Signatures of the Giant Pairing Vibration in ^{14}C and ^{15}C nuclei

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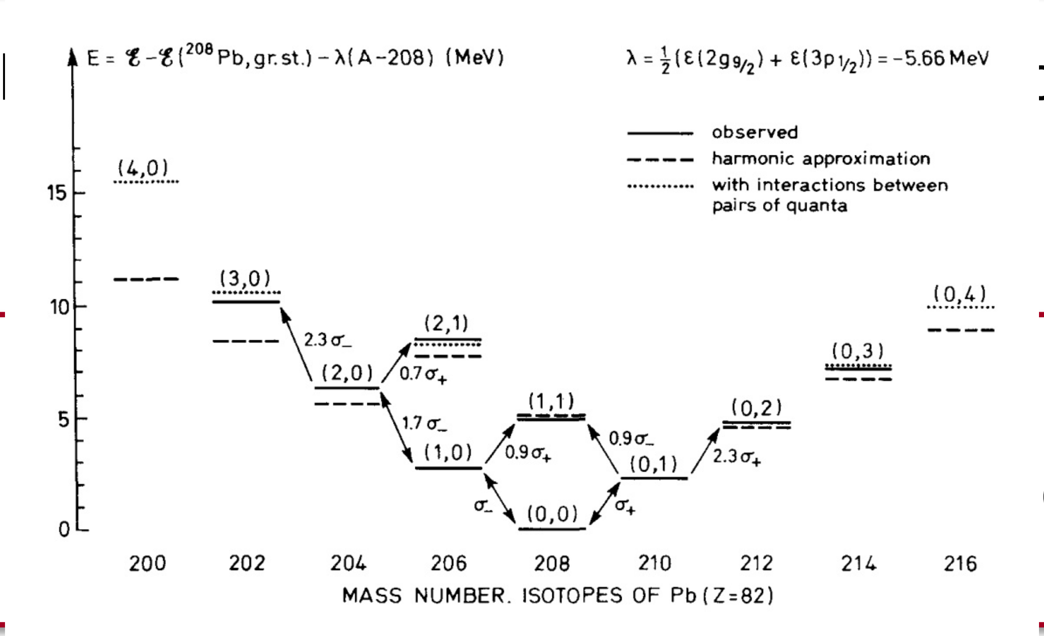
The 5th international conference on
"COLLECTIVE MOTION IN NUCLEI UNDER EXTREME CONDITIONS"
COMEX5

The pairing force

Effects of t omic nuclei



$$\delta B = \begin{cases} \Delta \\ 0 \\ -\Delta \end{cases}$$

ergy

Collective excitation modes

(p,t) and (t,p) reactions on Pb isotopes

Pairing vibrations and rotations

between 0^+ ground states differing by two neutrons

The pairing force

Large amount of theoretical and experimental activity → Still basic open questions

Giant Pairing Vibrations (*GPV*)

R.A. Broglia and D. Bes PLB 69 (1977) 129

- Excitation of a pair across major shells
- Analogy with **Giant Resonances** (GDR, GQR) ⟶ Small amplitude perturbations

Giant Resonances ⟷ Collective p-h excitations

Giant Pairing Vibrations (*GPV*) ⟷ Collective p-p or h-h excitations

What are the hypotheses?

Giant Pairing Vibrations (*GPV*)

- Mean field description of the system ground state (true for nuclei)
- Residual interaction in the pp channel
- Particle-hole symmetry (basic symmetry for systems of interacting fermions)

↓

We expect they exist!

GPV theoretical predictions

Several theoretical studies:

Predicted properties of the GPV (heavy nuclei)

- L = 0 multipolarity
- Excitation Energy ~ 72 $A^{-1/3}$

(~ 12 – 20 MeV)

- FWHM ~ 1-2 MeV
- Collectivity: B(GPV) ~ B(PV)
- Universality

✓ **on heavy nuclei (Pb and Sn isotopes)**

R.A. Broglia and D. Bes PLB 69 (1977) 129-133

L.Fortunato et al. EPJ A14, 37-42(2002)

✓ **with weakly bound exotic nuclei ((^{6}He,^{4}He) transfer reactions)**

W.von Oertzen and A.Vitturi,Rep.Prog.Phys.64 (2001) 1247

L.Fortunato, Phys.of Atomic Nuclei,Vol.66 (2003) 1445

✓ **on light nuclei (Oxygen isotopes)**

Excitation Energy ~ 20 MeV

E.Khan et al. PRC 69 (2004) 014314

B.Avez et al. PRC 78 (2008) 044318

Many experimental attempts:

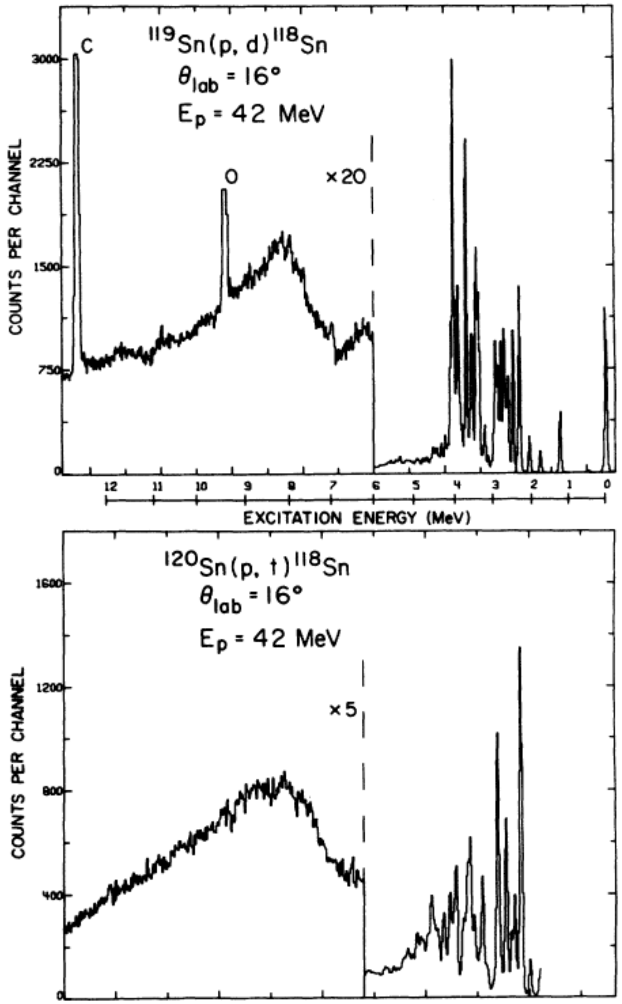
✓ **(p,t) and (t,p) reactions**

J. R. Shepard et al. NPA 322(1979)92
G. M. Crawley et al. PRC 22 (1980) 316
M. Matoba et al. PRC 27(1983) 2598
G. M. Crawley et al. PRL 39 (1977) 1451
G. M. Crawley et al. PRC 23 (1981) 589

A long story of unsuccesfull attempts

Experimental attempts

Deep hole state

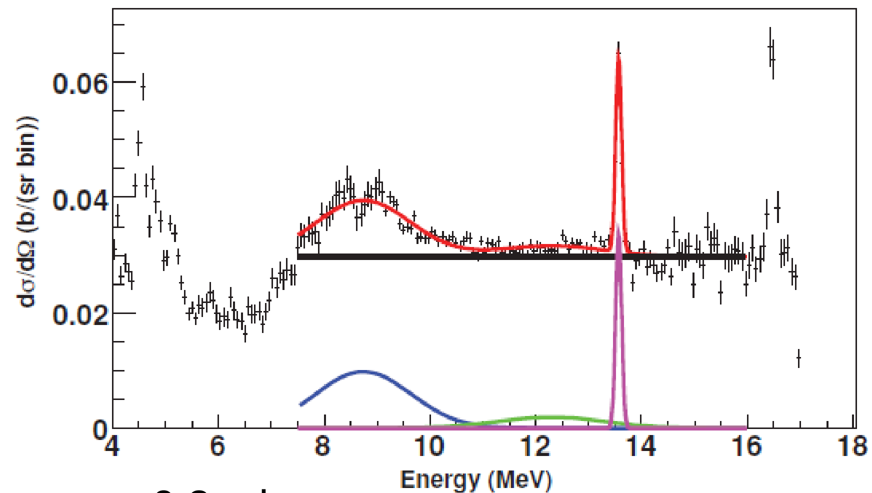


G.M. Crawley et al. PRL 39 (1977) 1451

Deep hole state

Small structure at ~12 MeV?

$^{120}Sn(p,t)^{118}Sn$ @ 50 MeV



σ_{max} < 0.2 mb

B. Mouginot et al. PRC 83 (2011) 037302

Many inconclusive experiments in 3 decades

Reaction mechanism

- **GPV** requires ***L* = 0** transfer
- In transfer reactions typically **large amount of angular momentum is transferred,** especially at high excitation energy

Incident energy →

1. Near the Coulomb barrier, weak sensitivity to angular momentum transfer
2. At high incident energy, large background
3. **Between 3-10 times the Coulomb barrier**

N. Anyas-Weiss et al. Phys. Rep. 12 (1974) 201

S. Kahana and A. J. Baltz Advances in Nuclear Physics Vol. 9

Projectile/target →

1. **Brink's matching conditions**

$$\Delta L = (\lambda_2 - \lambda_1) + \tfrac{1}{2} k_0 (R_1 - R_2) + Q_{eff} R/\hbar v \approx 0$$

D.M. Brink PLB 40 (1972) 37

2. Survival of a **preformed pair** in a transfer process favored if the initial and final orbitals are the same

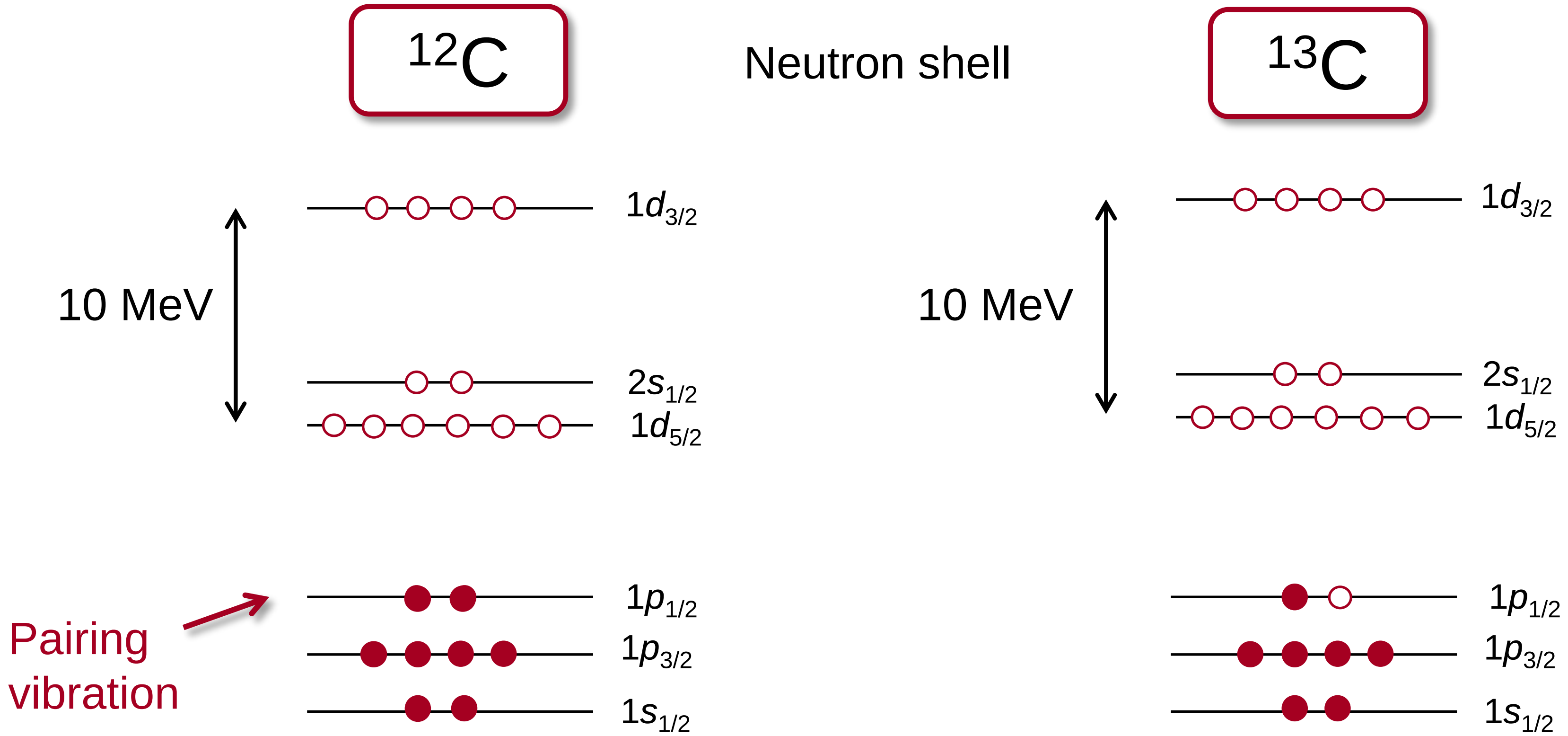
$(^{18}O,^{16}O)$ reactions

On light nuclei

Good candidates for ***L* = 0** transitions

- ✓ Favorable Brink matching conditions (D.M. Brink PLB 40 (1972) 37)
- ✓ Preformed neutron pair in ^{18}O
- ✓ At 3-5 times the Coulomb barrier good compromise between background, selectivity and sensitivity to low angular momentum transfer
- ✓ ^{14}C and ^{15}C good benchmarks

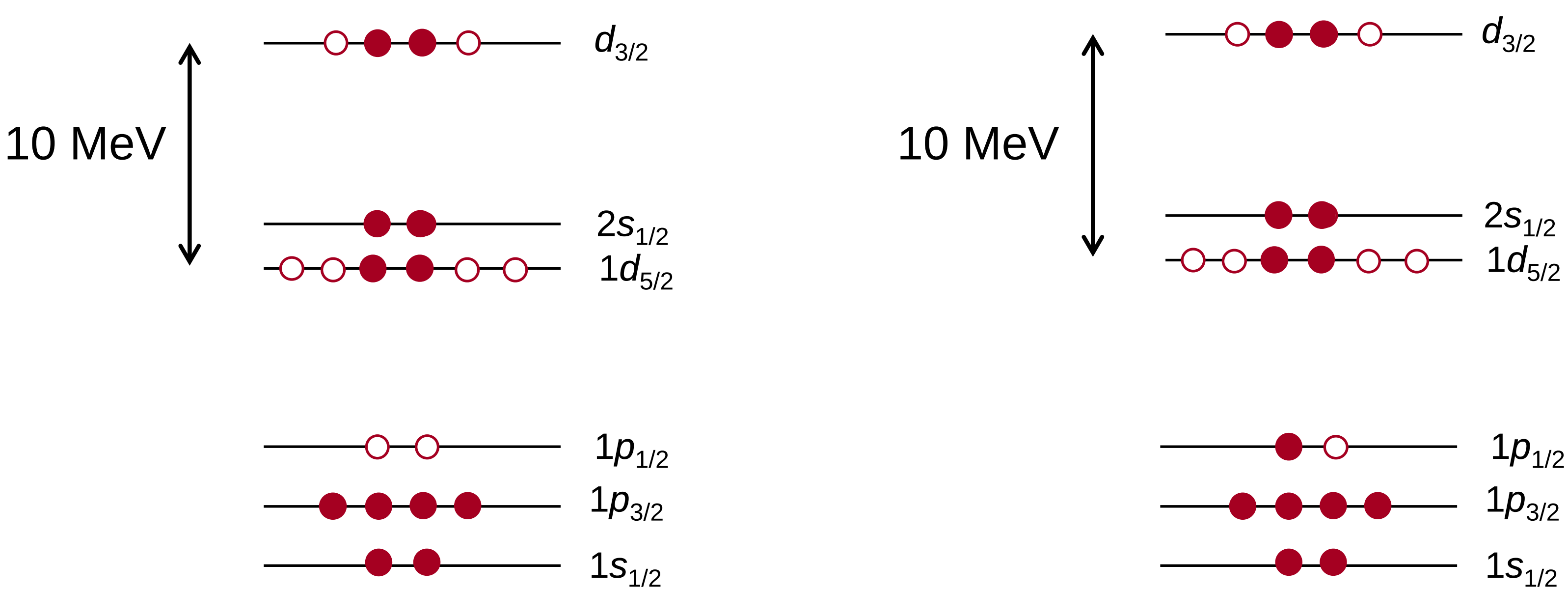
$L = 0$ transitions



$[1p_{1/2} \otimes 1p_{1/2}]^{0+}$ g.s.

