Overview of mean-field and beyond mean-field theoretical studies on giant resonances



Mean-field and/or Energy Density Functionals (EDFs)

$$E = \left\langle \Psi \middle| \hat{H} \middle| \Psi \right\rangle = \left\langle \Phi \middle| \hat{H}_{eff} \middle| \Phi \right\rangle = E[\hat{\rho}]$$

$$\Phi \left\rangle \text{ Slater determinant } \Leftrightarrow \hat{\rho} \text{ 1-body density matrix}$$

 $H_{eff} = T + V_{eff}$. If V_{eff} is well designed, the resulting g.s. (minimum) energy can fit experiment at best. Hartree-Fock or Kohn-Sham.

- Within a time-dependent theory (TDHF), one can describe harmonic oscillations around the minimum.
- The restoring force is: $v \equiv \frac{\delta^2 E}{\delta \rho^2}$



 hp^{-1}

$$X_{\rm ph}|ph^{-1}\rangle - Y_{\rm ph}$$

• The linearization of the equation of the motion leads to RPA¹. $\begin{pmatrix} A & B \\ -B^* & -A^* \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} = \hbar \omega \begin{pmatrix} X \\ Y \end{pmatrix}$

Modern functionals and techniques - I

- Skyrme (SEDF)
- **Gogny (GEDF)**
- **Relativistic MF or HF** (CEDF)

local functionals (evolved from $V_{eff} \div \delta(\mathbf{r}_1 - \mathbf{r}_2)$) non-local from V_{eff} having Gaussian shape covariant functionals

(Dirac nucleons exchanging effective mesons)

They are as "fundamental" as other models because of the KS theorem. They differ among one another (only) because of the ansatz about density dependence.

They are applicable to almost the whole isotope chart and (!) to highly excited states.





Fig. 4. The comparison of the uncertainties in the definition of two-proton and two-neutron drip lines obtained in CDFT and SDFT. The shaded areas are defined by the extremes of the predictions of the corresponding drip lines obtained with different parametrizations. The blue shaded area shows the area where the CDFT and SDFT results overlap. Non-overlapping regions are shown by dark vellow and plum colors for SDFT and CDFT, respectively. The results of the SDFT calculations are taken from the supplement to Ref. [2]. The two-neutron drip lines obtained by microscopic + macroscopic (FRDM [3]) and Gogny D1S DFT [5] calculations are shown by pretation of the references to color in this figure legend, the reader is referred to the web ersion of this Letter.)

A.V. Afanasjev *et al.*, Phys. Lett. B726, 680 (2013) CEDF

Modern functionals and techniques - II

• Several fully self-consistent spherical (quasi-particle) RPA codes. GC et al., Comp. Phys. Comm. 184, 142 (2013).

 Advances in deformed (Q)RPA.
 Example: ISGMR in ²⁴Mg
 The experimental strength function is reproduced by assuming prolate ground-state deformation.
 Y.K. Gupta *et al.*, PLB 748, 343 (2015).

Finite-amplitude method (FAM) quite instrumental !
T. Nakatsukasa *et al.*, PRC 76, 024318 (2007)

M. Kortelainen et al., arxiv:1509.02353 [nucl-th]

Octupole in ²⁴⁰Pu





Purposes of (Q)RPA studies

• **Test new models** – either new energy density functionals (EDFs), or models based on realistic forces that can be treated within linear response

• Find the **nature of elusive/new modes** ("pygmy" modes, toroidal modes ...)

• "Applications": nuclear Equation of State (EoS), astrophysics, matrix elements for $\beta\beta$ -decay ...





The nuclear matter incompressibility and the monopole resonance: what still ?

The relationship between K_{∞} and the energy of the GMR has been discussed for decades. Cf. J.P. Blaizot, Phys. Rep. 64, 171 (1980).



Mainly from ²⁰⁸Pb: $K_{\infty} = 240 \pm 20$. S. Shlomo et al., EPJA 30, 23 (2006). Density dependence of the functionals ?

- Open-shell nuclei seem to be "softer". Is the value from Pb biased or are we still unable to pin down K_{pairing} ? pn pairing ? P. Avogadro et al, PRC 88, 044319 (2013).
- Is there a "pygmy" monopole ? Cf. M. Vandebrouck.
- Monopole in a "bubble" nucleus ? A. Mutschler – Ph.D. thesis

Why do we strive to measure IV states ?

