Effect of thermal fluctuations in the pairing field on the width of giant dipole resonance

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1) Effect of thermal pairing on the GDR width:
   - thermal pairing
   - within the Phonon Damping Model (PDM)
   - within Thermal Shape Fluctuations Model (TSFM) that includes pairing fluctuations

2) PDM for the description of GDR in hot rotating nuclei

   - All the theoretical predictions are compared with the most recent experimental systematics.
Thermal pairing

In finite systems such as nuclei large thermal fluctuations smooth out the sharp superfluid-normal (SN) phase transition. As the result, pairing does not collapse at $T_c \approx 0.57 \Delta(T=0)$, but remains finite even at $T >> T_c$.

This has been shown within the following approaches:

1) Fluctuations of pairing field (Moretto, 1972)
2) SPA (Dang, Ring, Rossignoli, 1992)
3) SM (Zelevinsky, Alex Brown, Frazier, Horoi, 1996)
4) MBCS (Dang, Zelevinsky, 2001)
5) FTBCS1 (Dang, Hung, 2008)
6) Exact solutions of pairing problem (Volya, Alex Brown, Zelevinsky, 2001) embedded in the GCE, CE, and MCE (Dang, Hung, 2009)

\[
H = \sum_{jm} \epsilon_j \hat{N}_j - G \sum_{jj'} \hat{P}_j^\dagger \hat{P}_{j'},
\]

where the particle-number operator $\hat{N}_j$ and pairing operator $\hat{P}_j$ are given as

\[
\hat{N}_j = \sum_m a_{jm}^\dagger a_{jm}, \quad \hat{P}_j^\dagger = \sum_{m>0} a_{jm}^\dagger a_{j+m}^\dagger, \quad \hat{P}_j = (\hat{P}_j^\dagger)^\dagger,
\]
1) GDR photons are emitted in the early stage in competition with neutrons.
2) When $E^*$ becomes lower than $B_n$ slower $\gamma$ transitions take place.
3) Most of the angular momentum is carried off at the final stage of the decay by quadrupole radiation.
Phonon Damping Model (PDM)
NDD & Arima, PRL 80 (1998) 4145

\[ H = \sum_s E_s a_s^{\dagger} a_s + \sum_q \omega_q Q_q^{\dagger} Q_q + \sum_{ss'} F_{ss'}^{(q)} a_s^{\dagger} a_{s'}(Q_{q}^{\dagger} + Q_q) \]

\[ G_q(E) = \frac{1}{2\pi} [E - \omega_q - P_q(E)]^{-1} \]

\[ P_q(E) = \sum_{ss'} F_s^{(q)} F_{s'}^{(q)} \frac{f_s - f_{s'}}{E - E_s' + E_s}, \]
\[ \gamma_q(\omega) = \Im m P_q(\omega \pm i\varepsilon). \]

Quantal: ss' = ph
Thermal: ss' = pp', hh'

\[ \Gamma = \Gamma_Q + \Gamma_T = 2\gamma_q(E_{GDR}) \]

\[ E_{GDR} - \omega_q - P_q(E_{GDR}) = 0, \quad f_s = \left\{ \exp[(\varepsilon_s - \lambda)/T] + 1 \right\}^{-1} \]

GDR strength function:
\[ S_q(\omega) = \frac{1}{\pi} \frac{\gamma_q(\omega)}{[\omega - E_{GDR}]^2 + \gamma_q^2(\omega)}. \]
$ph \ (T = 0 \ & \ T \neq 0), \ pp' \ & \ hh' \ (T \neq 0)$
GDR width as a function of $T$

$^{63}\text{Cu}$
NDD, PRC 84 (2011) 034309

$^{120}\text{Sn}$ & $^{208}\text{Pb}$
NDD & Arima, PRL 80 (1998) 4145
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NDD & Arima, PRC 68 (2003) 044303

