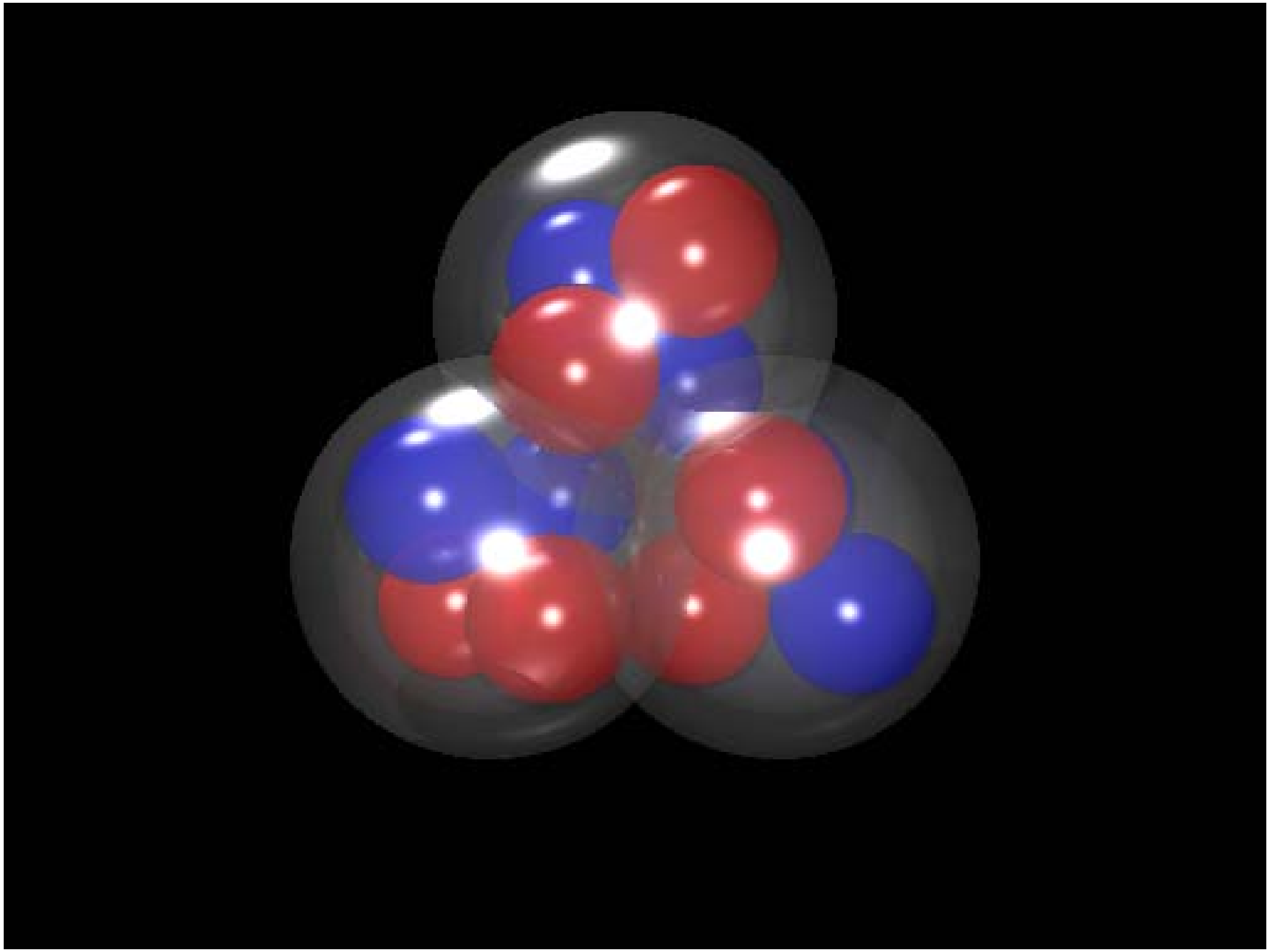

Alpha-Clustering in Light Nuclei

- Introduction
- Structure of ^{12}C: Hoyle band
- Alpha-cluster model (Wheeler, Brink, Robson, …)
- Algebraic Cluster Model
- Summary and conclusions

