A Quadrupole-octupole Collective Approach

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1. Construction of quadrupole+octupole collective model (outlook)
2. Symmetrization of collective solutions
3. Some preliminary results

Collective quadrupole+octupole approach in intrinsic frame

We construct our deformed collective model already in the intrinsic frame–contrarly to the usual procedure which starts from the spherical Hamiltonian expressed in laboratory coordinates BUT the resulting rotational symmetry in the intrinsic frame is conserved!

The set of collective variables in the intrinsic coordinate system:

$$\alpha_{20}, \alpha_{22}, \{\alpha_{3\nu}\}, \Omega$$

Nuclear surface in the intrinsic coordinate system:

$$\begin{aligned} R(\theta,\varphi) = R_0[1 + \\ &+\alpha_{20} Y_{20}(\theta,\varphi) + \alpha_{22}(Y_{22}(\theta,\varphi) + Y_{2,-2}(\theta,\varphi)) \\ &+ \sum_{\nu=0}^{3} 2\,\alpha_{3\nu} Re\,(Y_{3\nu}(\theta,\varphi))\,], \end{aligned}$$

Center-of-mass problem

It is believed that the dipole parameters $\{\alpha_{1\mu}\}, \mu = \{-1, 0, 1\}$ are responsible for the center of mass motion. Let us expand the c.m. vector up to the first order in $\alpha_{1\mu}$.

$$\vec{r}_{CM} = \vec{r}_{CM}(\alpha_{1\mu}, \alpha_{20}, \alpha_{22}, \{\alpha_{3\nu}\}). \tag{1}$$

This equation can be solved with respect to the variables $\alpha_{1\mu}$ with the condition

$$\vec{r}_{CM} = 0, \tag{2}$$

$$\alpha_{1\mu} = \alpha_{1\mu}(\vec{r}_{CM} = 0, \alpha_{20}, \alpha_{22}, \{\alpha_{3\nu}\}). \tag{3}$$

Obtained in this way $\alpha_{1\mu}$'s ensure the nuclear surface to be defined in the center of mass frame. The above consideration indicates that the quadrupole and octupole deformations are the only independent collective variables.

Intrinsic rotation group $\overline{\mathrm{G}}$

Action of the rotation intrinsic group $\bar{g} \in \overline{\mathrm{SO}(3)}$.

Transformations of coordinates:

$$(\alpha^{lab}_{\lambda\mu})' = \bar{g}\alpha^{lab}_{\lambda\mu} = \alpha^{lab}_{\lambda\mu}$$

$$(\alpha_{\lambda\mu})' = \bar{g}\alpha_{\lambda\mu} = \sum_{\mu'} D^{\lambda}_{\mu'\mu}(g^{-1})\alpha_{\lambda\mu'}$$

$$\Omega' = \bar{g}\Omega = \Omega g.$$

Action in the space of functions of intrinsic variables:

$$\bar{g}\psi(\alpha_{\lambda\mu}, \Omega) = \psi(\bar{g}\alpha_{\lambda\mu}, \bar{g}^{-1}\Omega)$$

Relation between intrinsic and laboratory frame

The relation between collective laboratory and intrinsic shape variables

$$\alpha^{lab}_{\lambda\mu}(\alpha_{\lambda\nu}) = \sum_{\nu=-\lambda}^{\lambda} D^{\lambda *}_{\mu\nu}(\Omega)\,\alpha_{\lambda\nu}$$

with, at least, additional 3 conditions:

$$f_k(\alpha_{\lambda\mu}, \Omega) = 0, \;\; \{k = 1, 2, 3\},$$

which determine the orientation of both intrinsic vs laboratory frame.

Intrinsic frame

The transformation from the laboratory to intrinsic coordinate system is, in general, non-reversible.

It means that, for one given set of laboratory variables $\{\alpha_{\lambda\nu}^{lab}\}$ usualy may correspond several sets of intrinsic variables $\{\alpha_{\lambda\mu}, \Omega\}$, (well known problem e.g. for the Bohr Hamiltonian)

$$\alpha_{\lambda\nu}^{lab}(\alpha_{\lambda\nu}, \Omega) = \alpha_{\lambda\nu}^{lab}(\alpha_{\lambda\nu}^{'}, \Omega^{'})$$

where $(\alpha_{\lambda\nu}, \Omega) \neq (\alpha_{\lambda\nu}^{'}, \Omega^{'})$

How to omit this disadvantage?