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\textbf{pTSFM}
(Kusnezov, Alhassid, Snover)
\textbf{AM}
(Ormand, Bortignon, Broglia, Bracco)
\textbf{FLDM}
(Auerbach, Shlomo)

$T_c \approx 0.57\Delta(0)$
**pTSFM**
(Kusnezov, Alhassid, Snover)

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120Sn & 208Pb
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New measurements at VECC (Kolkata):
$\alpha$ induced fusion reactions $^4\text{He} + ^{115}\text{In} \rightarrow ^{119}\text{Sb}^*$
at beam energies of 30, 35, and 42 MeV
Mukhopadhyay et al. PLB 709 (2012) 9

VECC data for $^{119}\text{Sb}$
Others: Data for tin region
$^{201}$TI

New data at low $T$:
D. Pandit et al. PLB 713 (2012) 434

Exact canonical pairing gaps

NDD & N. Quang Hung PRC 86 (2012) 044333
201\textsuperscript{TI}

New data at low $T$:
D. Pandit et al. PLB 713 (2012) 434

NDD & N. Quang Hung PRC 86 (2012) 044333
Sudhee Banerjee’s group at VECC Kolkata

N. Quang Hung
(TanTao U.)

$^97\text{Tc}$
Pairing effect in the TSFM on the GDR width
Rhine Kumar, Arumugam, NDD, PRC 90 (2014) 044308, PRC 91 (2015) 044305

Total free energy at a fixed deformation = Liquid-drop energy + Nilsson-Strutinsky shell correction:

\[ F_{\text{TOT}} = E_{\text{LDM}} + \sum_{p,n} \delta F \]

\[ \delta F = F - \tilde{F} \]

\[ F = \langle H_0 \rangle - \lambda N - TS = \sum_i (e_i - \lambda - E_i) - 2T \sum_i \ln[1 + \exp(-E_i/T)] + \frac{\Delta^2}{G}, \]

\[ \tilde{F} = 2 \sum_i (e_i - \lambda) \tilde{n}_i - 2T \sum_i \tilde{s}_i + 2\gamma_s \int_{-\infty}^{\infty} \tilde{f}(x) x \sum_i n_i(x) dx - \frac{\Delta^2}{G} \]

\[ \tilde{f}(x) = \frac{1}{\sqrt{\pi}} \exp(-x^2) \sum_{m=0}^{p} C_m H_m(x); \]

\[ C_m = \frac{(-1)^{m/2}}{2^m (m/2)!} \text{ if } m \text{ is even}, \quad 0 \text{ if } m \text{ is odd}, \]

\[ x = \frac{e - e_i^{\omega}}{}, \quad \gamma_s \text{ is defined from } d\tilde{F} / d\gamma_s = 0 . \]

\[ \tilde{n}_i = \int n_i \tilde{f}(x) dx, \quad n_i = [1 + \exp(\beta E_i)]^{-1}, \quad \tilde{s}_i = \int s_i \tilde{f}(x) dx, \quad s_i = -2 \left[ n_i \ln n_i + (1 - n_i) \ln(1 - n_i) \right], \quad S = \sum_i s_i \]
Averaged cross-section

GDR Hamiltonian:

\[ H = H_{\text{osc}} + \eta \, D^\dagger D + \chi \, P^\dagger P \]

\[ \sigma(E_\gamma) = \sum_i \frac{\sigma_{mi}}{1 + (E_\gamma^2 - E_{mi}^2)^2 / E_\gamma^2 \Gamma_i^2}, \quad \Gamma_i \approx 0.026 E_i^{(1.8 \sim 1.9)} \]

\[ \sigma_m = 60 \frac{2}{\pi} \frac{N \, Z}{A} \frac{1}{\Gamma} (1 + \alpha). \]