$L = 0$ transitions

^{14}C Neutron shell ^{15}C



$(d_{3/2})^2$ and above missing states

$[1d_{5/2}\otimes 1d_{5/2}]^{0+}$	6.59
$[2s_{1/2}\otimes 2s_{1/2}]^{0+}$	9.75
$[1d_{3/2}\otimes 1d_{3/2}]^{0+}$	?

(t,p) reactions

$[(1p_{1/2}{}^{-1})^{1/2-}\otimes(1d_{5/2}\otimes 1d_{5/2})^{0+}]^{1/2-}$	3.10
$[(1p_{1/2}{}^{-1})^{1/2-}\otimes(2s_{1/2}\otimes 2s_{1/2})^{0+}]^{1/2-}$	5.87
$[(1p_{1/2}{}^{-1})^{1/2-}\otimes(1d_{3/2}\otimes 1d_{3/2})^{0+}]^{1/2-}$	?

S.Mordechai, et al., Nucl. Phys. A301 (1978) 463

S.Truong and H.T.Fortune, PRC 28 (1983) 977

About the experiment

Experimental setup

- $^{18}O^{7+}$ beam from Tandem at 84 MeV
- ^{12}C and ^{13}C thin targets (50 μg/cm²)
- Ejectiles detected by the MAGNEX spectrometer
- Angular settings $\theta_{opt} = 6°, 12°, 18°$

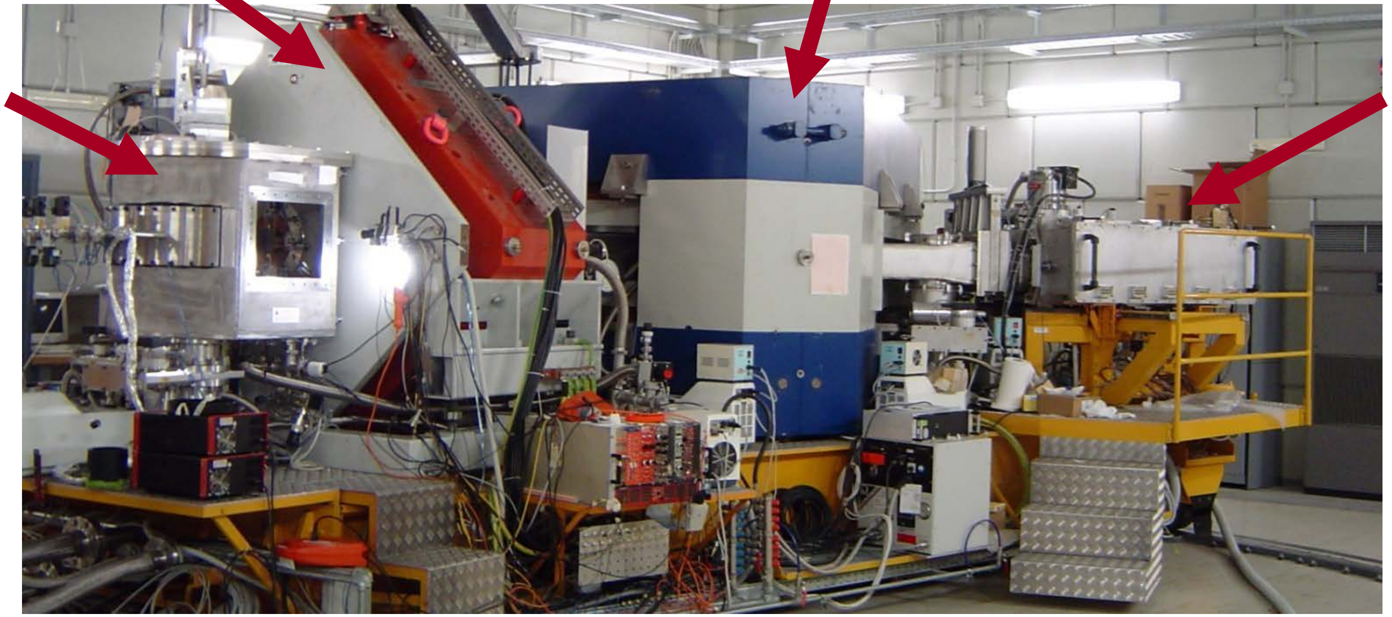
  $3° < \theta_{lab} < 24°$

MAGNEX

Optical characteristics	Actual values
Maximum magnetic rigidity (Tm)	1.8
Solid angle (msr)	50
Momentum acceptance	-14%, +10%
Momentum dispersion (cm/%)	3.68
First order momentum resolution	5400

Good compensation of the aberrations

F. Cappuzzello et al. Nova Publisher Inc (2011) 1



About the reaction mechanism

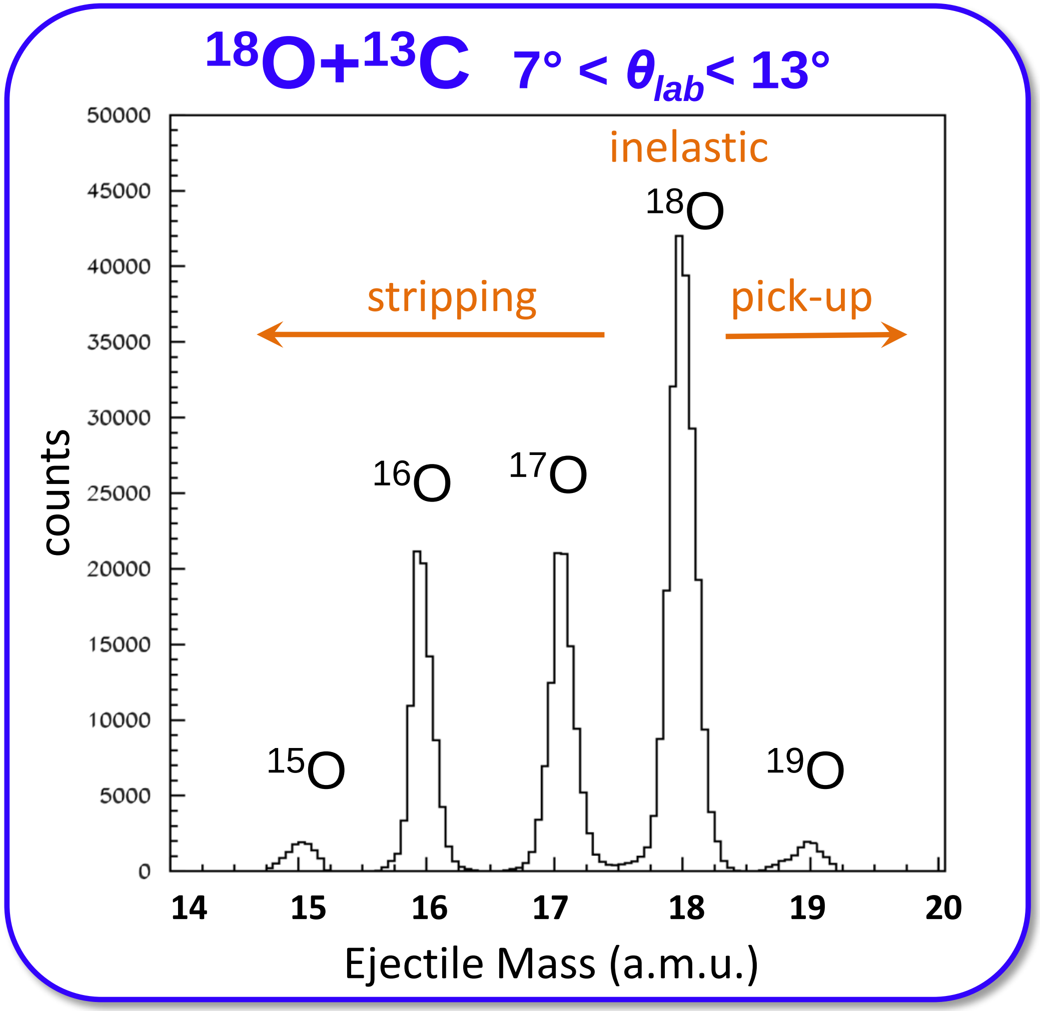
1. Transfer yields

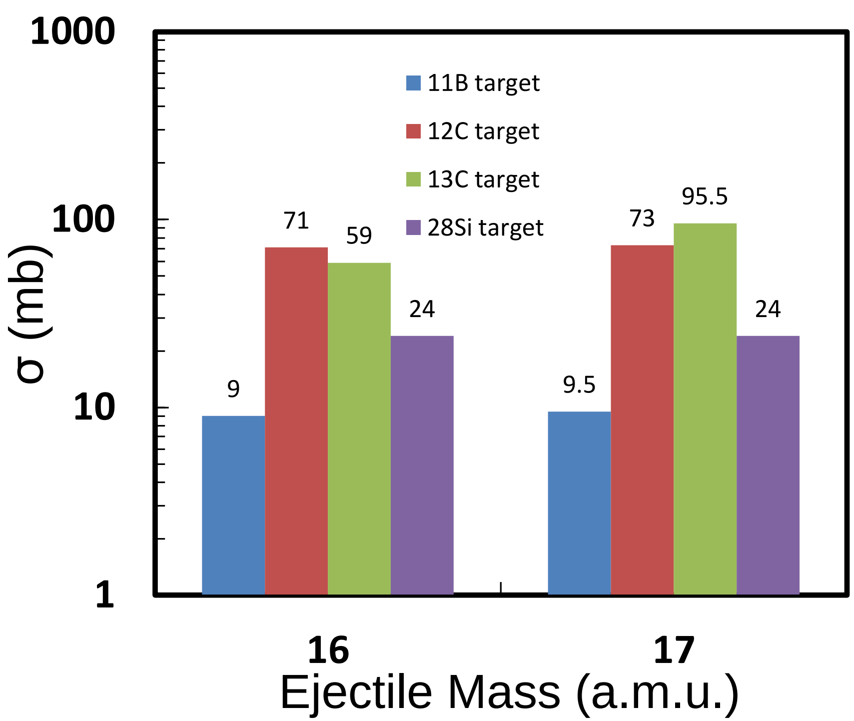
We compared the transfer yields for inelastic scattering, one-, two-, three-neutron transfer in the same conditions

Enhancement of the two-neutron transfer channel

The 2n transfer is not a 2nd order process

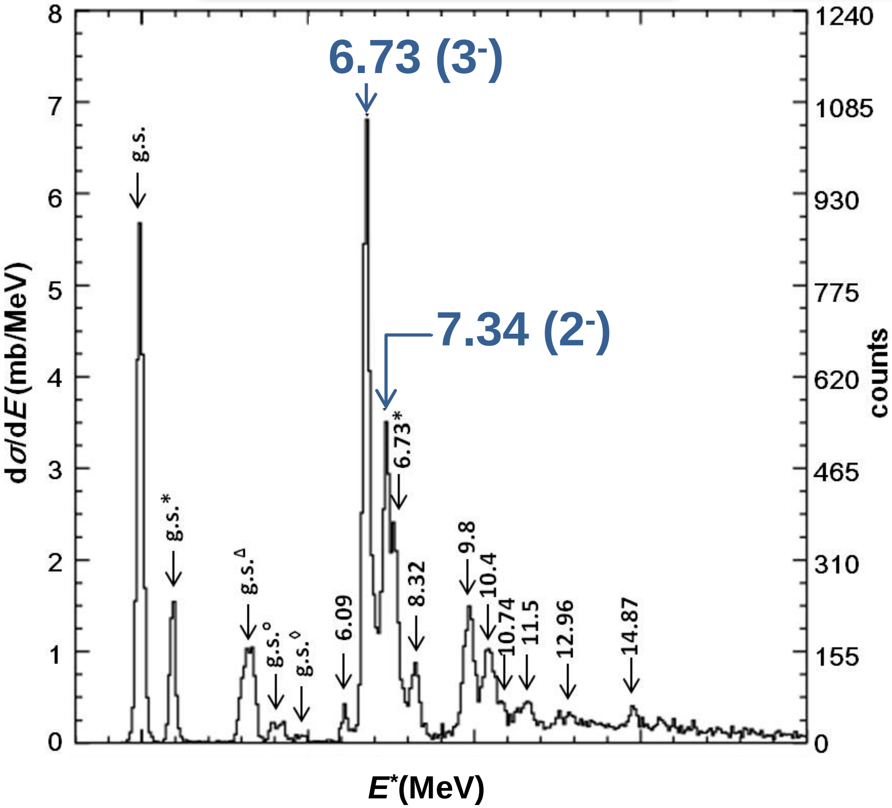
TRANSFER OF A CORRELATED PAIR

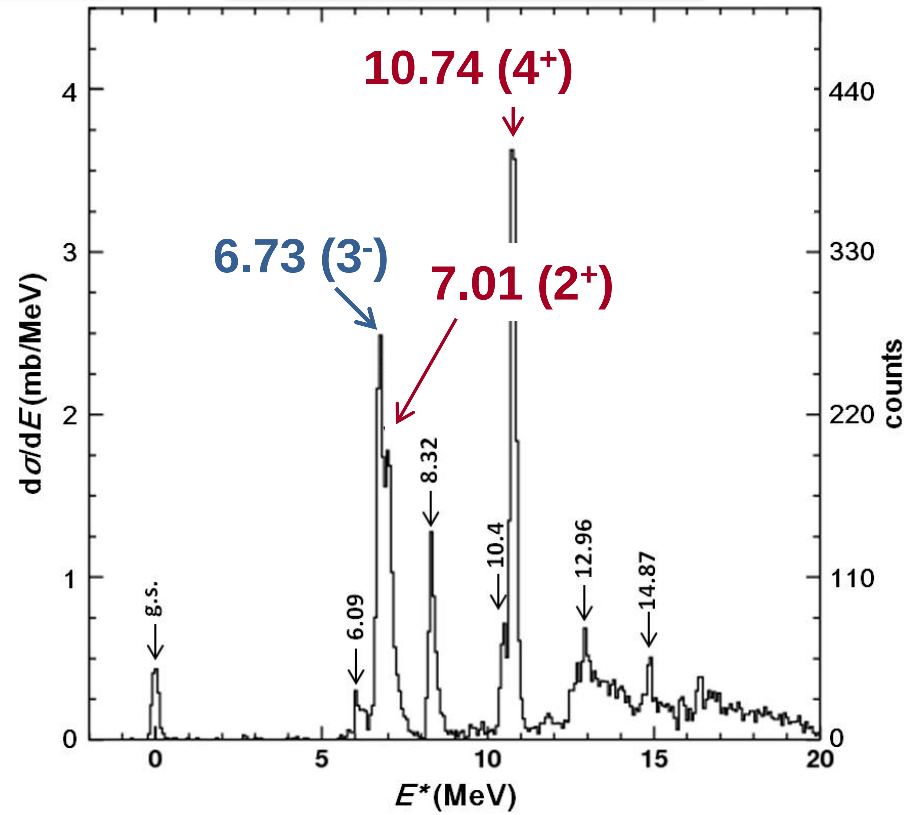




^{14}C spectrum via one- and two-neutron transfer

$^{13}C(^{18}O,^{17}O)^{14}C$ | $3° < \theta_{lab} < 5°$ | $^{12}C(^{18}O,^{16}O)^{14}C$