• Because they are interesting *per se* ... and they provide, in principle, access to the **SYMMETRY ENERGY**



Extraction of symmetry energy parameters and neutron skins - I



L = 59 ± 16 MeV [W.G. Newton at al., EPJA 50, 41 (2014); B.A. Li, NUSYM15]

X. Roca-Maza et al. (in preparation)

- Neutron skins correlated with the product $\alpha_{\rm D} J$ (cf. Droplet Model)
- Functionals that reproduce $\alpha_{\rm D}$ in ^{208}Pb do it also in ^{68}Ni and ^{120}Sn
- From RCNP/GSI data 20 < L < 66 MeV
- Warning: GRs sensitive to a combination of J and L $S(\rho_A) = J + \frac{L}{3\rho_0} \left(\rho \rho_A\right)$

Extraction of symmetry energy parameters and neutron skins - II

The correlation between L and the neutron skin is well accepted.

R.J. Funstahl, NPA 706, 65 (2002) B.A. Brown, PRL 85, 5296 (2002) B.A. Brown, S. Typel, PRC 64, 027202 (2001)

²⁰⁸Pb

2001 2004	0.180 ± 0.030
2004	
	0.120 ± 0.070
2010	0.194 ± 0.024
2011	0.156 ± 0.025
2012	0.168 ± 0.022
2012	0.330 ± 0.170
2013	0.216 ± 0.048
2013	0.190 ± 0.028
2014	0.150 ± 0.030
2015	0.254 ± 0.062
2015	0.218 ± 0.015
	2010 2011 2012 2012 2013 2013 2013 2014 2015 2015

From collective modes: 0.17 fm < neutron skin < 0.25 fm

Correlations – effect of the fitting protocol

- When the constraint on a property A included in the fit is relaxed, correlations with other observables B become larger.
- When a strong constraint is imposed on A, the correlations with other properties become very small.

X. Roca-Maza et al., JPG 42, 034033 (2015)

The debated nature of the "pygmy" dipole

Courtesy: A. Zilges

D. Savran et al., PPNP 76, 210 (2013)

A. Bracco et al., EPJA 51, 99 (2015)

- Many experiments have identified strength (well) below the GDR region.
- Is this a "skin mode", possessing some degree of collectivity ?
- Or does it just have single-particle character?

- Several theoretical calculations support the picture that the transition density of the "pygmy" states is mainly ISOSCALAR in the inner part of the nucleus while NEUTRONS dominate at the surface.
- There is a gradual transition to ISOVECTOR states that belong to the GDR tail.
- "Details" are model-dependent, as the amount of collectivity is.

Exclusive measurements

Gamma-decay

- Example: data from M. Scheck *et al.*, PRC 87, 051304(R) (2013).
- Cf. also talks by A. Bracco, S.G. Pickstone, J. Isaak.
- Comparison with microscopic models (e.g. QPM) do not seem to provide a simple picture so far.
- This should be pursued ! Example: isospin character from 2⁺₁ vs. 2⁺₂ decay.

Neutron-decay

- It can shed light on the structure of GRs. Fine structure ? Disentangle the GR tail and the "pygmy" part ?
- Mentioned in A. Bracco's talk.

FIG. 4. (Color online) Observed relative intensities, I_{rel} , for the decays of the spin-1 states to the given final levels.

Charge-exchange and Gamow-Teller Resonances

 $\varepsilon_{\rm ph}^{(II)}, \varepsilon_{\rm ph}^{(I)}$

 $\varepsilon_{\rm ph}^{(II)} - \varepsilon_{\rm ph}^{(I)} = \varepsilon_{j_{<}} - \varepsilon_{j_{>}}$ Unperturbed GT energy related to the spin-orbit splitting

Highest and lowest particle-

hole transitions in the picture

$$\hbar\omega\approx\varepsilon_{\rm ph}+\langle V_{\rm res}\rangle$$

RPA GT energy related also to V in $\sigma\tau$ channel

Osterfeld, 1982:

Using empirical Woods-Saxon s.p. energies, the GT energy is claimed to determine g_0 '

$$V_{\rm res} = g_0' \delta(\vec{r}_1 - \vec{r}_2) \sigma_1 \sigma_2 \tau_1 \tau_2$$

Fully microscopic calculations

- Different theories can reproduce the E_{GTR} in stable nuclei with quite a different picture behind them.
- In RMF the <u>pion</u> is playing the main role and a fit of the associated constant is needed.
- In RHF the dominant terms are <u>exchange</u> terms including the <u>isoscalar</u> σ , ω mesons.

Explore more extended isotopic chains including neutron-rich nuclei Consistent results for other charge-exchange modes (spin-dipole ...) Decay ?