Expectation value of an observable including thermal shape and pairing fluctuations:

\[ \langle O \rangle_{\beta, \gamma, \Delta_P, \Delta_N} = \frac{\int \mathcal{D}[\alpha] \exp[-F_{\text{TOT}}(T; \beta, \gamma, \Delta_P, \Delta_N)/T] O}{\int \mathcal{D}[\alpha] \exp[-F_{\text{TOT}}(T; \beta, \gamma, \Delta_P, \Delta_N)/T]} \]

\[ \mathcal{D}[\alpha] = \beta^4 |\sin 3\gamma| \, d\beta \, d\gamma \, \Delta_P \, \Delta_N \, d\Delta_P \, d\Delta_N \]
$^{120}$Sn Without pairing

$T = 0.1$ MeV

$T = 0.4$ MeV

$T = 0.6$ MeV

$T = 0.8$ MeV

$^{120}$Sn With pairing

$T = 0.1$ MeV

$T = 0.4$ MeV

$T = 0.6$ MeV

$T = 0.8$ MeV
\[ H = H_0 - \gamma \hat{M}, \quad \hat{M} = \sum_{k>0} m_k (N_k - N_{-k}), \quad \hat{N} = \sum_{k>0} (N_k + N_{-k}), \quad N_{\pm k} = a_{\pm k}^\dagger a_{\pm k}, \]

\[ H_0 = \sum_{k>0} (\varepsilon_k - \lambda) N_k + \sum_{k>0} (\varepsilon_k - \lambda) N_{-k} + \sum_q \omega_q Q_q^+ Q_q + \sum_{k,k',>0,q} \mathcal{F}_{kk'}^{(q)} (a_k^+ a_{k'} + a_{-k}^+ a_{-k'}) (Q_q^+ Q_q) \]

\[ G_q(E) = \frac{1}{2\pi} \frac{1}{E - \tilde{\omega}_q}, \quad \tilde{\omega} = \omega_q + P_q(E), \]

\[ P_q(E) = \sum_{kk'} \left[ \mathcal{F}_{kk'}^{(q)} \right]^2 \left[ \frac{f_{k'}^+ - f_k^+}{E - E_k^- + E_{k'}^-} + \frac{f_{k'}^- - f_k^-}{E - E_k^+ + E_{k'}^+} \right]. \]

\[ f_k^\pm = \left[ \exp(E^\mp / T) + 1 \right]^{-1}, \quad E^\mp = \varepsilon_k - \lambda \mp \gamma m_k, \]

\[ \gamma_q(\omega) = \pi \sum_{kk'} \left[ \mathcal{F}_{kk'}^{(q)} \right]^2 \left\{ (f_{k'}^- - f_k^-) \delta(\omega - E_k^- + E_{k'}^-) ight. \]
\[ + (f_{k'}^- - f_k^-) \delta(\omega - E_k^+ + E_{k'}^+) \right\}, \]
$^{88}$Mo

$\bar{T} = 2.04$ MeV
$J = 38 \hbar$

$E^* = 126$ MeV

$\bar{T} = 3.06$ MeV
$J = 38 \hbar$

$E^* = 262$ MeV

Ciemala, PhD thesis (2013)

See also NDD, Ciemala, K miecik, Maj, PRC 87 (2013) 054313
GDR in nuclei produced during evaporation from $^{88}$Mo

Ciemala et al., PRC 91 (2015) 0454313
Conclusions

1. Thermal pairing plays an important role in quenching the GDR width at low T, leading to a nearly constant GDR width at $T \leq 1$ MeV. This has been showed within the PDM and the TSFM that includes pairing fluctuations, whose predictions agree well with the most recent experimental systematics. This means the TSFM can describe correctly the GDR width at low T even in open shell nuclei if thermal pairing is properly included.

2. The fact that both models show the same effect of thermal pairing means that it is robust and model independent.

3. Shell effects is crucial to describe the GDR width at low T only in closed shell nuclei. In open shell nuclei shell effects are negligible.

4. The PDM also successfully describes the GDR in hot nuclei at high angular momentum when pairing effect vanishes or negligible.