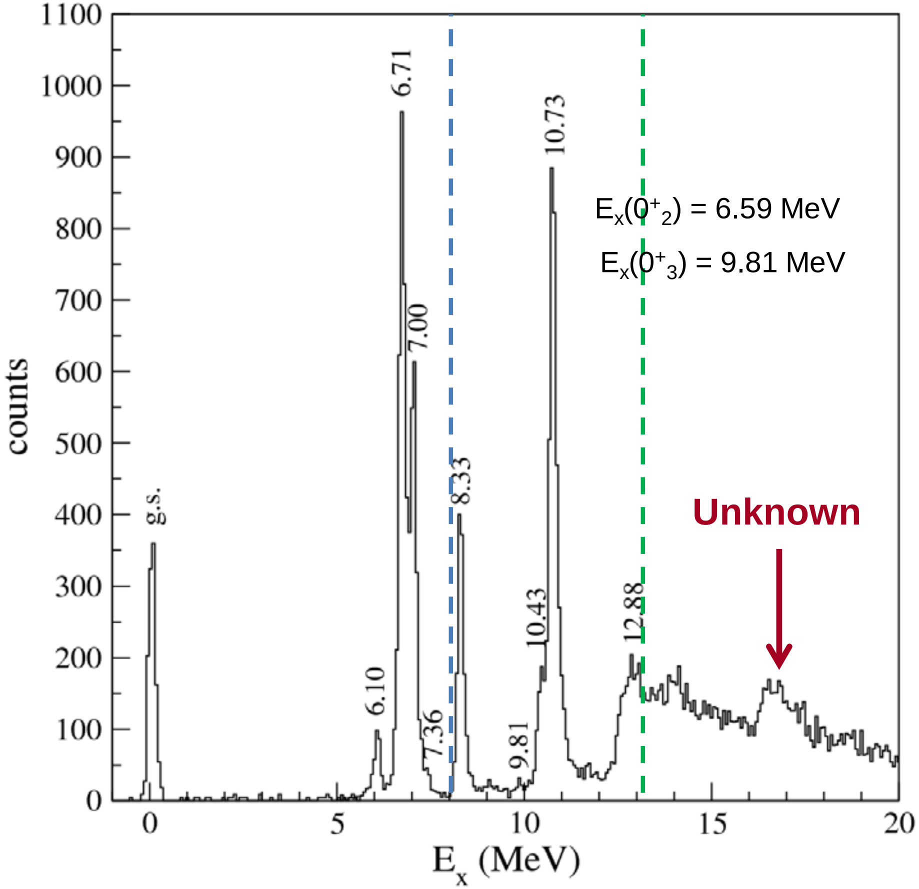
$$|[(^{13}C_{g.s.})^{1/2^-} \otimes (1d_{5/2})^{5/2^+}]^{2^-,3^-} >$$

$$|[(^{12}C_{g.s.})^{0^+} \otimes (1d_{5/2}, 2s_{1/2})^{2^+,4^+}]^{2^+,4^+} >$$

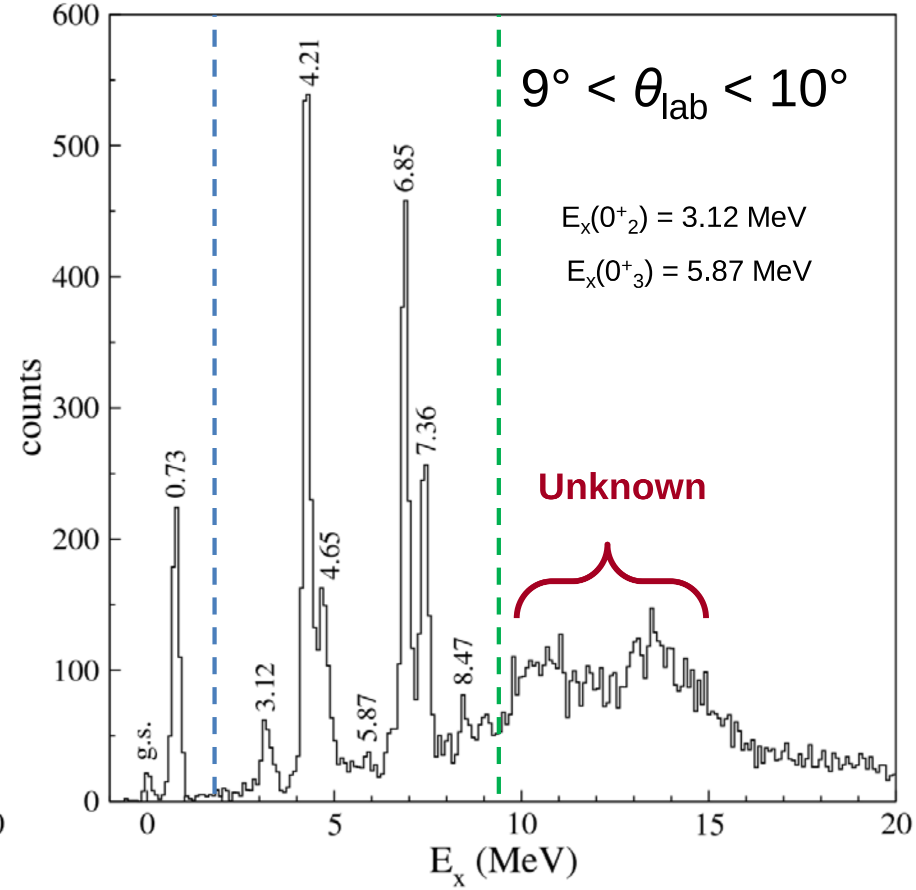
- Selectivity of natural parity states with large 2n⊗core overlap for the (^{18}O,^{16}O) reaction

Features of the (^{18}O,^{16}O) energy spectra

$^{12}C(^{18}O,^{16}O)^{14}C$



$^{13}C(^{18}O,^{16}O)^{15}C$



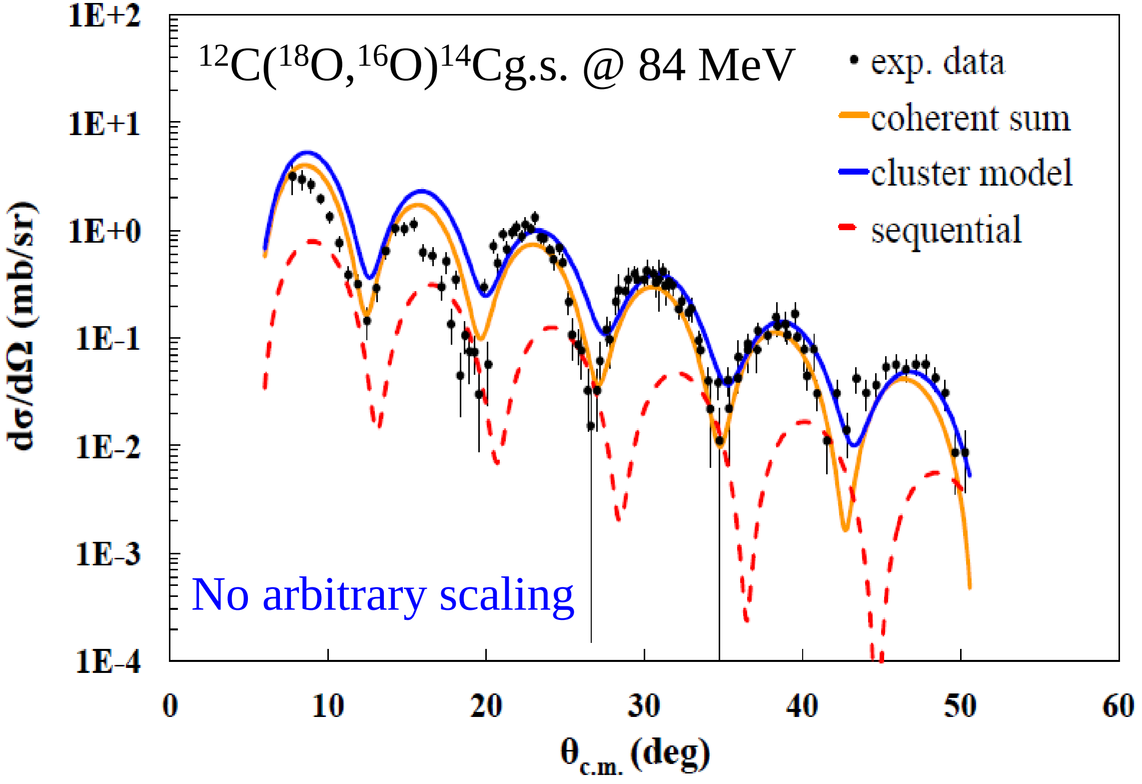
S_n

S_{2n}

- Almost complete suppression of the 0^+_2 , 0^+_3 states

3. Angular distribution

M. Cavallaro, et al., PRC 88 (2013) 054601



Extreme Cluster Model

(CRC)

❖ Relative motion of the 2n system frozen and separated by the c.m.

❖ Only the term with the 2n coupled to S = 0 participates to the transfer

Sequential transfer

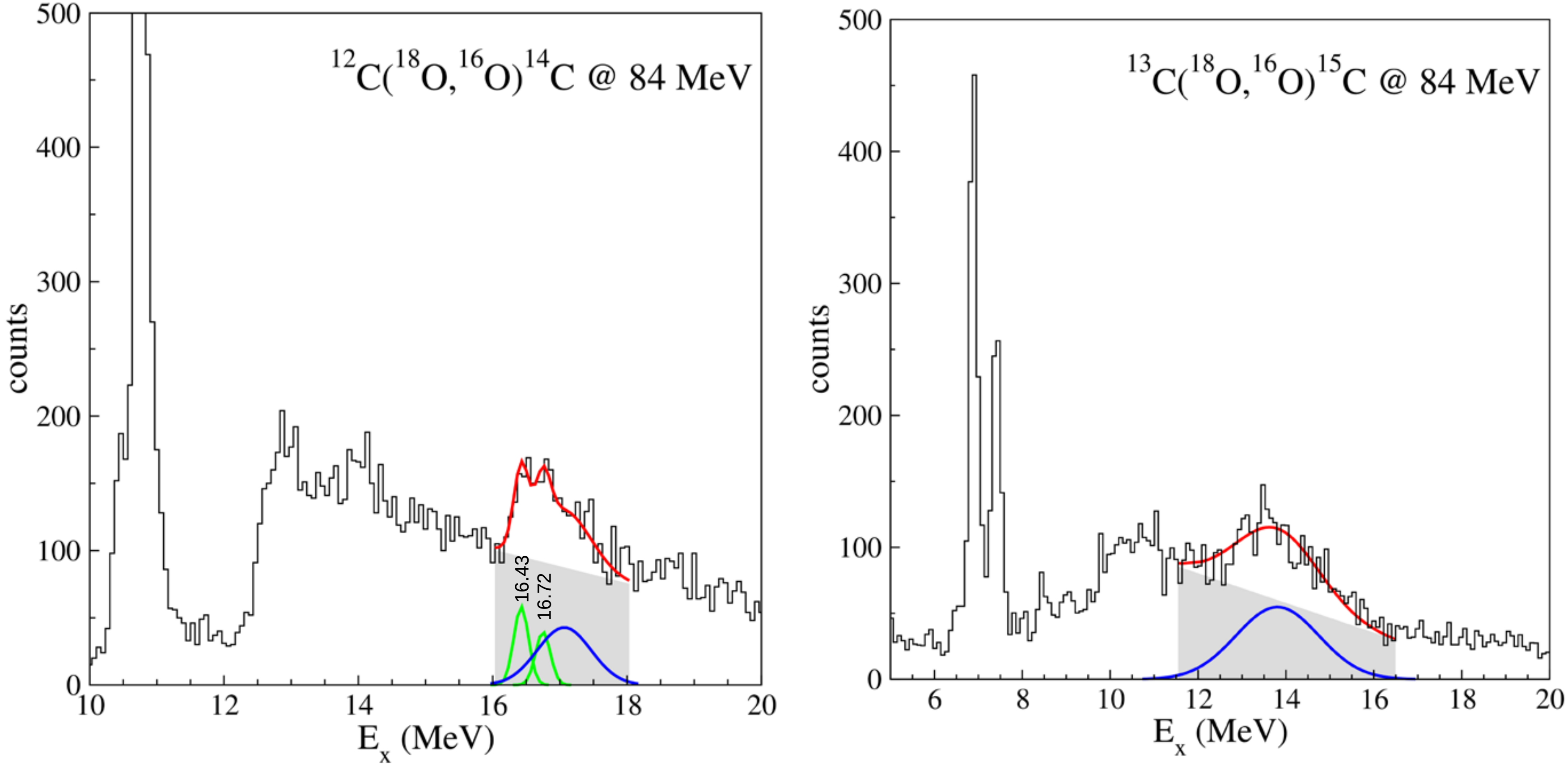
(DWBA)

Introducing the $^{17}O + ^{13}C$ intermediate partition

Coherent sum

Dominance of correlated transfer versus sequential two-step mechanism

Energy and width of the bumps



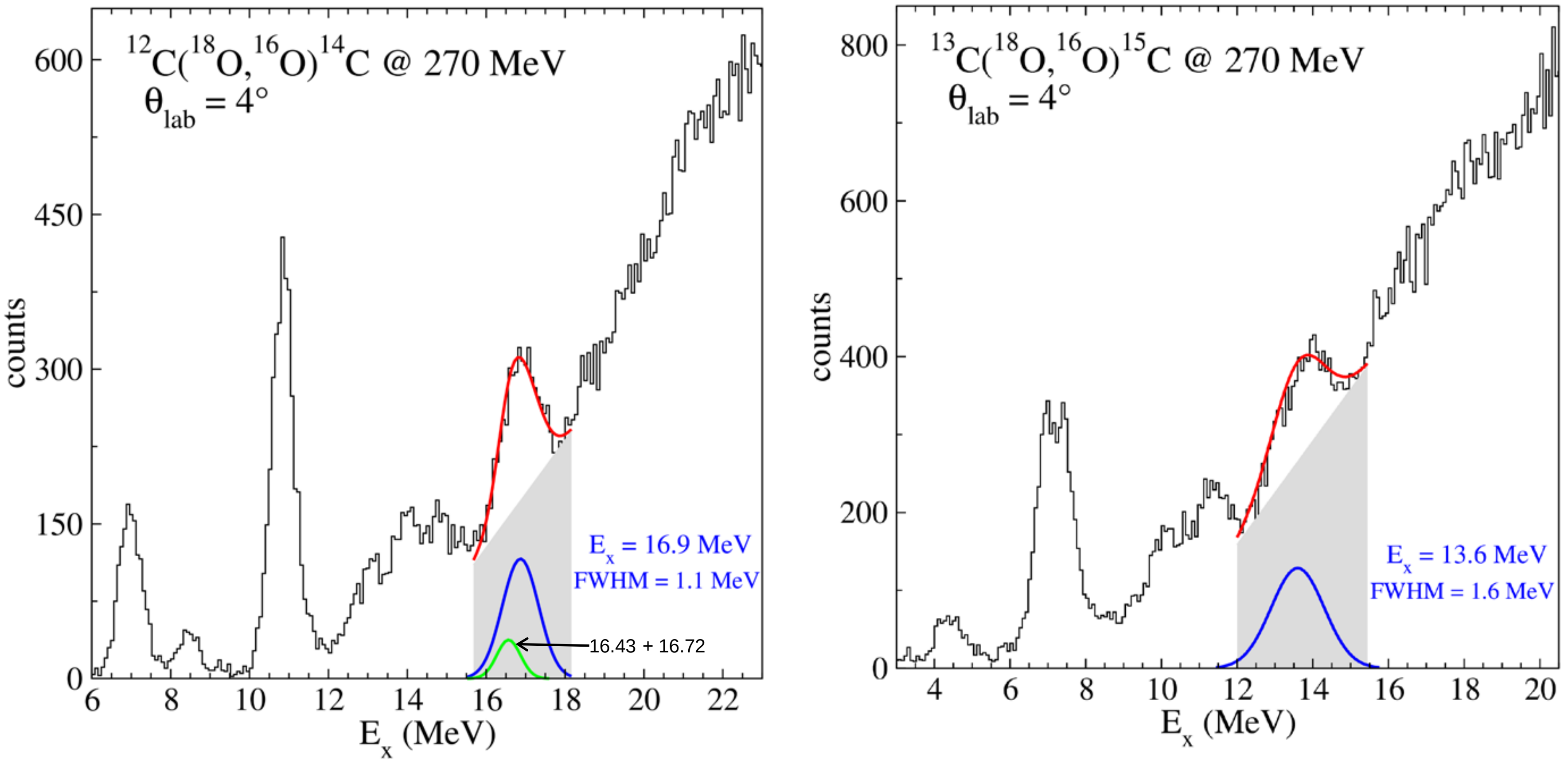
Gaussian model superimposed on a linear background