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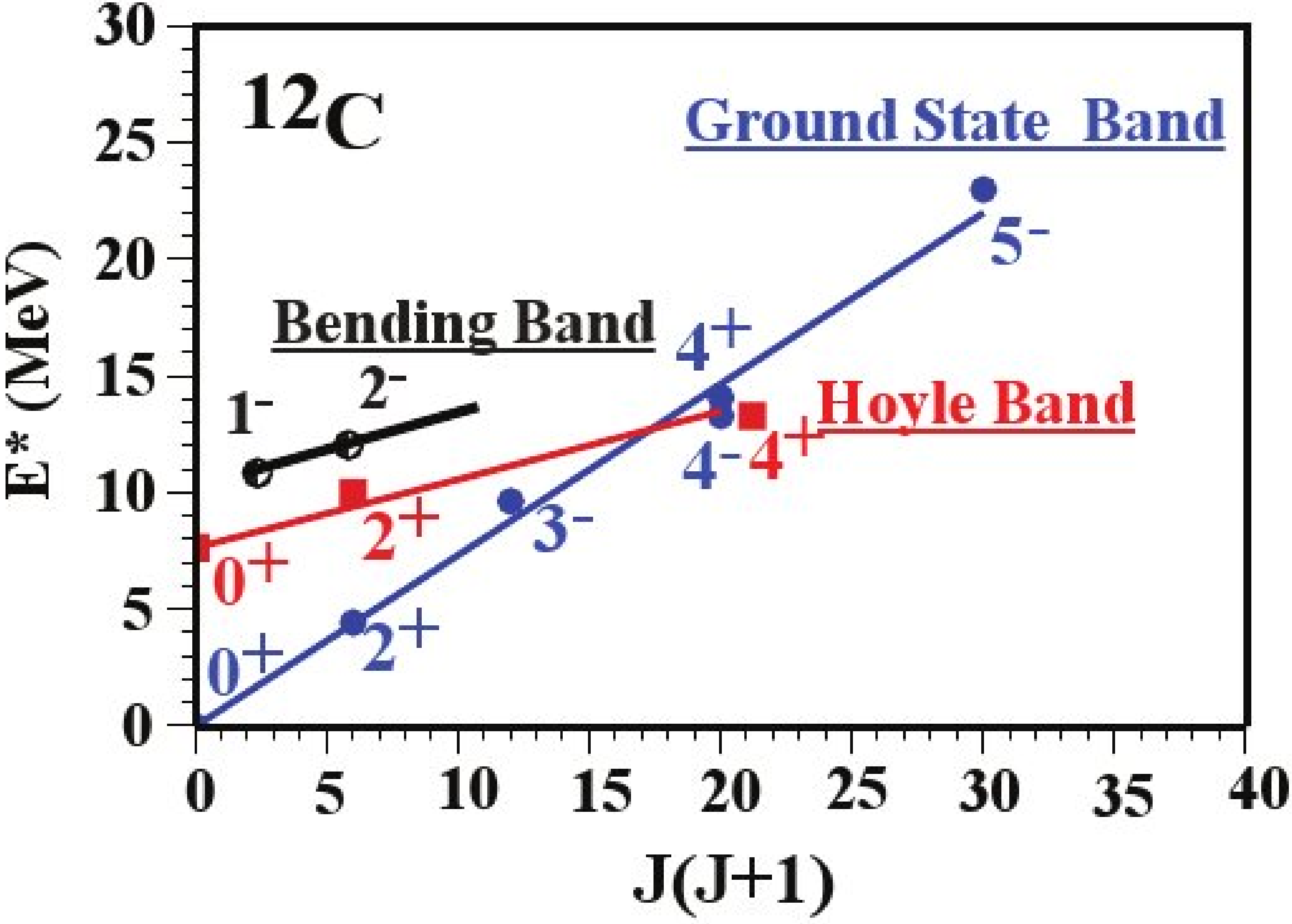
bijker@nucleares.unam.mx



12C
Ground State Band
Bending Band
Hoyle Band
E* (MeV)
J(J+1)
0
5
10
15
20
25
30
35
40
1-
2-
0+
2+
3-
4+
4-
4+
5-