Symmetrization group and uniqueness of eigensolutions

It is possible to find the **intrinsic transformation group** of the intrinsic variables which does not change the transformation relation between intrinsic and laboratory variables

$$\alpha^{lab}_{\lambda\nu}(\bar{g}(\alpha_{\lambda\nu},\Omega)) = \alpha^{lab}_{\lambda\nu}(\alpha_{\lambda\nu},\Omega)$$

The set of all transformations $\bar{g}$ forms the so called **symmetrization group** G_s.

REMARK: generally while working in the intrinsic frame, for most of square integrable functions $\Psi(\alpha_{\lambda\mu},\Omega) \neq \Psi((\alpha_{\lambda\mu})',\Omega')$.

The symmetrization condition for states. For all $\bar{g} \in \overline{\mathrm{G}}_s$:

$$\bar{g}\Psi(\alpha_{\lambda\mu},\Omega) = +1 \cdot \Psi(\alpha_{\lambda\mu},\Omega)$$

Construction of the collective basis

Initial (before projection) H.O. one-phonon basis functions

$$\Psi_k^{(\pm)}(\alpha_2,\alpha_3,\Omega) = u_0(\eta_2,\alpha_{20}-\mathring{\alpha}_{20})u_0(\sqrt{2}\eta_2,\alpha_{22}-\mathring{\alpha}_{22})u_{n_0}(\eta_3,\pm\alpha_{30}-\mathring{\alpha}_{30})$$
$$u_{n_1}(\sqrt{2}\eta_3,\pm\alpha_{31}-\mathring{\alpha}_{31})u_{n_2}(\sqrt{2}\eta_3,\pm\alpha_{32}-\mathring{\alpha}_{32})$$
$$u_{n_3}(\sqrt{2}\eta_3,\pm\alpha_{33}-\mathring{\alpha}_{33})R^J_{MK}(\Omega)$$

with $R^J_{MK}(\Omega)=\sqrt{2J+1}\,D^J_{MK}*(\Omega)$ and

$$\sum_{k=0}^{3} n_k = 1$$

Basis functions of good (positive or negative) parity

$$\Psi_k(\alpha_2,\alpha_3,\Omega;\pi=+1)=\frac{1}{2}(\Psi_k^{(+)}(a\alpha_2,\alpha_3,\Omega)+\Psi_k^{(-)}(\alpha_2,\alpha_3,\Omega))$$
$$\Psi_k(a\alpha_2,\alpha_3,\Omega;\pi=-1)=\frac{1}{2}(\Psi_k^{(+)}(\alpha_2,a\alpha_3,\Omega)-\Psi_k^{(-)}(\alpha_2,\alpha_3,\Omega))$$

Applying the projection operator onto the scalar irreducible representation of the symmetrization group one obtains the basis function

$$\Psi_k^{(A1)} \equiv \hat{P}^{(A1)}(\bar{g})\Psi_k = \sum_{l=1}^{card(G_s)} \frac{1}{8} u_0(\eta_2, \hat{\bar{g}}_l\alpha_{20} - \mathring{\alpha}_{20}) u_0(\sqrt{2}\eta_2, \hat{\bar{g}}_l\alpha_{22} - \mathring{\alpha}_{22}) \times$$
$$u_{n_0}(\eta_3, \hat{\bar{g}}_l\alpha_{30} - \mathring{\alpha}_{30}) u_{n_1}(\sqrt{2}\eta_3, \hat{\bar{g}}_l\alpha_{31} - \mathring{\alpha}_{31}) \times$$
$$u_{n_2}(\sqrt{2}\eta_3, \hat{\bar{g}}_l\alpha_{32} - \mathring{\alpha}_{32}) u_{n_3}(\sqrt{2}\eta_3, \hat{\bar{g}}_l\alpha_{33} - \mathring{\alpha}_{33}) R^J_{MK}(\bar{g}\Omega)$$

where

$$R^J_{MK}(\Omega g^{-1}) = \sqrt{2J+1} \sum_{K'=-J}^{J} D^J_{KK'}(g) D^{J*}_{MK'}(\Omega)$$

but after projection it may happen that

$$\langle \Psi_k^{(A1)} | \Psi_{k'}^{(A1)} \rangle \neq \delta_{kk'}.$$

How to orthogonalize them efficiently?