^{14}C **E_x = 16.9 ± 0.1 MeV FWHM = 1.2 ± 0.3 MeV**

^{15}C **E_x = 13.7 ± 0.1 MeV FWHM = 1.9 ± 0.3 MeV**

Changing incident energy

New experiment @ 270 MeV



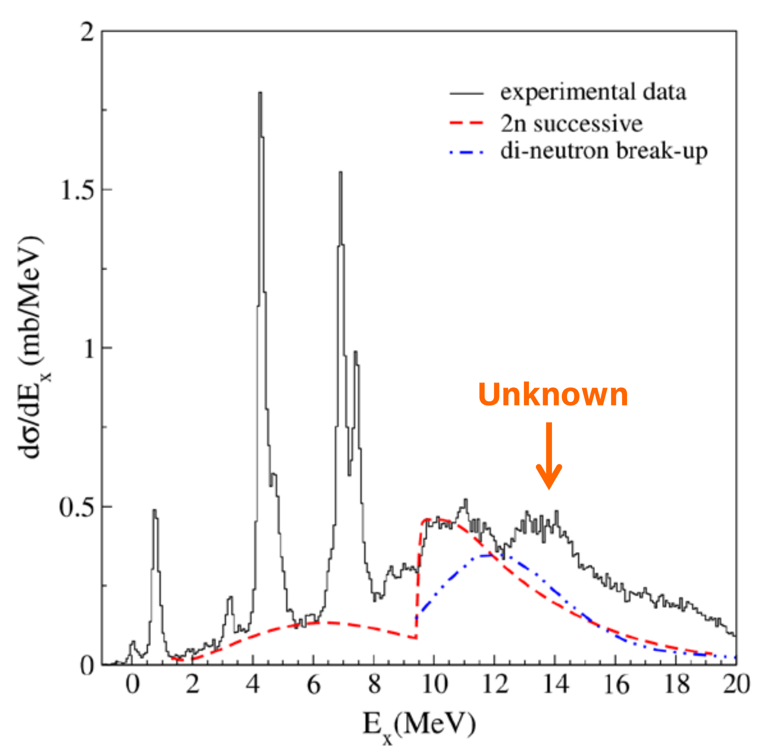
@ 84 MeV incident energy

$^{14}C_{GPV}$ E_x = 16.9 ± 0.1 MeV
FWHM = 1.2 ± 0.3 MeV

$^{15}C_{GPV}$ E_x = 13.7 ± 0.1 MeV
FWHM = 1.9 ± 0.3 MeV

Projectile break-up contribution

$^{13}C(^{18}O,^{16}O)^{15}C$ @ $7° < \theta_{lab} < 17°$



Two independent semi-classical models

1) Removal of two independent neutrons from the projectile
   - Transfer to the continuum of the target+n+n
   - Two-step mechanism
   - No n-n correlations
   - Optical model S-matrix for the n-target interaction

F. Cappuzzello et al., PLB 711 (2012) 347

2) Towing of a di-neutron system
   - Extreme hypothesis of the removal of a di-neutron from projectile
   - TDSE approach

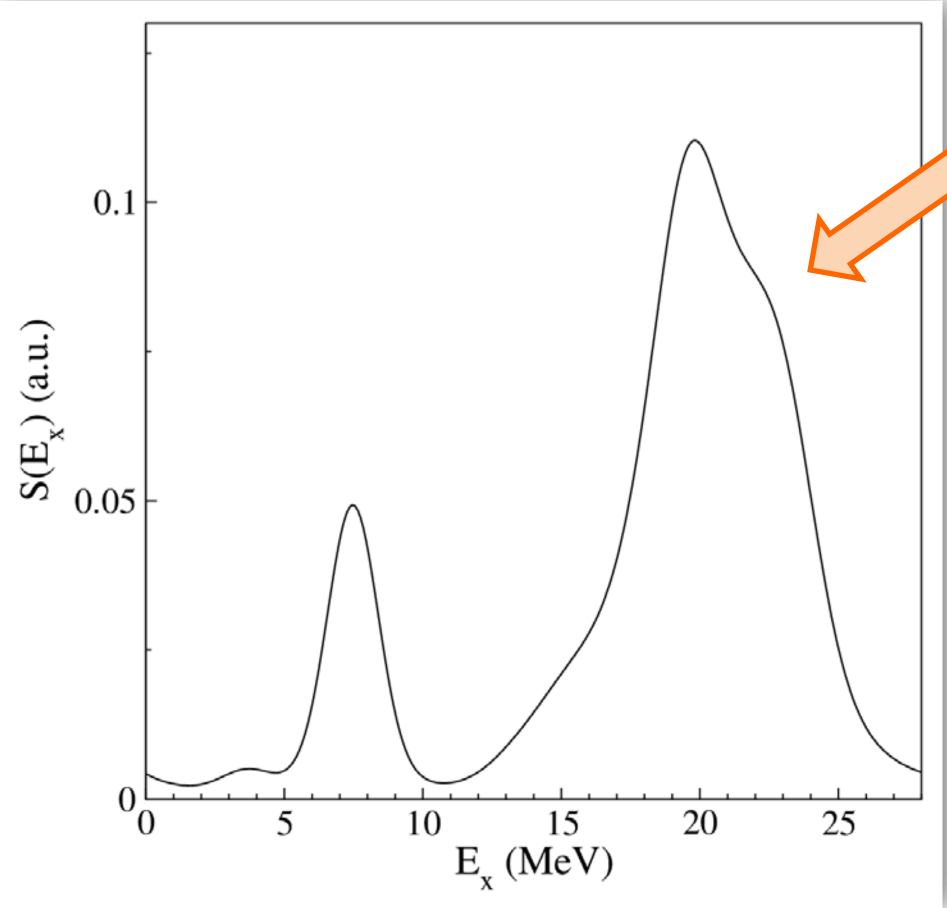
J.A. Scarpaci et al., PLB 428 (1998) 241

The ^{15}C bump at 13.7 ± 0.1 MeV is not reproduced

Similar results for ^{14}C case

Bumps energy and width

cQRPA calculations



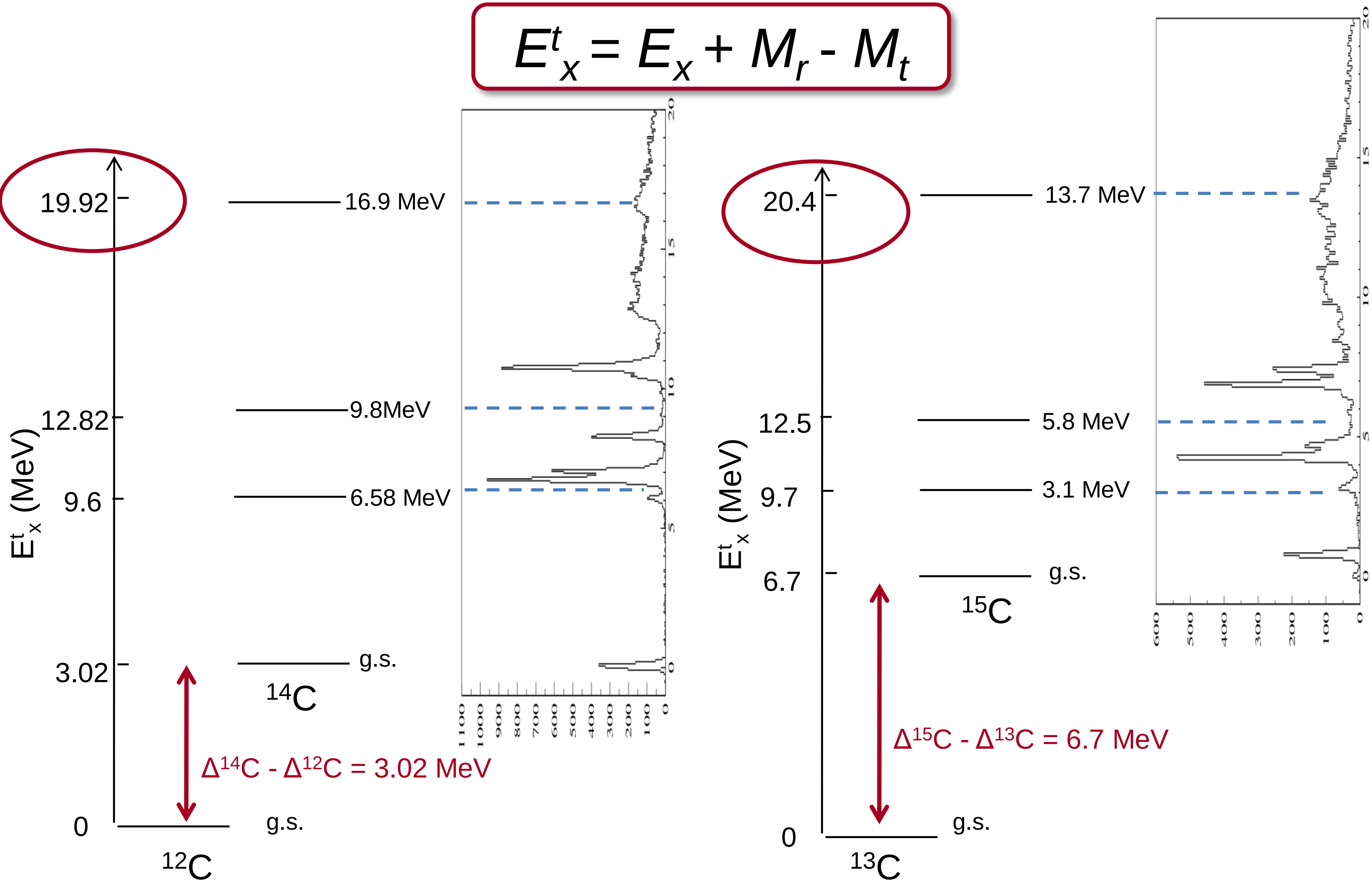
Collective structure at ~ 20 MeV with respect to the $^{12}C_{g.s.}$

E.Khan et al. PRC 69 (2004) 014314

B.Avez et al. PRC 78 (2008) 044318

Pairing energy scale

$$E^t_x = E_x + M_r - M_t$$



Measured widths

^{14}C E_x = 16.9 ± 0.1 MeV **FWHM = 1.2 ± 0.3 MeV**
^{15}C E_x = 13.7 ± 0.1 MeV **FWHM = 1.9 ± 0.3 MeV**

- Consistent with the discussions about the GPV
  (W.von Oertzen and A.Vitturi, Rep.Prog.Phys.64(2001)1247)
- ^{15}C bump has shorter half life

- We can speculate on the different contributions to the width

Escape width → ^{15}C bump is higher in the continuum

~ 3.77 MeV higher above S_n
~ 0.6 MeV higher above S_{2n}

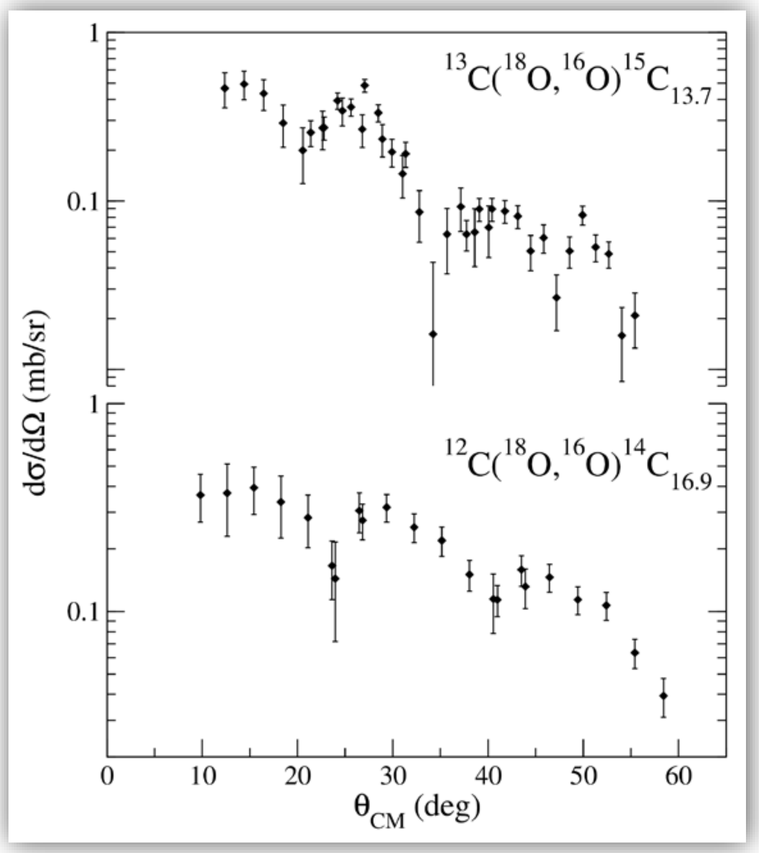
^{14}C S_n = 8.176 MeV S_{2n} = 13.122 MeV
^{15}C S_n = 1.218 MeV S_{2n} = 9.349 MeV

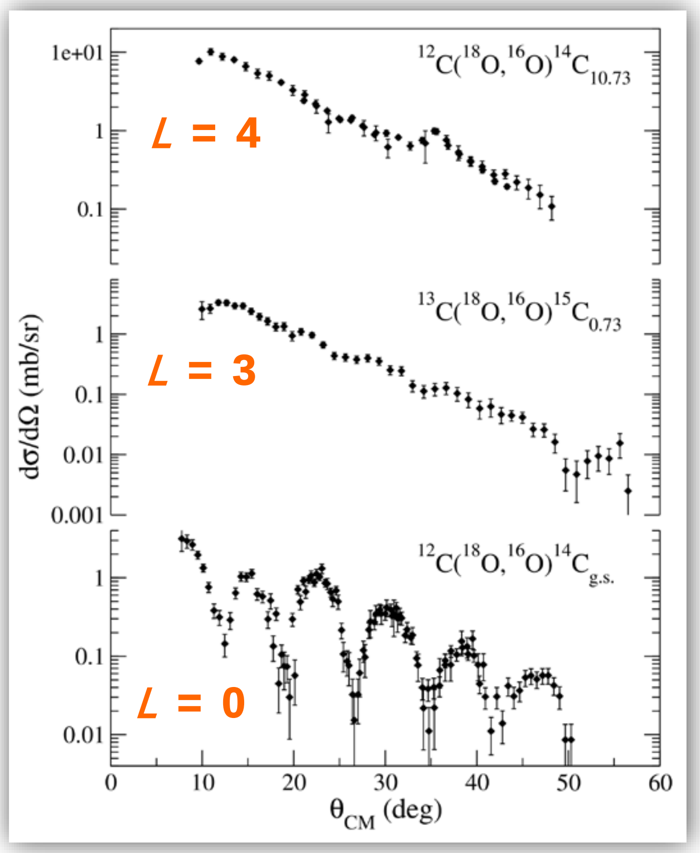
Spreading width → ^{15}C bump coupling with 3p-2h excitations due to the unpaired neutron

Landau width → ^{14}C bump coupling with *h-h* 0^+ excitations

Multipolarity

Multipolarity: angular distributions





First *L* = 0 indication

Equal population of the *M*-states in heavy-ion reactions near the Coulomb barrier

- L ≠ 0 transitions: featureless shape
- *L* = 0 transitions: oscillations clearly appear

S. Kahana and A. J. Baltz Advances in Nuclear Physics Vol. 9

Multipolarity: calculations for ^{14}C GPV

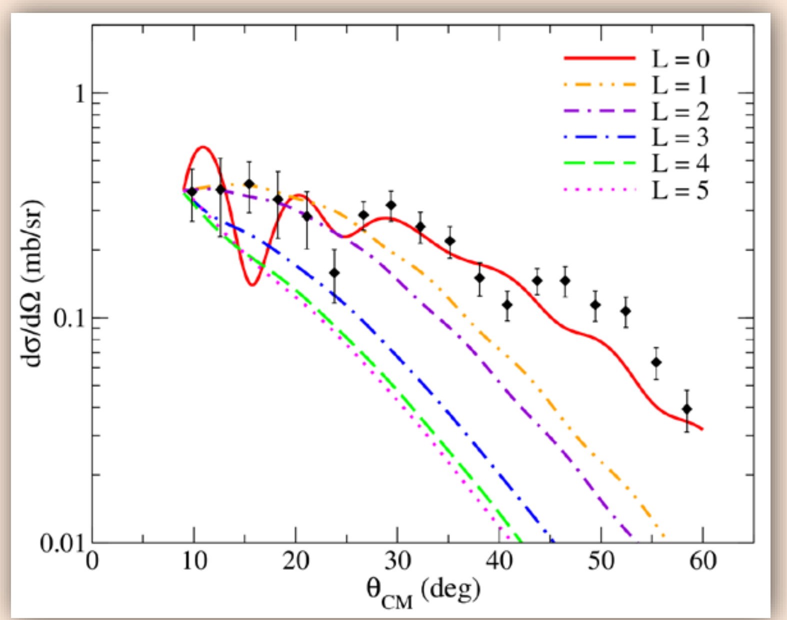
Common ingredients: a. Sao-Paulo parameter free double folding potential
b. Extreme cluster model approximation for the two neutrons