Experimental Studies

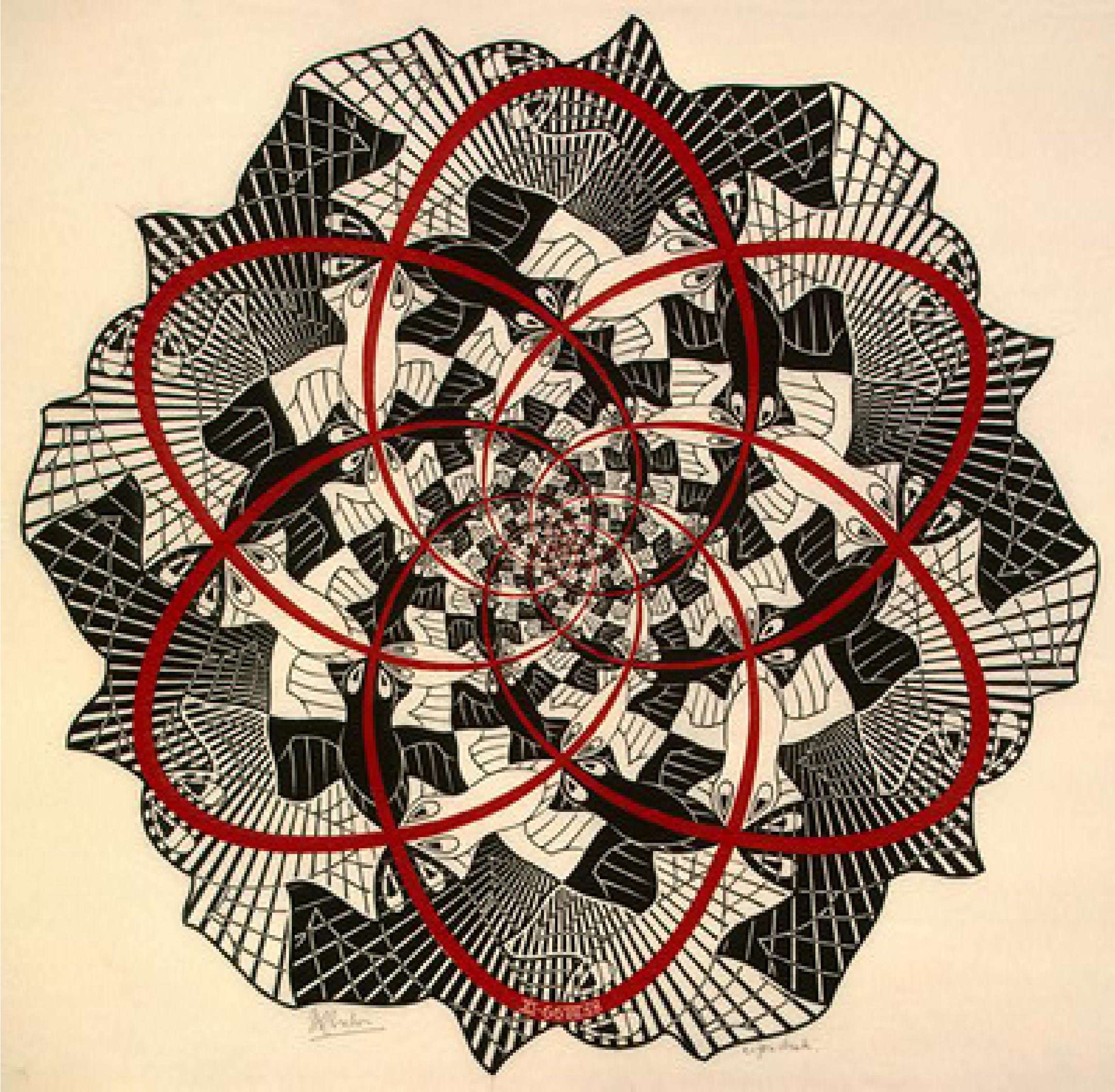
gs	3^-	Kokalova et al, PRC 87, 057307 (2013)
gs	4^-	Freer et al, PRC 76, 034320 (2007)
		Kirsebom et al, PRC 81, 064313 (2010)
gs	5^-	Marín-Lámbarri et al, PRL 113, 012502 (2014)
Hoyle	2^+	Itoh et al, PRC 84, 054308 (2011)
		Freer et al, PRC 86, 034320 (2012)
		Zimmerman et al, PRL 110, 152502 (2013)
Hoyle	4^+	Freer et al, PRC 83, 034314 (2011)
Hoyle	$3^-, 4^-$	Some evidence for negative parity strengths between 11 and 14 MeV Freer et al, PRC 76, 034320 (2007)

Recent Theoretical Work

- Alpha-cluster model (Wheeler 1937, Brink 1966, Robson, 1978)