1. 1 Standard Gram-Shmidtt procedure,
2. 2 Solving the eigenproblem of the overlap operator (as in the general GCM method).

Collective Hamiltonian

A realistic collective Hamiltonian with variable mass tensor

$$\mathcal{H}_{coll}(\alpha_2, \alpha_3, \Omega) = \frac{-\hbar^2}{2\sqrt{|g|}} \sum_{\{i,j\}=2}^{3} \frac{\partial}{\partial \alpha^i_\mu} \sqrt{|g|} \left[B(\alpha_{2\mu}, \alpha_{3\mu'})^{-1} \right]^{ij} \frac{\partial}{\partial \alpha^j_{\mu'}} + \hat{H}_{rot}(\Omega) + \hat{V}(\alpha_2, \alpha_3)$$

where g is the metric tensor corresponding to $\alpha_{\lambda\mu}$ manifold.

The collective 6D potential $\hat{V}(\alpha_2, \alpha_3)$ is obtained through the macroscopic-microscopic Strutinsky-like method.

Rotor Hamiltonian

The generalized rotor Hamiltonian $\hat{H}_{rot}$ of given symmetry G ($g \in G$) and rank n can be constructed out of the angular momentum operators in the following way:

$$\hat{H}_{rot}(\Omega) \equiv \sum_{\mu=-\lambda}^{\lambda} c_{\lambda\mu}(n)\, \hat{\mathcal{T}}_{\lambda\mu}(n,\Omega),$$

where

$$\hat{\mathcal{T}}_{\lambda\mu}(n;\lambda_2,\lambda_3,\dots,\lambda_{n-1},\Omega) \equiv \left[\left(\left(\left(\hat{I} \otimes \hat{I}\right)_{\lambda_2} \otimes \hat{I}\right)_{\lambda_3} \otimes \dots \otimes \hat{I}\right)_{\lambda_{n-1}}\right]_{\lambda\mu}$$

and

$$(\hat{I} \otimes \hat{I})_{\lambda_2\mu_2} = \sum_{\mu=1}^{I} \sum_{\mu'=1}^{I} (1,\mu;1,\mu'|\lambda_2\mu_2)\hat{I}_{1\mu}\,\hat{I}_{1\mu'}; \quad \lambda_2 = \{0,1,2\}$$

and

$$\hat{I}_{10} = \hat{I}_z, \quad \hat{I}_{1+1} = -\frac{1}{\sqrt{2}}(\hat{I}_x - i\hat{I}_y), \quad \hat{I}_{1-1} = +\frac{1}{\sqrt{2}}(\hat{I}_x + i\hat{I}_y).$$

Preliminary estimates of intraband $B(E2)$ transitions

Experiment, ILL Grenoble on ^{156}Gd (PRL 104, 222502, 2010), "*Ultrahigh-Resolution-Ray Spectroscopy of 156 Gd: A Test of Tetrahedral Symmetry*", M. Jentschel et al.

Nucleus	Transition, $I_i^\pi \to I_j^\pi$	No. of state	B(E2) (W.u.)	Dominating excitation
^{156}Gd	$2^+ \to 0^+$		211 (exp.187(5))	
	$4^+ \to 2^+$		183 (exp.263(5))	
	$5^- \to 3^-$	$(1 \to 1)$	168 (exp.293^{+61}_{-134})	$\alpha_{30} \to \alpha_{30}$
	$5^- \to 3^-$	$(2 \to 2)$	170	$\alpha_{32} \to \alpha_{32}$
	$5^- \to 3^-$	$(6 \to 4)$	179	$\alpha_{31} \to \alpha_{31}$
	$5^- \to 3^-$	$(10 \to 7)$	175	$\alpha_{33} \to \alpha_{33}$