1. Discretized continuum scheme calculations

A.M. Moro and F.M. Nunes, Nucl. Phys. A 767 (2006) 138

- Three body assumption ⇨ finer details not accurate
- Global features: L = 0 cross section absolute value is found consistent with the experimental **without any scaling factor**

2. CRC calculations



- Same approach used to describe transitions to bound and resonant states in ^{14}C
  M. Cavallaro et al., PRC 88 (2013) 054601
- Calculations for various L components
- Artificial energy value of 12 MeV (below S_{2n})
- No spectroscopic amplitudes available
  ⇨ Renormalization at θ_{CM} = 9°
- Shape of the L = 0 calculation consistent with the experimental angular distribution

Both approaches suggest L = 0 transfer for the ^{14}C resonance at 16.9 MeV

Collectivity

Sum rules

•Unfortunately no exact formulation of a sum rule in the particle-particle L=0 channel

•Typically the **transfer probability** is analyzed to *evaluate* the collectivity

Transfer probability

The GPV strength is predicted to be similar to that of the *L* = 0 transition to the ground state in Pb and Sn even-even isotopes

Semi-classical description of the relative motion

$$\left(\frac{d\sigma(\theta)}{d\Omega}\right)_{tr} = \left(\frac{d\sigma(\theta)}{d\Omega}\right)_{sc} P_{tr}(\theta)F(Q,L)$$

$$\eta = \frac{Z_1 Z_2 e^2 p}{2E\hbar} = 3.5$$

$F(Q,L)$ Quantal corrections

$L = 0$

$$F(Q) = \exp[-C_1 \Delta Q^2]$$

$$\Delta Q = -\frac{1}{2}\frac{m_x(m_b - m_A)}{m_a + m_A}v^2 + m_{12}\ddot{r}\, m_x\left(\frac{R_A - R_a + R_0}{m_A} - \frac{R_b - R_B + R_0}{m_b}\right)$$

$$C_1 = \frac{R_0 m_{12}(1/\alpha)}{4(2E - E_B)\hbar^2}$$

$$\frac{P_{tr}(^{14}C_{GPV})}{P_{tr}(^{14}C_{g.s.})} = 3.2 \pm 1.7$$

$$\frac{P_{tr}(^{12}C_{GPV})}{P_{tr}(^{14}C_{g.s.})} = 3.5 \pm 0.8$$

$\left(\frac{d\sigma(\theta)}{d\Omega}\right)_{sc}$ Elastic scattering → $P_{tr}(\theta)$

Transfer probability

W. von Oertzen and A. Vitturi, Rep. Prog. Phys. 64 (2001) 1247
R.A. Broglia and A. Winther, Heavy Ion Reactions, (Addison-Wesley, 1991)

^{14}C E_x = 16.9 MeV FWHM = 1.2 MeV

^{15}C E_x = 13.7 MeV FWHM = 1.9 MeV

- ✓ Right energy
- ✓ Right width
- ✓ Right strength
- ✓ L = 0 mode

GPV population

Particle-hole symmetry confirmation

nature COMMUNICATIONS

ARTICLE

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OPEN

Signatures of the Giant Pairing Vibration in the ^{14}C and ^{15}C atomic nuclei

F. Cappuzzello[1,2], D. Carbone[2], M. Cavallaro[2], M. Bondì[1,2], C. Agodi[2], F. Azaiez[3], A. Bonaccorso[4], A. Cunsolo[2], L. Fortunato[5,6], A. Foti[1,7], S. Franchoo[3], E. Khan[3], R. Linares[8], J. Lubian[8], J.A. Scarpaci[9] & A. Vitturi[5,6]

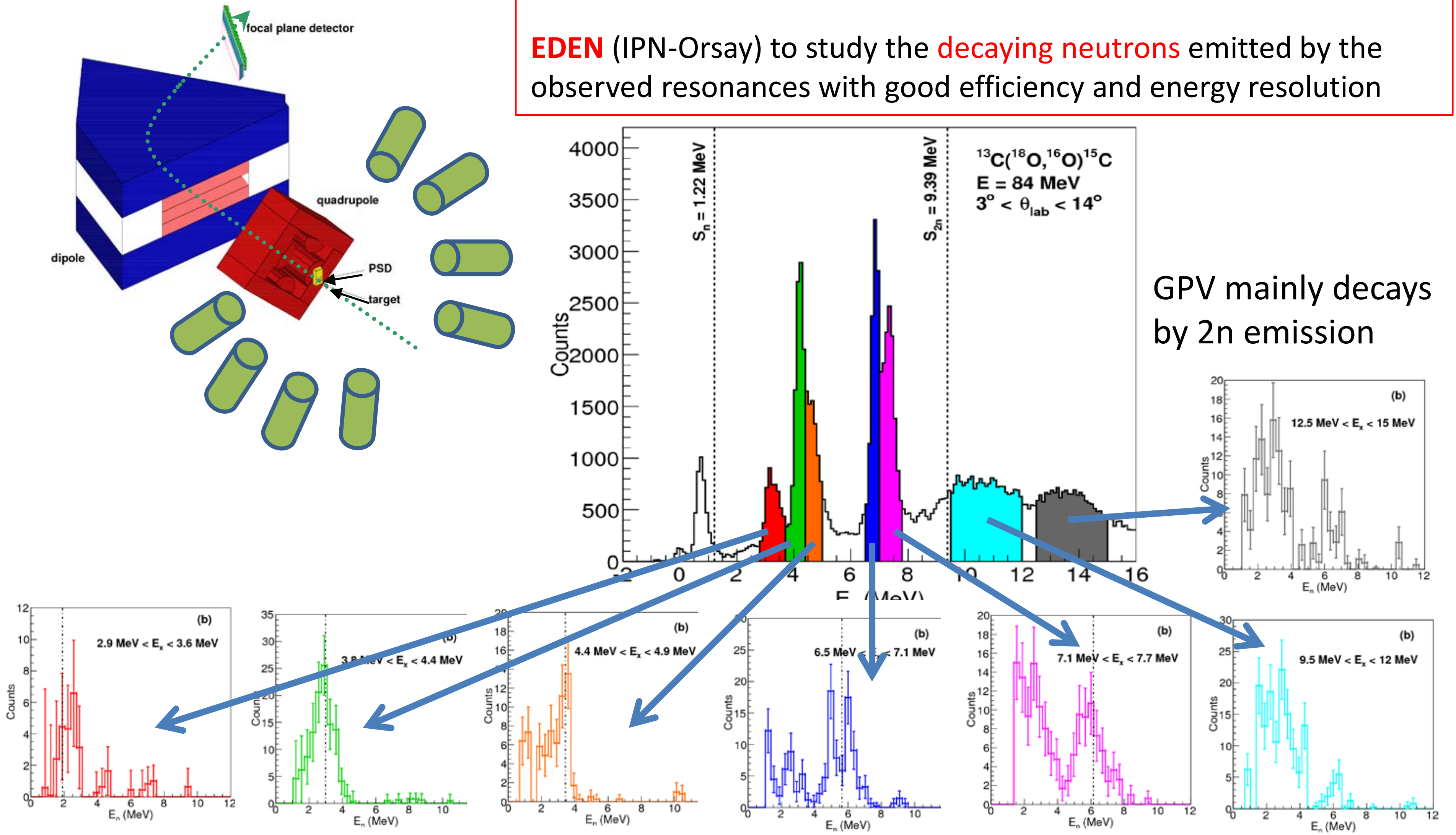
Resonance decay

Neutron decay of ^{15}C by time-of-flight

MAGNEX to measure high resolution energy spectra for well identified reaction products

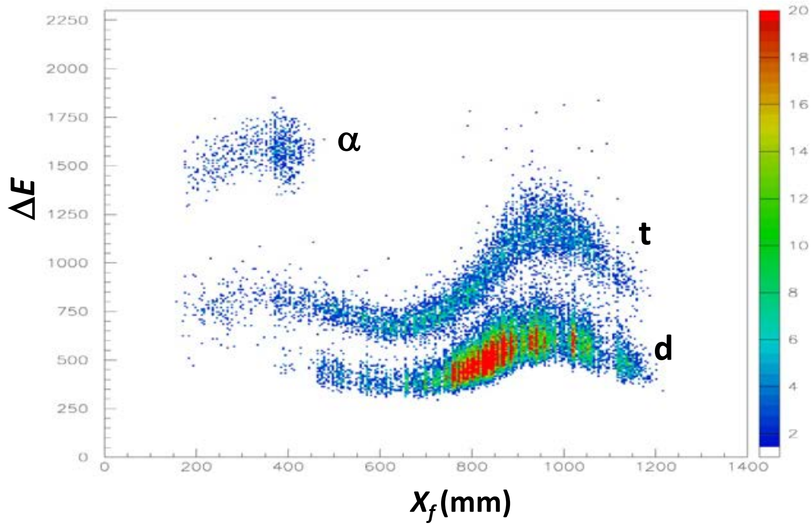
EDEN (IPN-Orsay) to study the decaying neutrons emitted by the observed resonances with good efficiency and energy resolution

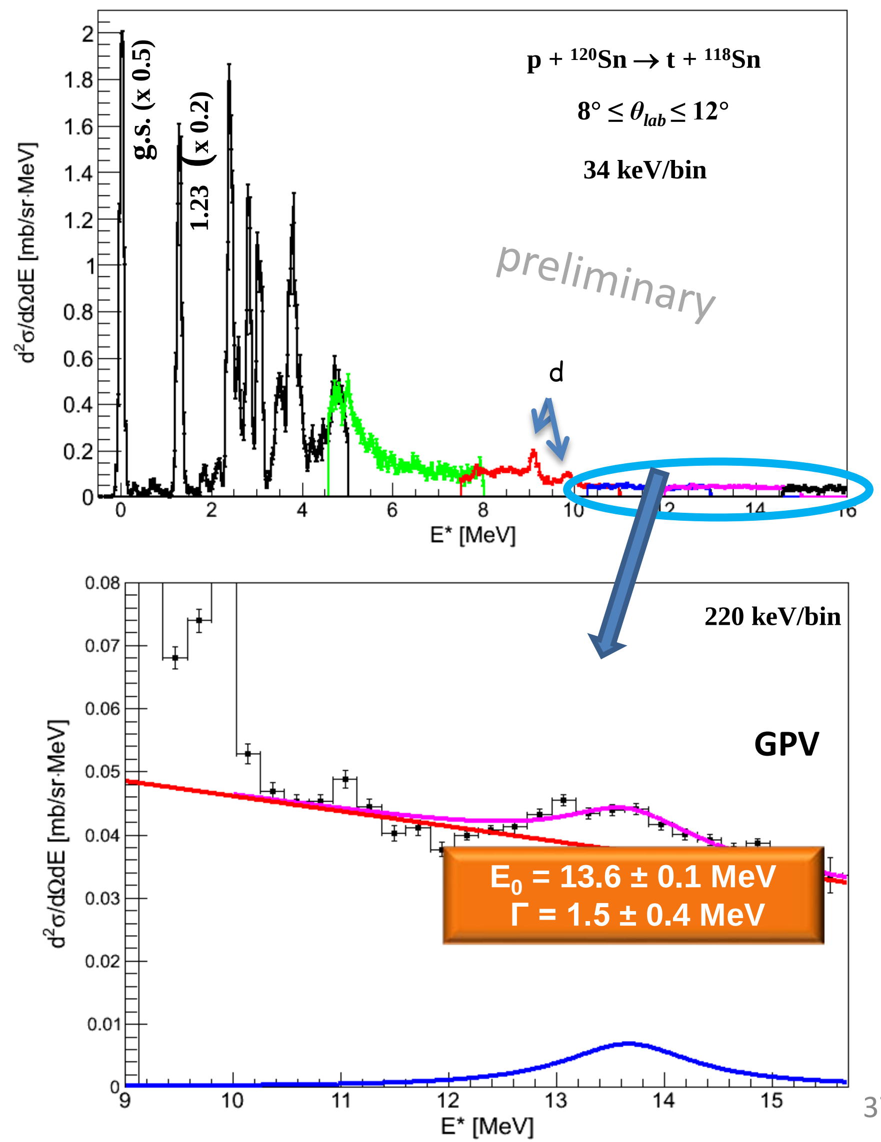
GPV mainly decays by 2n emission



Other systems

$^{120}Sn(p,t)^{118}Sn$ at 35 MeV (MAGNEX data)





The **Cross Section** of the GPV candidate in the range $8^\circ \leq \vartheta_{lab} \leq 12^\circ$ is $\sigma = 1.1 \pm 0.1$ μb

Agreement with B. Mouginot et al. PRC 83 (2011) 037302

Such a **small value** explains the historical difficulty to observe the GPV by (p,t) reactions.

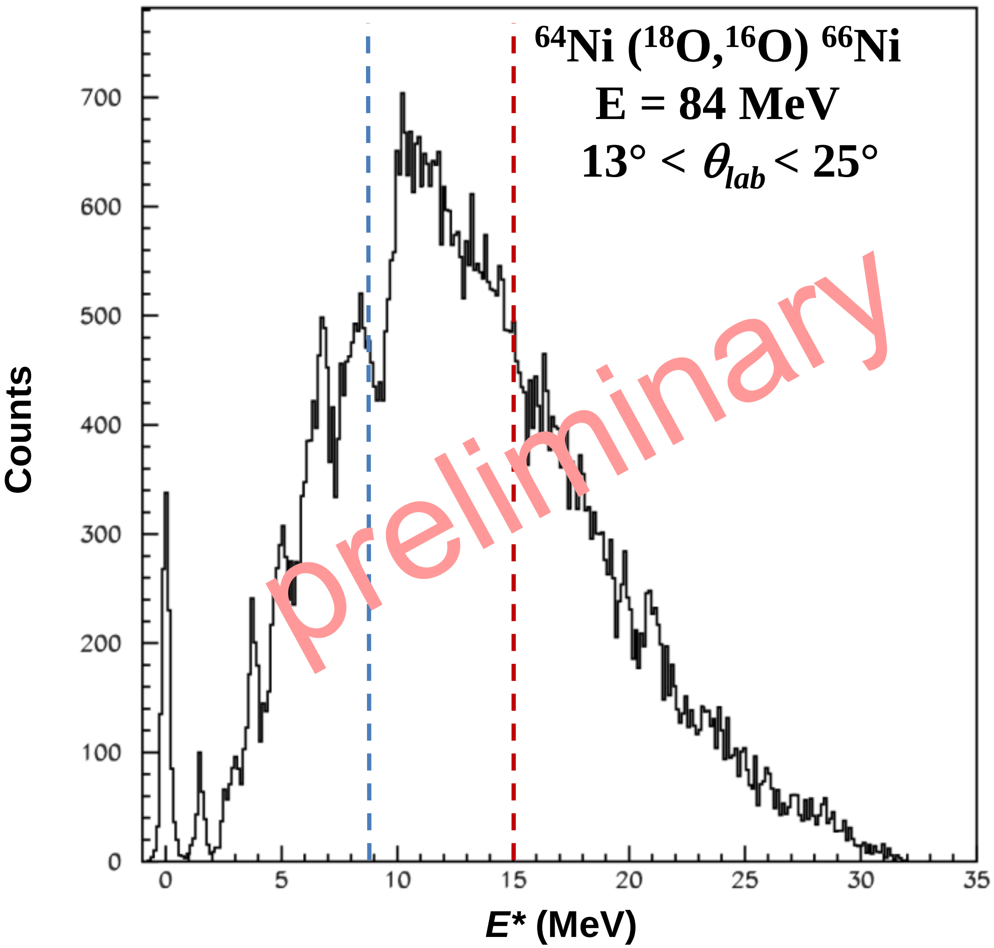
Conclusions and outlooks

✓ **First signature of the GPV**

GPV signals in T = 1 and T = 0 np pairing?