- AMD (Kanada-Enyo, PTP, 2007)
- FMD model (Chernykh et al, PRL, 2007)
- BEC-like cluster model (Funaki et al, PRC, 2009)
- Ab initio no-core shell model (Roth et al, PRL, 2011)
- Lattice EFT (Epelbaum et al, PRL, 2011, 2012)
- No-core symplectic model (Dreyfuss et al, PLB, 2013)
- Algebraic Cluster Model (2002, 2014)
- and many others

- Recent reviews: Freer & Fynbo, PPNP 78, 1 (2014), Jenkins & Courtin, JPG 42, 034010 (2015)



Algebraic Cluster Model (ACM)

- For k dof introduce a SGA of U(k+1)
- 2-body systems: U(4) model
- 3-body systems: U(7) model
- 4-body systems: U(10) model
- Applications: hadrons, molecules, alpha-cluster nuclei

ACM for Three-Body Systems

Two relative Jacobi vectors

$$\vec{\rho} = \frac{1}{\sqrt{2}}(\vec{r}_1 - \vec{r}_2) \ , \qquad \vec{\lambda} = \frac{1}{\sqrt{6}}(\vec{r}_1 + \vec{r}_2 - 2\vec{r}_3)$$

Building blocks $b_\rho^\dagger \ , \ b_\lambda^\dagger \ , \ s^\dagger$

Total number of bosons $N = n_s + n_\rho + n_\lambda$

Identical Clusters

Explicit construction of harmonic oscillator states with good permutation symmetry

Kramer & Moshinsky, Nucl. Phys. 82, 241 (1966)

In ACM, wave functions are generated numerically, the permutation symmetry S_3 is determined by the interchange P(12) and the cyclic permutation P(123)

Wave Functions

- Labeled by $\left|[N],\alpha,L_t^P\right\rangle$
- Total number of bosons: N
- Angular momentum L and parity P
- Permutation symmetry: t=S(ymmetric) for alpha-cluster nuclei

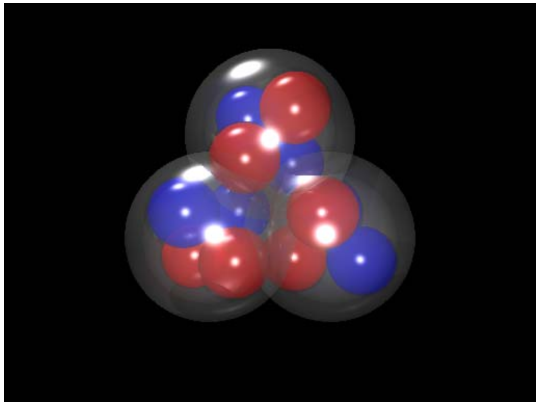
Bijker & Iachello, PRC 61, 067305 (2000)
Bijker & Iachello, AP 298, 334 (2002)

Three-body Clusters: Oblate Top

$$
\begin{aligned}
H &= \xi_1 \, (R^2 \, s^\dagger s^\dagger - b_\rho^\dagger \cdot b_\rho^\dagger - b_\lambda^\dagger \cdot b_\lambda^\dagger) \, (\text{h.c.}) \\
&\quad + \xi_2 \left[(b_\rho^\dagger \cdot b_\rho^\dagger - b_\lambda^\dagger \cdot b_\lambda^\dagger) \, (\text{h.c.}) + 4 (b_\rho^\dagger \cdot b_\lambda^\dagger)(\text{h.c.}) \right] \\
&\quad + \kappa_1 \, \vec{L} \cdot \vec{L} + \kappa_2 \, (b_\rho^\dagger \cdot \tilde{b}_\lambda - b_\lambda^\dagger \cdot \tilde{b}_\rho) \, (\text{h.c.})
\end{aligned}
$$

$$
\begin{aligned}
R^2 = 0 &\quad : \quad \text{anharmonic oscillator} \\
R^2 = 1, \ \xi_1 > 0, \ \xi_2 = 0 &\quad : \quad \text{deformed oscillator} \\
R^2 \neq 0, \ \xi_1, \ \xi_2 > 0 &\quad : \quad \text{oblate top}
\end{aligned}
$$

Equilibrium shape:
equilateral triangle



Bijker & Iachello, Ann. Phys. 298, 334 (2002)