$$E_\gamma(2^+ \to 0^+) \approx 200\, keV\, (exp.88\, keV)$$

$$E_\gamma(4^+ \to 2^+) \approx 350\, keV\, (exp.199\, keV)$$

$$E_\gamma(5^- \to 3^-) \approx 0.23 - 0.28\, MeV\, (exp.0.13\, MeV)$$

$$E(3_1^-) \approx 1.07\, MeV\, (exp.1.27\, MeV)$$

Collective quadrupole electric transition operators

The intrinsic frame is chosen to fix quadrupoles in the principal axes frame.

The quadrupole intrinsic operator:

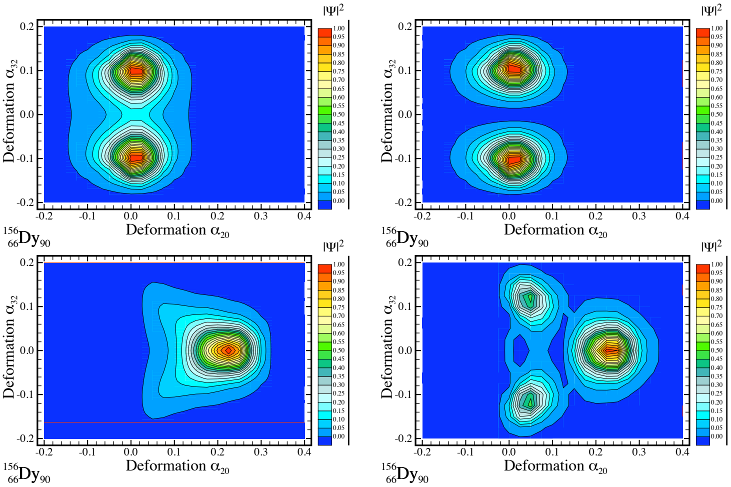
$$\hat{Q}_{20}^{intr} = \frac{3ZR_0^2}{4\pi}\Big\{\alpha_{20} +$$

$$\frac{1}{\sqrt{5\pi}}\Big(\frac{10}{7}\alpha_{20}\alpha_{20} - \frac{20}{7}\alpha_{2-2}\alpha_{22} + \frac{4}{3}\alpha_{30}\alpha_{30} - 2\alpha_{3-1}\alpha_{31} + \frac{10}{3}\alpha_{3-3}\alpha_{33}\Big)\Big\}$$

$$= \hat{Q}_{20}^{quadr}(1^{st}) + \hat{Q}_{20}^{quadr}(2^{nd}) + \hat{Q}_{20}^{oct}(2^{nd})$$

Four possible types of solutions of the ATDHF 2-D quadr.+oct. Hamiltonian (IJMP E, vol.20, 2011, p. 500-505)

$|\Psi_{vib}|^2$ as function of α_{20} and α''_{32} for ^{156}Dy.



SUMMARY:

- We have constructed a realistic collective model able to give the collective eigenfunctions and reduced transition probabilities $B(E\lambda)$
- This model treats all quadrupole and octupole degrees of freedom in the same footing.
- The basis in which the Hamiltonian is diagonalized contains zero- and one–phonon H.O. shifted 6D solutions.
- Since only real parts of complex $\alpha_{\lambda\mu}$ collective variables are considered the symmetrization group is no longer the octahedral group $\bar{O}$ as for purely quadrupole vibrations but its subgroup $\bar{D}_4$.
- The intraband $B(E2, 4^+ \to 2^+)$ transition is too small compared to the experimental one.
- We have generated the fragment of the lowest odd-spin negative-parity band in ^{156}Gd nucleus.
- Considering the structure of this band and the only measured intraband transition $B(E2, 5^- \to 3^-)$ we are not able to judge which kind of 1-ph excitation is dominating in both those states

FUTURE:

- The collective potential may be additionally minimized with respect to the higher multipole degrees of freedom which do not break the D_4 symmetry.
- The basis set may be extended to account for the 2-ph, 3-ph,...collective exitations
- Testing the credibility of the model using other nuclei.

COLLABORATION

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