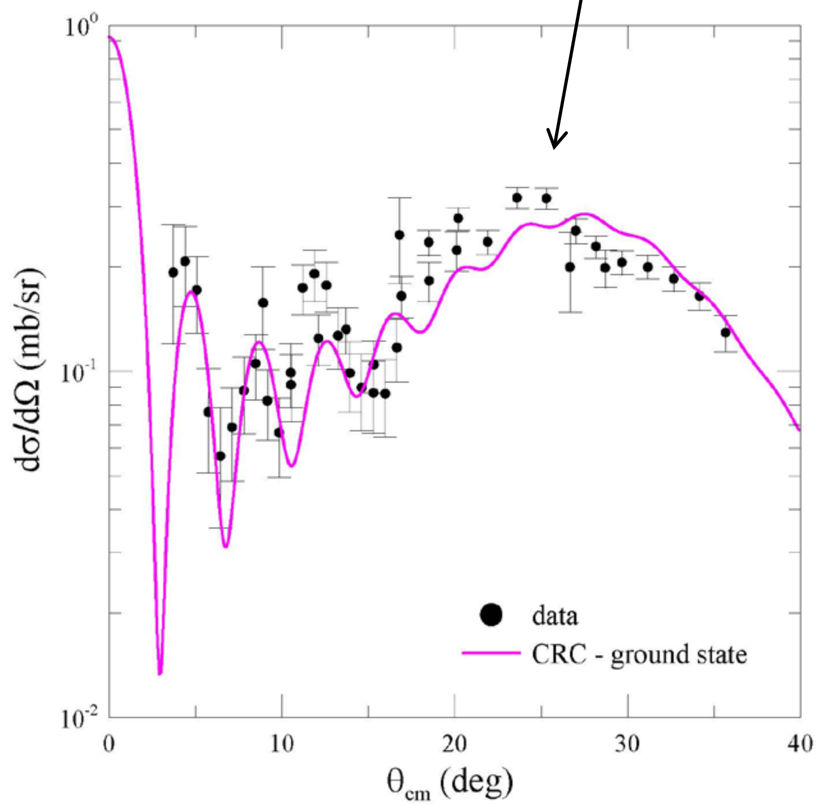
No reason why they should not be there

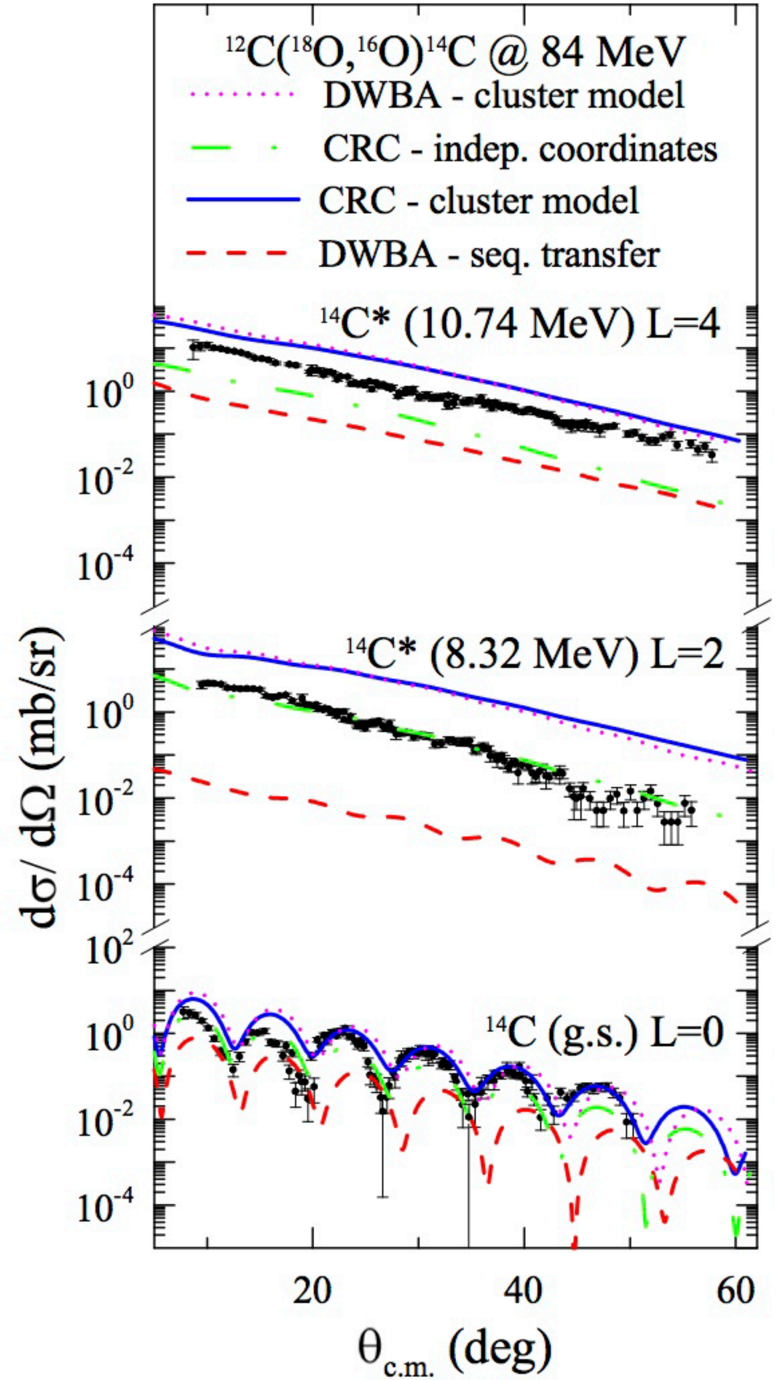
^{66}Ni energy spectra



^{64}Ni(^{18}O,^{16}O)^{66}Ni$_{g.s.}$

grazing angle





2n transfer

$^{12}C(^{18}O,^{16}O)^{14}C$

Angular distributions

M. Cavallaro et al., accepted by PRC

QRPA-calculations

Response function for the transfer of a neutron pair on ^{12}C

1) $^{12}C_{g.s.}$ with HFB:

- Mean field: Skyrme interaction
- Pairing interaction: zero-range density dependent
- Quasi-particle

$$V_{pair} = V_0 \left[1 - \left(\frac{\rho(r)}{\rho_0} \right)^{\alpha} \right] \delta(\boldsymbol{r}_1 - \boldsymbol{r}_2)$$

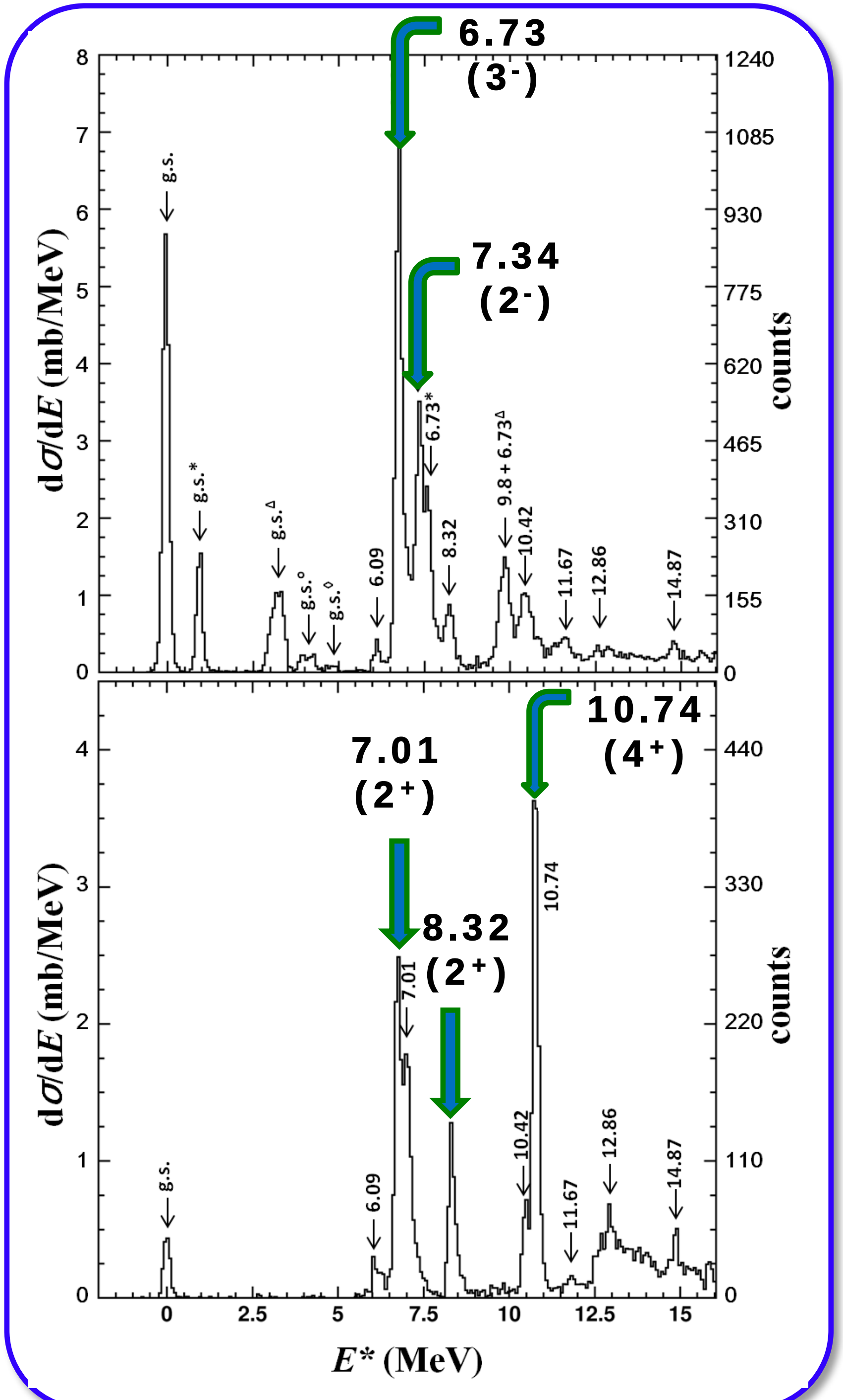
→ **Unperturbed response function G_0**

2) QRPA:

- Residual interaction $H = T + V = T + U_{HFB} + (V - U_{HFB}) = T + U_{HFB} + V_{res}$
- *p*-*p* excitations
- Linear response function approach

→ **Perturbed response function *G***

2. Energy spectra



$^{13}C(^{18}O,^{17}O)^{14}C$

$$\left| \left[\left(^{13}C_{gs}\right)^{1/2^-} \otimes \left(1d_{5/2}\right)^{5/2^+} \right]^{2^-,3^-} \right\rangle$$

In the (^{18}O,^{16}O), the **suppression of s.p. states**, which would require an uncorrelated transfer of 2n and the breaking of the initial pair in the $^{18}O_{g.s.}$, reveals the minor role of the **two-step dynamics**

$^{12}C(^{18}O,^{16}O)^{14}C$

$$\left| \left[\left(^{12}C_{gs}\right)^{0^+} \otimes \left(1d_{5/2},2s_{1/2}\right)^{2^+,4^+} \right]^{2^+,4^+} \right\rangle$$

Break-up calculations

Sequential transfer to the continuum of uncorrelated neutrons (two independent break-up processes)

$E_x > S_n$: $^{18}O + ^{13}C \rightarrow ^{16}O + ^{14}C_{g.s.} + n$

$E_x > S_{2n}$: $^{18}O + ^{13}C \rightarrow ^{16}O + ^{13}C_{g.s.} + n + n$

1) Calculation of the S-Matrix → Transfer probability

$$\frac{dP}{d\varepsilon_f}(j_f, j_i) = \sum_{j_f} \left(|1 - \bar{S}_{j_f}|^2 + 1 - |\bar{S}_{j_f}|^2 \right) B(j_f, j_i)$$

Elastic break-up Neutron absorption

2) Total transfer cross section

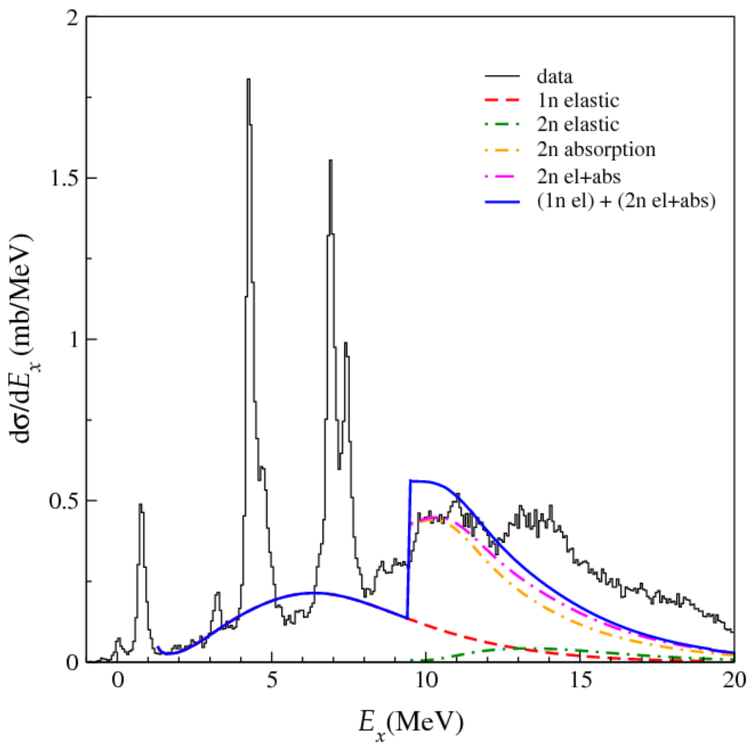
$$\frac{d\sigma_{1n}}{d\varepsilon_f} = C^2 S \int_0^\infty b\,db\, \frac{dP(b)}{d\varepsilon_f} P_{el}(b)$$

