- $^6\text{He}, 0^+ \rightarrow ^6\text{Li}, 1^+$, $\log ft = 2.9$
- $^{18}\text{Ne}, 0^+ \rightarrow ^{18}\text{F}, 1^+$, $\log ft = 3.1$
- $^{42}\text{Ti}, 0^+ \rightarrow ^{42}\text{Sc}, 1^+$, $\log ft = 3.2$

(smaller $\log ft \rightarrow$ larger $B(\text{GT})$)
Super-Multiplet State
*proposed by Wigner (1937)

In the limit of null $L \cdot S$ force, SU(4) symmetry exists. We expect:
   a) GT excitation strength is concentrated in a low-energy GT state.
   b) excitation energies of both the IAS and the GT state are identical.

$\rightarrow$ Super-Multiplet State

In $^{54}$Co, we see a broken SU(4) symmetry. In $^{42}$Sc, we see a good SU(4) symmetry.
$\rightarrow$ attractive IS residual int. restores the symmetry!
$\rightarrow$ 0.611 MeV state in $^{42}$Sc has a character close to Super-Multiplet State!

We call this state the
Low-energy Super GT state!
SM Configurations of GT transitions

particle-particle int. (attractive)  
(T=0, IS p-n int. is attractive)

particle-hole int. (repulsive)

Isoscalar interaction can play Important roles!

$N=Z$ LS-closed Core + 2 nucleon system!
GT transitions forming Low-Energy Super GT state

J^\pi = O^+ \rightarrow 1^+ 

2n \quad \rightarrow \quad ^2\text{H} \ (d) 

\begin{align*}
\text{g.s.} & \quad \text{B}(\text{GT}) = 6.0 \ ? \ \text{Large} ! \\
\Sigma \ (\text{Sum rule}) & = 3 \times |N-Z| = 6 \\
\text{g.s.} & \quad \text{B}(\text{GT}) = 4.73 \\
\text{g.s.} & \quad \text{B}(\text{GT}) = 3.09 \\
\text{1st E_x state (IAS is the g.s.)} & \quad \text{Smaller} ! \\
\end{align*}
$^{18}$O($^3$He, t)$^{18}$F at 0°

Low-energy collective GT excitation: $B(GT) = 3.1$

Low Energy Super GT state
$^6$He $\beta^-$-decay & $^6$Li(p,n)$^6$Be

\[ \Rightarrow 2p + \alpha \]
\[ \Gamma = 92 \text{ keV} \]

$^6$Be

$^6$Li(p,n)$^6$Be

$E_p = 186$ MeV

Low Energy Super GT state

$\beta$-decay

log $ft = 2.9$

[B(GT) = 4.7]
$^{90}\text{Zr}$: Fermi & GT transitions

Schematic Picture of Single-Particle Transitions

GT Giant Resonance

GT low-lying state

p-h nature of configurations

Fermi transition

Gamow-Teller transitions
Discrete States and GTR in $^{90}\text{Nb}$
Formation of GT-GR in $^{90}$Nb

*in $^{90}$Zr$\rightarrow$$^{90}$Nb transitions

*\(\sigma_{\tau}\) int. : repulsive nature

*both configurations : p-h nature (repulsive)
$^{42}\text{Ca}(^{3}\text{He},t)^{42}\text{Sc}$ in 2 scales

*strong attractive $p-n$ interaction in $^3S, J=1, T=0$ (IS) channel!

*contribution of the Tensor force?
GT transitions forming Low-Energy Super GT state

\[ J^\pi = O^+ \rightarrow 1^+ \]

\[ \Sigma (\text{Sum rule}) = 3 \times |N-Z| = 6 \]

\[ B(\text{GT}) = 6.0 \text{? Large !} \]

\[ B(\text{GT}) = 4.73 \text{ g.s.} \]

\[ B(\text{GT}) = 3.09 \text{ g.s.} \]

\[ B(\text{GT}) = 2.17 \text{ 1}^{\text{st}} \text{ E}_x \text{ state (IAS is the g.s.)} \]
$^{42}\text{Ca}(^{3}\text{He},t)^{42}\text{Sc}$ in 2 scales

*strong attractive $p-n$ interaction in $^{3}\text{S}$, $J=1$, $T=0$ (IS) channel!

*contribution of the Tensor force?

Do we see the Screening Effect of Nuclear Medium?
**Summary**

GT (\(\sigma T\)) operator: a simple operator!

* GT transitions: sensitive to the structure of |i> and |f>

High resolution of the \((^3\text{He},t)\) reaction

* Fine structures of GT transitions
  (Precise comparison with mirror \(\beta\)-decay results)

\[\Rightarrow\] Low-energy Super GT state (LES GT state)

We got a key to study the IS \(pn\)-interaction!
(May be connected to Tensor?)
GT-study Collaborations

Bordeaux (France) : $\beta$ decay
GANIL (France) : $\beta$ decay
Gent (Belgium) : $(^3\text{He}, t), (d, ^2\text{He}), (\gamma, \gamma')$, theory
GSI, Darmstadt (Germany) : $\beta$ decay, theory
ISOLDE, CERN (Switzerland) : $\beta$ decay
iThemba LABS. (South Africa) : $(p, p'), (^3\text{He}, t)$
Istanbul (Turkey): $(^3\text{He}, t), \beta$ decay
Jyvaskyla (Finland) : $\beta$ decay
Koeln (Germany) : $\gamma$ decay, $(^3\text{He}, t)$, theory
KVI, Groningen (The Netherlands) : $(d, ^2\text{He})$
Leuven (Belgium) : $\beta$ decay
LTH, Lund (Sweden) : theory
Osaka University (Japan) : $(p, p'), (^3\text{He}, t)$, theory
Surrey (GB) : $\beta$ decay
TU Darmstadt (Germany) : $(e, e')$, $(^3\text{He}, t)$
Valencia (Spain) : $\beta$ decay
Michigan State University (USA) : theory, $(t, ^3\text{He})$
Muenster (Germany) : $(d, ^2\text{He}), (^3\text{He}, t)$
Univ. Tokyo and CNS (Japan) : theory, $\beta$ decay
Review

Spin–isospin excitations probed by strong, weak and electro-magnetic interactions

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