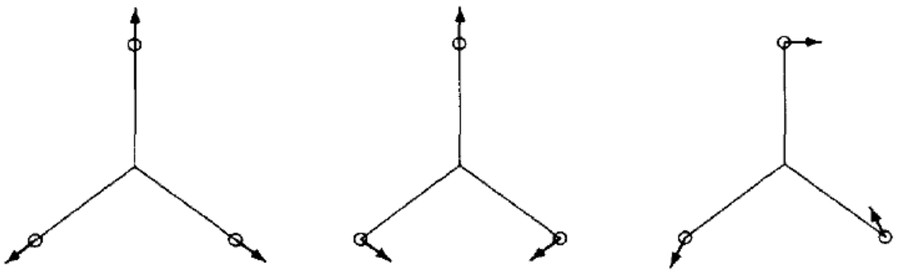
Rotations and Vibrations

- Excitations of an oblate top

$$E \approx \omega_1(\nu_1 + \frac{1}{2}) + \omega_2(\nu_2 + 1) + \kappa_1 L(L+1) + \kappa_2(K \mp 2\ell_2)^2$$

- Frequencies

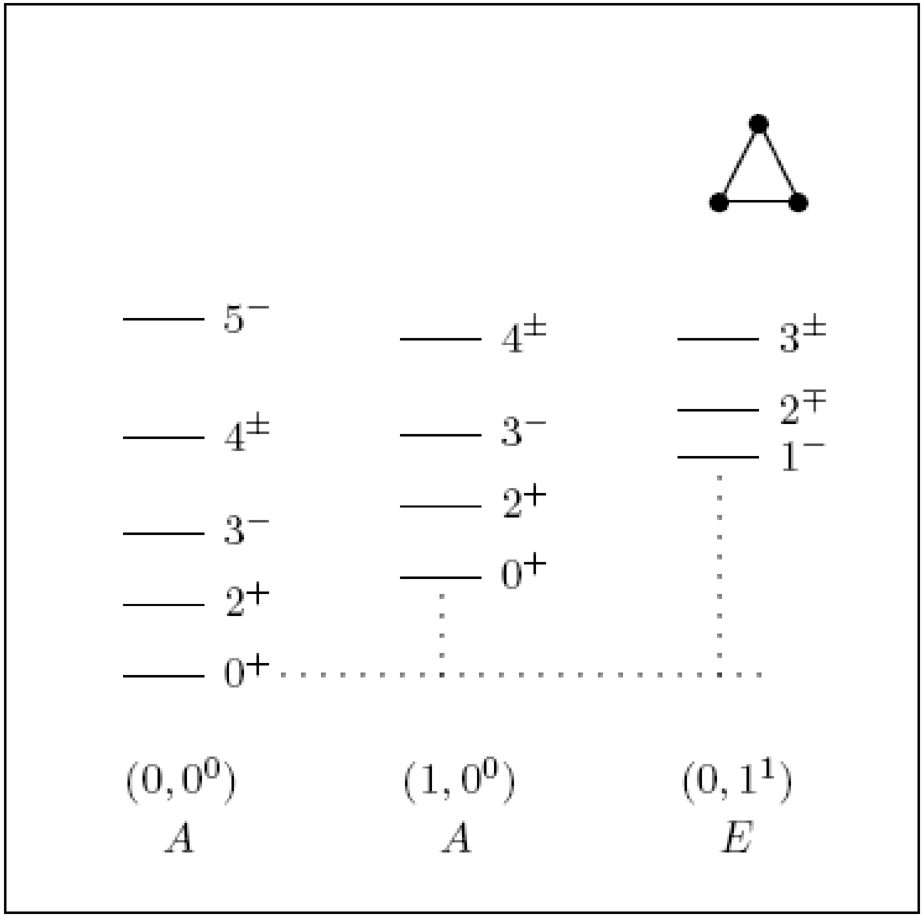
$$\omega_1 = 4NR^2\xi_1 \ , \quad \omega_2 = \frac{4NR^2}{1+R^2}\xi_2$$



$\nu_1(A_1)$ Breathing vibration

$\nu_2^{l_2}(E)$ Bending vibration

Energy Spectrum



Ground state and Hoyle band (breathing vibration)