Semi-classical treatment of the relative motion

Initial state spectroscopic factor Core-target elastic scattering

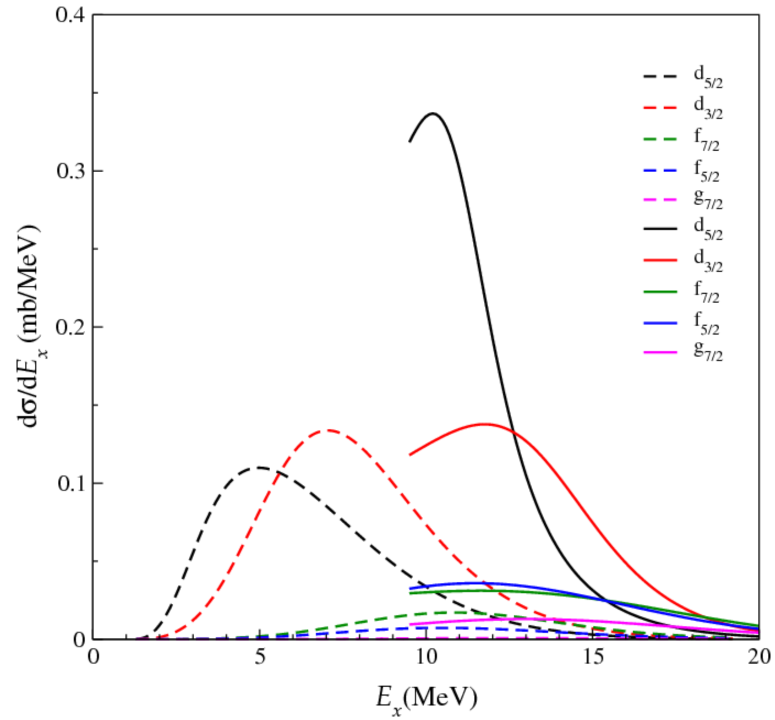
Break-up calculations

$^{13}C(^{18}O,^{16}O)^{15}C$



Above S_{2n}
the absorption term dominates compared to the elastic BU

Partial wave decomposition

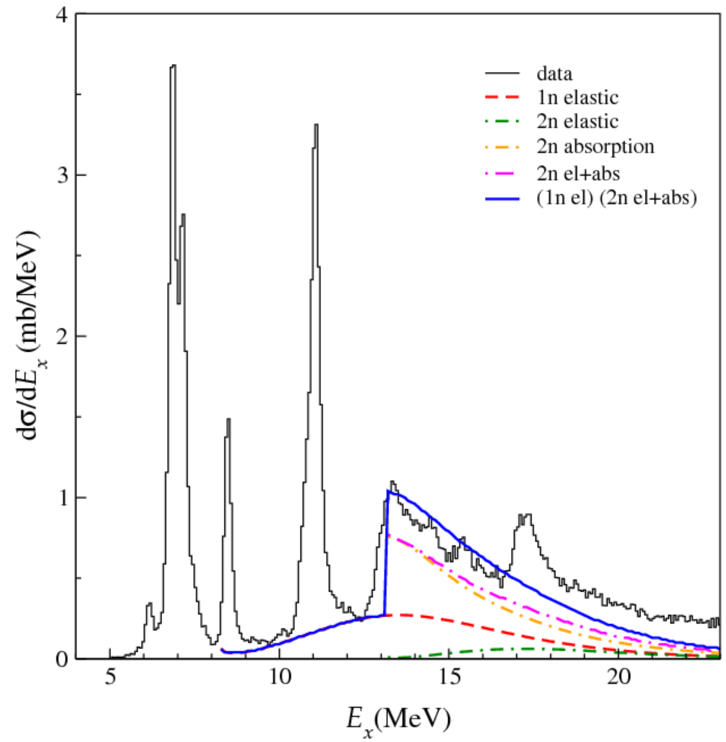


Absorption in the *d* shell

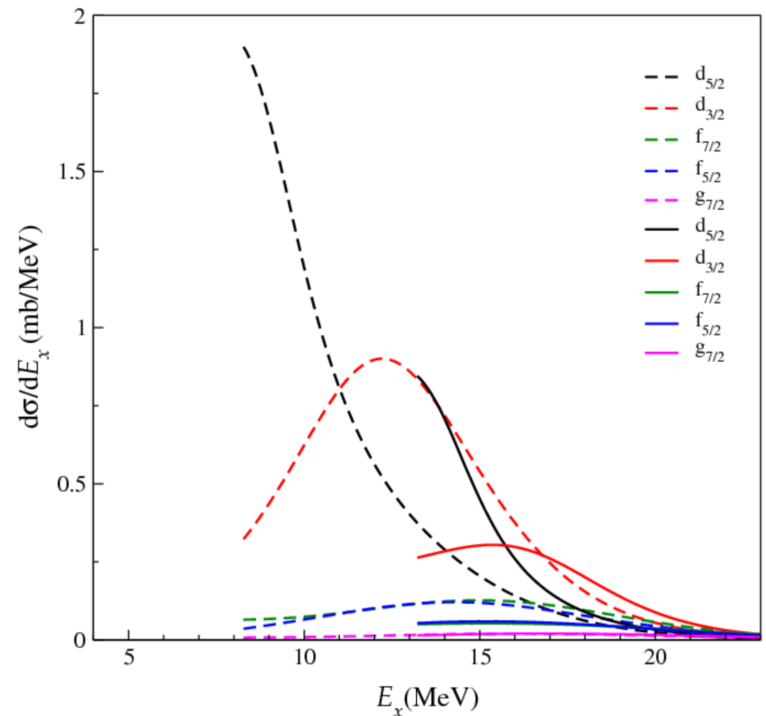
F. Cappuzzello et al., PLB 711 (2012) 347

Break-up calculations

$^{12}C(^{18}O,^{16}O)^{14}C$



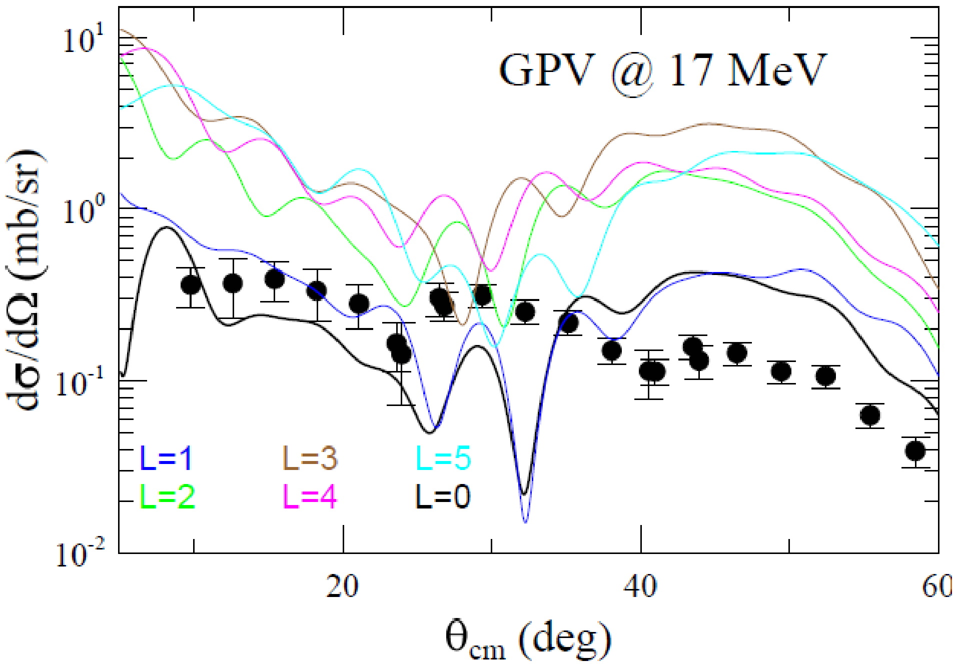
Partial wave decomposition



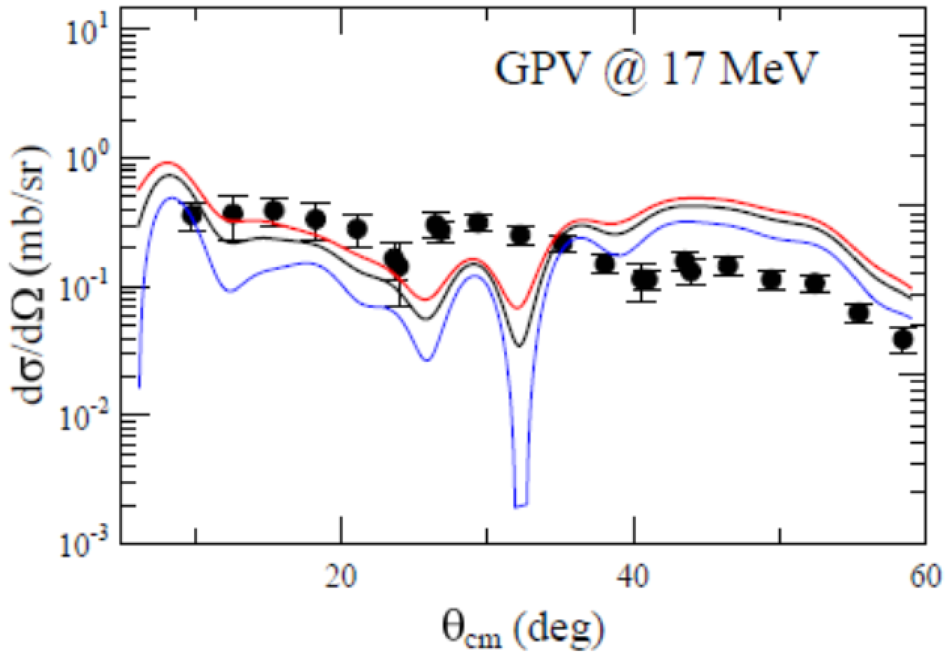
CD-DWBA calculations

- •No continuum-continuum coupling
- •Assumption of three-body continuum

•L=0,1 best agreement



•No scaling of the calculation

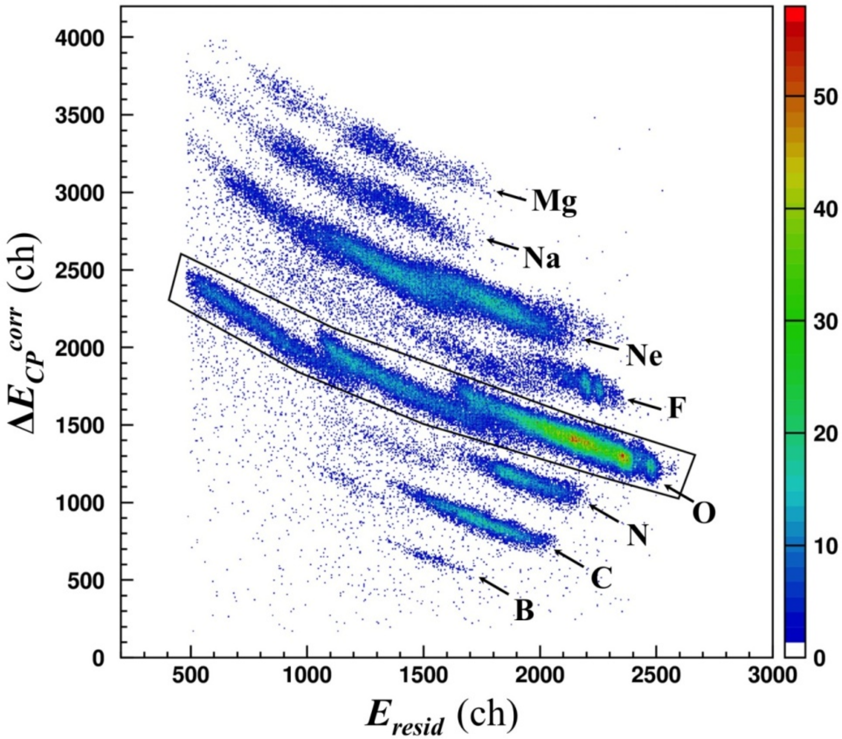


- •L=1 isovector mode (GDR) is at about 25 MeV
- •L=0 GPV is the most likely mode from these calcualtions

Data reduction

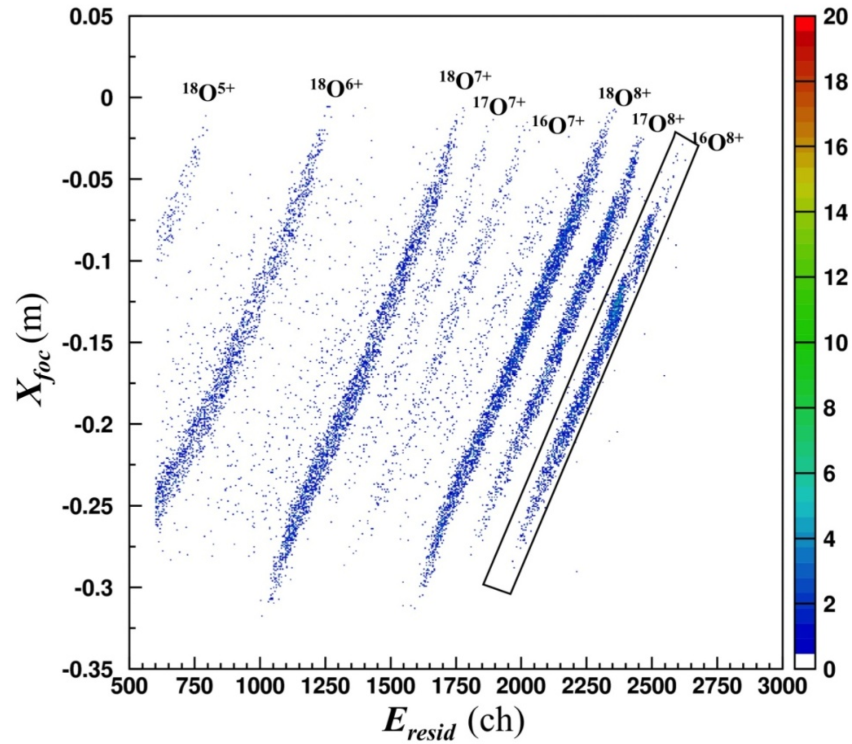
Particle Identification

Z identification



A identification

$$B\rho = \frac{p}{q} \rightarrow X_{foc}^2 \propto \frac{m}{q^2} E_{resid}$$

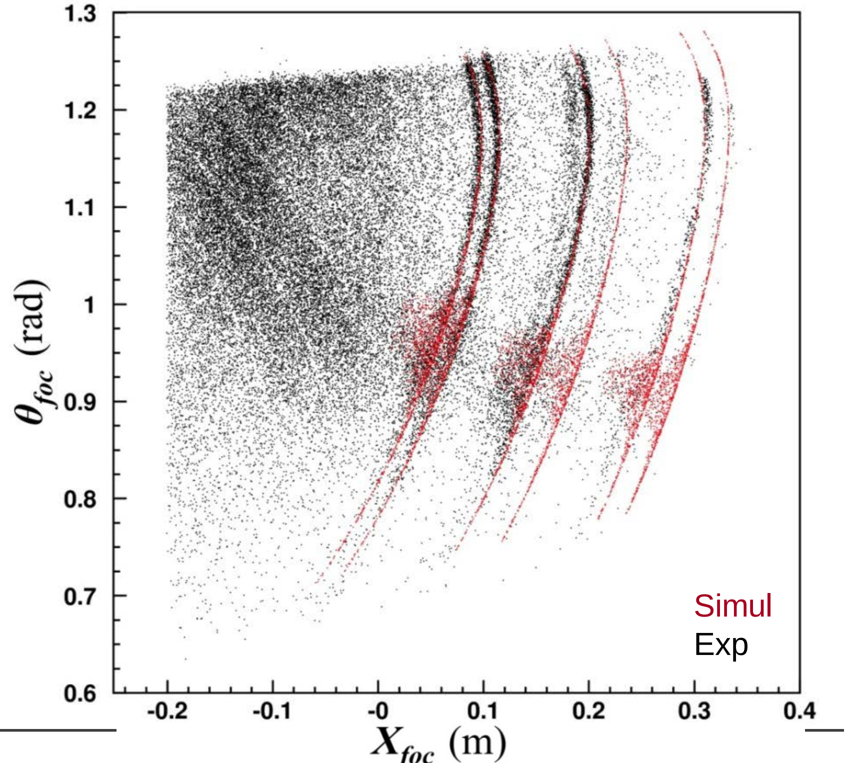


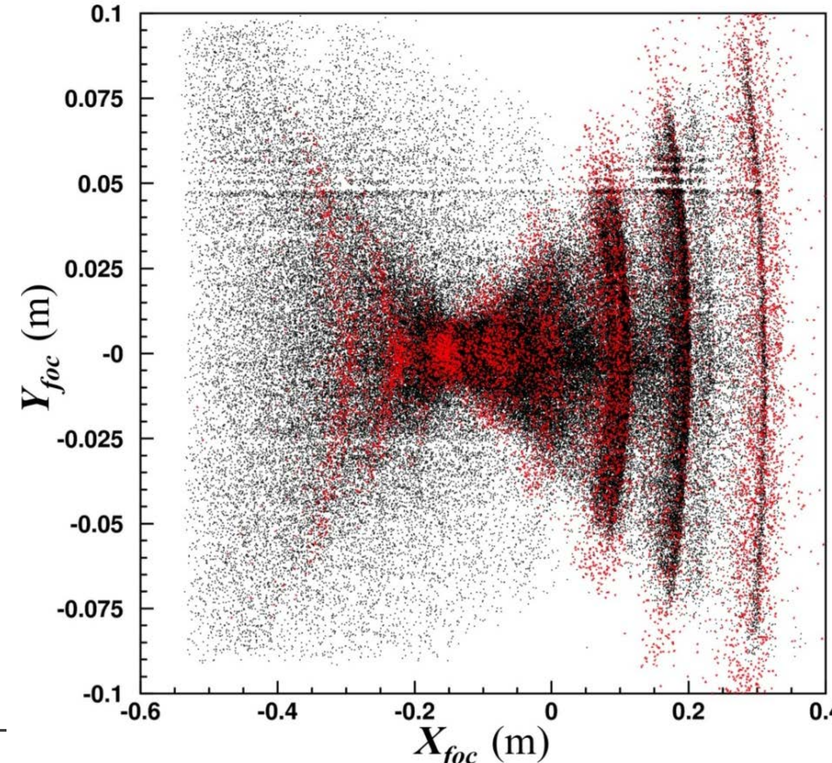
F. Cappuzzello et al., NIMA 621 (2010) 419