Collectivity of very neutron rich light nucleus ⁸He

Taken from : H. Sakai, talk at IInd Topical Workshop on Modern Aspects in Nuclear Structure, Bormio, 19 - 22 February 2014

Second RPA calculations

• The wave function of the vibrational states is enriched by adding 2 particle-2 hole components on top of the 1 particle-1 hole already present in RPA.

$$X_{\rm ph}|ph^{-1}\rangle - Y_{\rm ph}|hp^{-1}\rangle + X^{(2)}_{\rm php'h'}|ph^{-1}p'h'^{-1}\rangle - Y^{(2)}_{\rm php'h'}|hp^{-1}hp'^{-1}\rangle$$

• If one projects on the 1p-1h space, assuming the "complicated" states are not interacting, one gets a very manageable equation

$$\begin{pmatrix} A+\Sigma(E) & B\\ -B & -A-\Sigma^*(-E) \end{pmatrix} \quad \Sigma_{\rm php'h'}(E) = \sum_{\alpha} \frac{\langle ph|V|\alpha\rangle\langle\alpha|V|p'h'\rangle}{E-E_{\alpha}+i\eta}$$

• Recently, full calculations by D. Gambacurta et al. go beyond this approximation.

Matrix elements of the type $\langle \pi \nu | V | \nu \pi \rangle$ are very strong ! NEED TO RE-FIT THE INTERACTION

(Q)RPA plus particle-vibration coupling

$$\begin{pmatrix} A + \Sigma(E) & B \\ -B & -A - \Sigma^*(-E) \end{pmatrix} \quad \Sigma_{php'h'}(E) = \sum_{k=1}^{n} \sum_{k=1}^{$$

One first solves self-consistent Hartree-Fock plus Random Phase Approximation (HF-RPA).

One adds the self-energy contribution (the state α is 1p-1h plus one phonon).

The scheme is known to be effective to produce the spreading width of GRs.

One reduces to collective phonons. No free phenomenological parameters.

 $\frac{\langle ph|V|\alpha\rangle\langle\alpha|V|p'h'\rangle}{E-E_{\alpha}+in}$ h h р h' **p'** h' ď (2) (1) h р h р h' **p**' h' b' (3) (4)

PVC = TBA / non charge-exchange states

- TBA = Time-blocking approximation. • Same diagrams as shown above.
- **Continuum included**

hand)

N. Lyutorovich et al., PLB 749, 292 (2015)

PVC model for Gamow-Teller Resonances

• The PVC calculations reproduce the lineshape of the GT response quite well.

Y. Niu et al., PRC 90, 054328 (2014).

Application of PVC to β -decay

PVC can strongly affect the half-lives:

$$T_{1/2} = \frac{D}{g_A^2 \int_{E_c}^{Q_\beta} S(E) f(Z, E) dE}$$

As already seen, it produces fragmentation and downward shift of the RPA peaks. Then, there is more strength in the decay window. The effect is enhanced by the phase-space factor. \rightarrow Better agreement with experiment.

Relativistic TBA calculations

E. Litvinova et al.

- Results available for both non chargeexchange and charge-exchange excitations.
- Upper panel: photobsorbtion cross section [50 Years of Nuclear BCS, World Sciencientific, 2013];
- Lower panel: spin-dipole strength [PLB 706, 477 (2012)].

Fig. 1. Total dipole photoabsorption cross section in stable medium-mass nuclei,

- RMF Lagrangians employed.
- Pairing included in the case of openshell systems.
- Extension beyond TBA in progress: Phys. Rev. C 91, 034332 (2015). Cf. also: M. Baldo *et al.*, J. Phys. G 42 (2015) 085109.

Short conclusion – mainly apologies

- It is hard to give a really complete and fair overview.
- Many groups are active.
- Technical progress/new physics goals.
- Apologies for repetitions of material already included in other talks.
- Moreover, some IMPORTANT aspects have not been covered like e.g. finite-temperature, large amplitude motion, particle-particle RPA and pairing modes, matrix elements for ββ-decay from QRPA ...

Outlook

THEORETICAL TOOLS

- Long way still to beyond mean-field calculations for openshell/deformed nuclei possibly including continuum effects
- Design effective interactions suited for calculations beyond mean-field (THEOS within ENSAR2)

SOME PHYSICS QUESTIONS

- Nature of the "pygmy" states description of n and γ decay
- Unified picture for GRs towards drip lines and very soft modes or two-neutrons and two-protons decay [K. Hagino et al., EPJA 51, 102 (2015)]
- Spin-isospin modes (extreme isospin and/or weak binding, relationship with weak interaction processes...)