Trajectory reconstruction technique

Large acceptance → Aberrations → Hardware design

Aberrations → Software ray-reconstruction



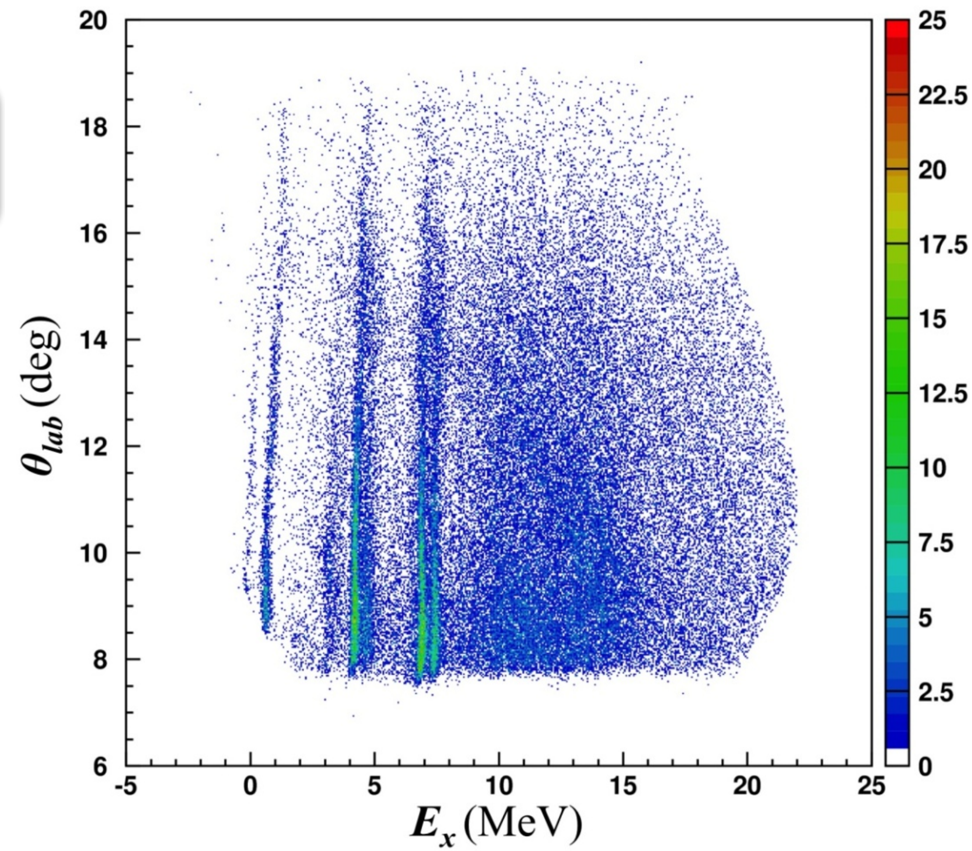


4. Application of the inverse map to the data → $E_{kin} \rightarrow Q$

F. Cappuzzello et al., NIMA 638 (2011) 74

Reconstructed parameters

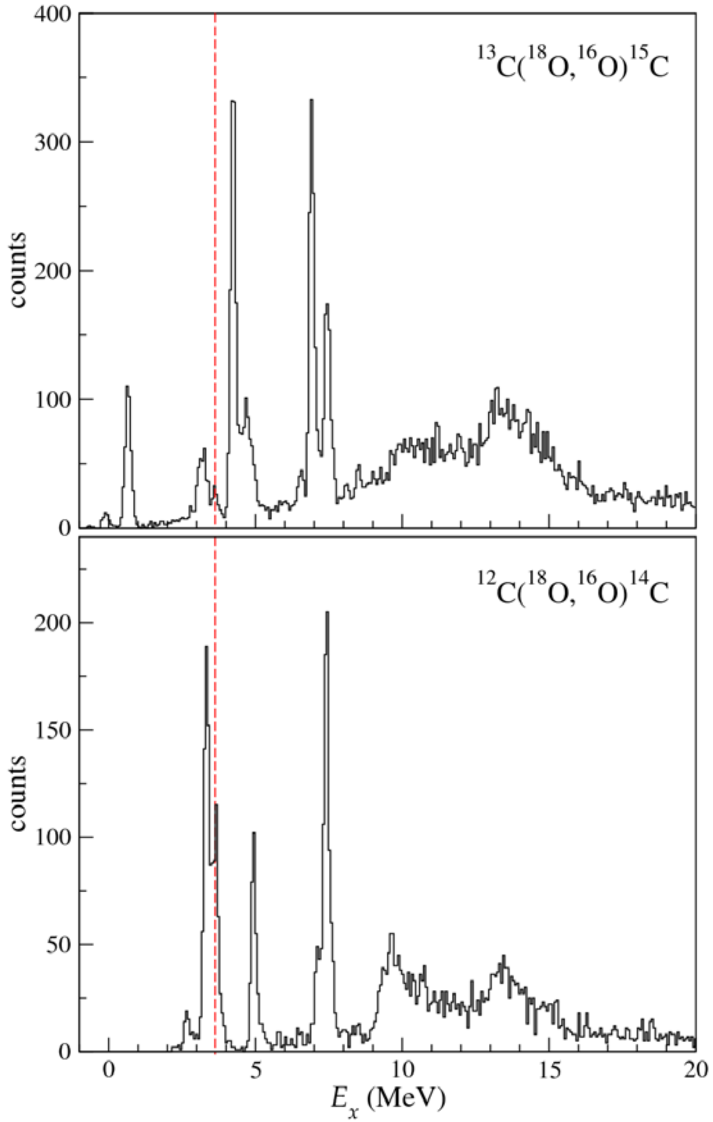
$$\theta_{lab} = arccos\frac{\cos(\theta_{opt}) - sin(\theta_{opt})tan(\theta_i)}{\sqrt{1 + tan^2(\theta_i) + tan^2(\phi_i)}}$$

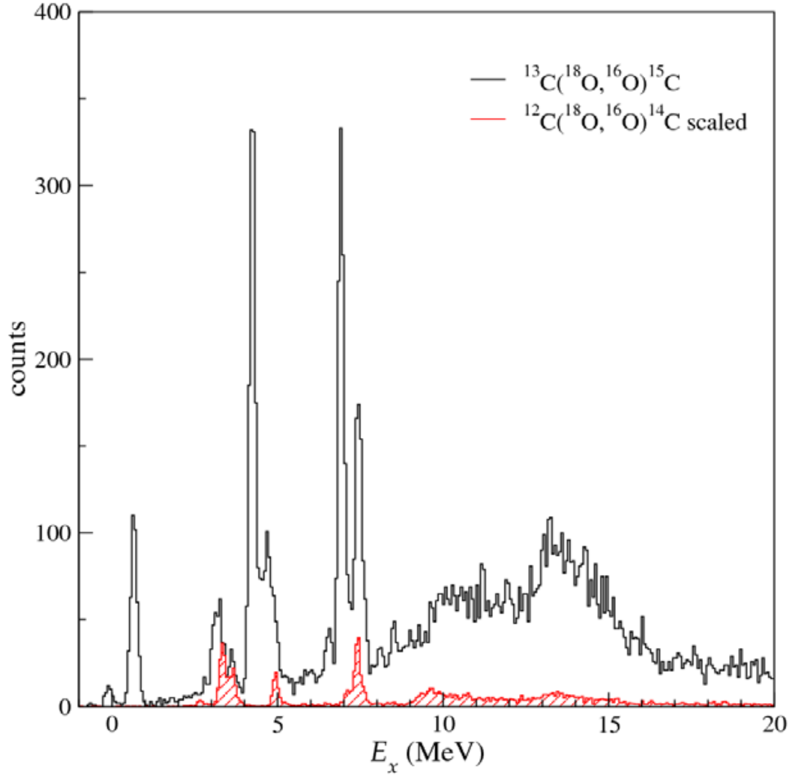


Q_0= -2.793 MeV

$$E_x = Q_0 - Q = Q_0 - K\left(1 + \frac{M_e}{M_r}\right) + E_{beam}\left(1 - \frac{M_b}{M_r}\right) + 2\frac{\sqrt{M_b M_e}}{M_r}\sqrt{E_{beam}K}cos\theta_{lab}$$

Background subtraction in ^{15}C spectra





$$N_{tot} = N_{^{13}C} - S_{^{12}C} \cdot N_{^{12}C}$$

QRPA calculations

Linear response theory

Time dependent Hartree-Foch equation

$$\hbar\omega \frac{\partial R}{\partial t} = [h(R) + F(t), R(t)]$$

Weak external field including *p-h* and *p-p* operators

$$F = \sum_{ij} F_{ij}^{11} a_i^+ a_j + \sum_{ij} (F_{ij}^{12} a_i^+ a_j^+ + F_{ij}^{21} a_i a_j)$$

Small changes in the nuclear density

Bethe-Salpeter equation

$$\boldsymbol{\rho}' = \boldsymbol{G}\boldsymbol{F} \longrightarrow \boldsymbol{G} = \boldsymbol{G}_0 + \boldsymbol{G}_0\boldsymbol{V}\boldsymbol{G} = \frac{\boldsymbol{G}_0}{1 - \boldsymbol{G}_0\boldsymbol{V}}$$

Two-nucleon transfer $S(\omega) = -\frac{1}{\pi} Im \int F^{12*}(\boldsymbol{r})\boldsymbol{G}^{22}(\boldsymbol{r},\boldsymbol{r}';\omega)F^{12}(\boldsymbol{r}')d\boldsymbol{r}d\boldsymbol{r}'$

Reaction mechanism

Transfer yields

Comparison between

- Two-neutron transfer (^{16}O)
- One-neutron transfer (^{17}O)
- Inelastic scattering (^{18}O)

Reaction	**Inelastic** (^{18}O) (counts × 10^5)	**1*n*-transfer** (^{17}O) (counts × 10^5)	**2*n*-transfer** (^{16}O) (counts × 10^5)
$^{18}O + {}^{12}C$	1.694 ± 0.006	0.950 ± 0.005	0.924 ± 0.005
$^{18}O + {}^{13}C$	1.869 ± 0.007	1.284 ± 0.007	0.814 ± 0.006

Equiprobable processes

Cross section calculations

Complete treatment of the transfer process

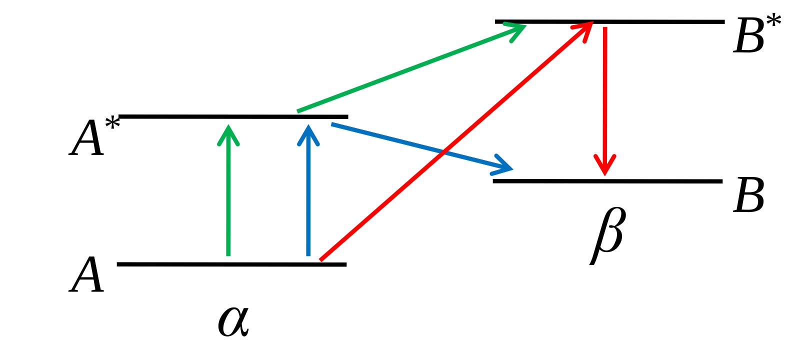
1. One-step channel

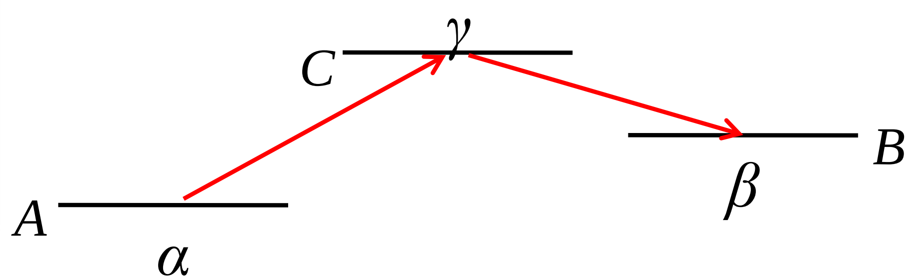
   (α → β)

2. Sequential channel

   (γ partitions)

3. Non-orthogonal term





Previous calculations

2nd order DWBA for $^{A}Sn(p,t)^{A-2}Sn$

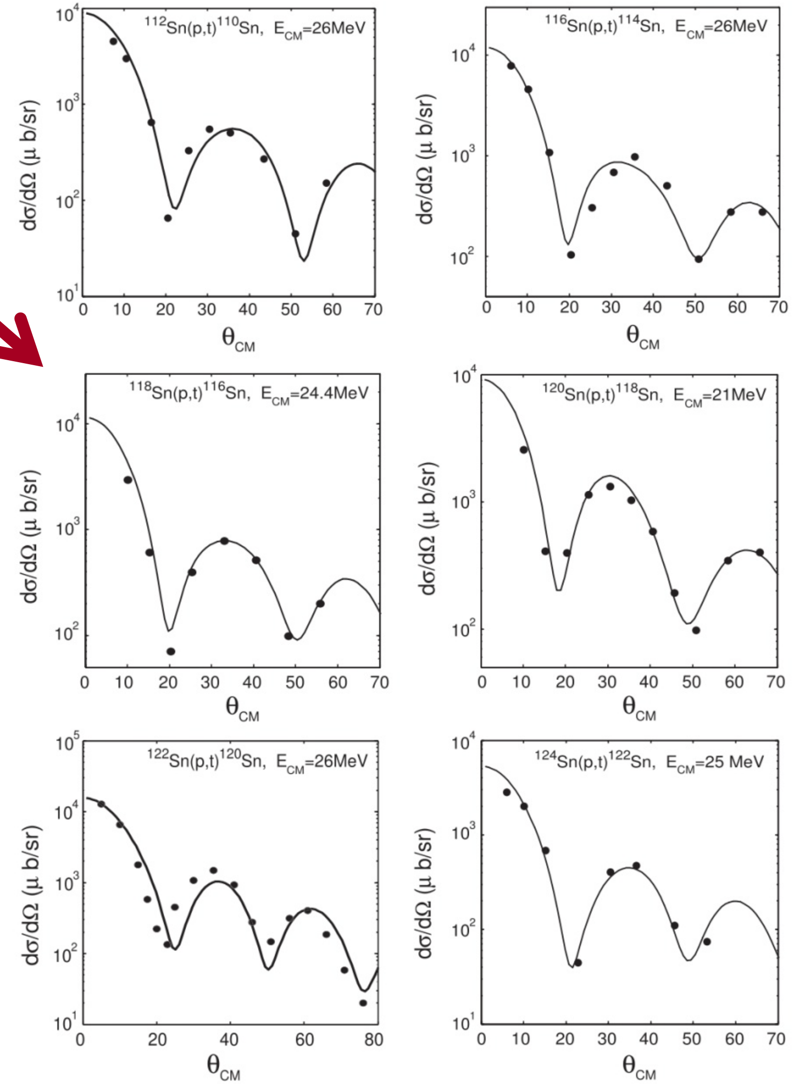
(Potel et al. PRL 107 (2011) 092501)

EFR-CCBA for $^{74}Ge(^{18}O,^{16}O)^{76}Ge$ and $^{76}Ge(^{16}O,^{18}O)^{78}Ge$

(Lemaire and Low PRC16 (1977) 183)

Importance of inelastic channels

$^{18}O^{*}$ 2^{+} state at 1.98 MeV



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CRC calculations

Sao-Paulo optical potential

$$V_{LE}(R,E) = V_F(R)e^{-\frac{4v^2(R)}{c^2}}$$

L.C.Chamon et. al. PRC 66 (2002) 014610
D. Pereira et al. PLB 670 (2009) 330

- Double-folding potential $V_F(R) = \int \rho_1(r_1)\rho_2(r_2)v_{NN}(\boldsymbol{R} - \boldsymbol{r}_1 + \boldsymbol{r}_2)d\boldsymbol{r}_1 d\boldsymbol{r}_2$

v_{NN} → nucleon-nucleon interaction: **M3Y**

$\rho(r)$ → wide and systematic dataset

- Pauli non-locality $e^{-\frac{4v^2(R)}{c^2}}$

- Imaginary part $W(R) = 0.6 \cdot V_{LE}\ (R)$