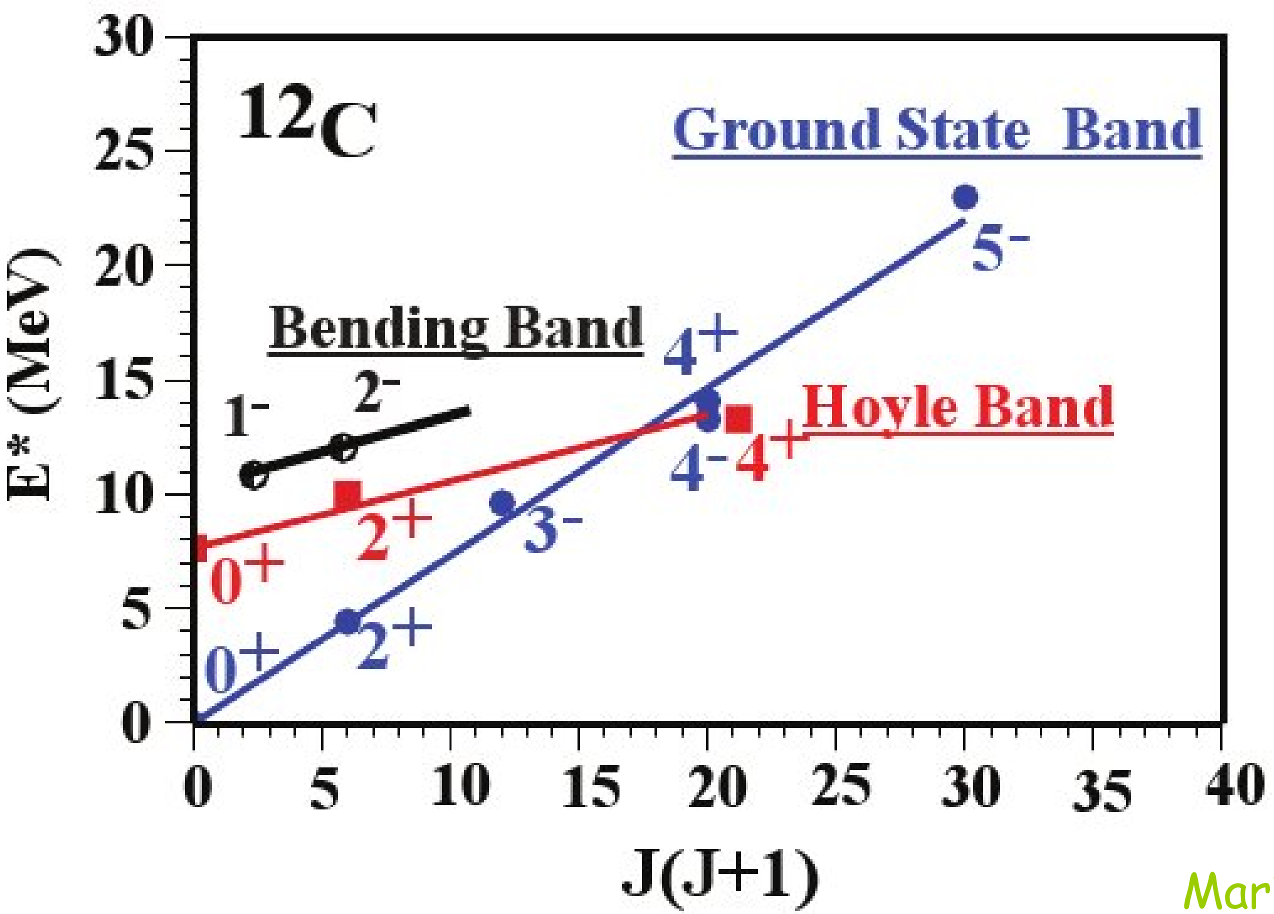
$L^P = 0^+, 2^+, 3^-, 4^\pm, 5^-, \ldots$

Bending vibration

$L^P = 1^-, 2^\pm, 3^\pm, \ldots$

Fingerprint of triangular shape with D_{3h} symmetry

Rotational Bands



Marín-Lámbarri et al, PRL 113, 012502 (2014)

Energy Formula

$$
\begin{aligned}
E = E_0 \; &+ \; \omega_1(\nu_1 + \frac{1}{2}) \left(1 - \frac{\nu_1 + 1/2}{N}\right) \\
&+ \; \omega_2(\nu_2 + 1) \left(1 - \frac{\nu_2 + 1}{N + 1/2}\right) \\
&+ \; \kappa_1 \, L(L+1) + \kappa_2 \, (K \mp 2\ell_2)^2 \\
&+ \; \left[\lambda_1 \, (\nu_1 + \frac{1}{2}) + \lambda_2 \, (\nu_2 + 1)\right] L(L+1) \; .
\end{aligned}
$$

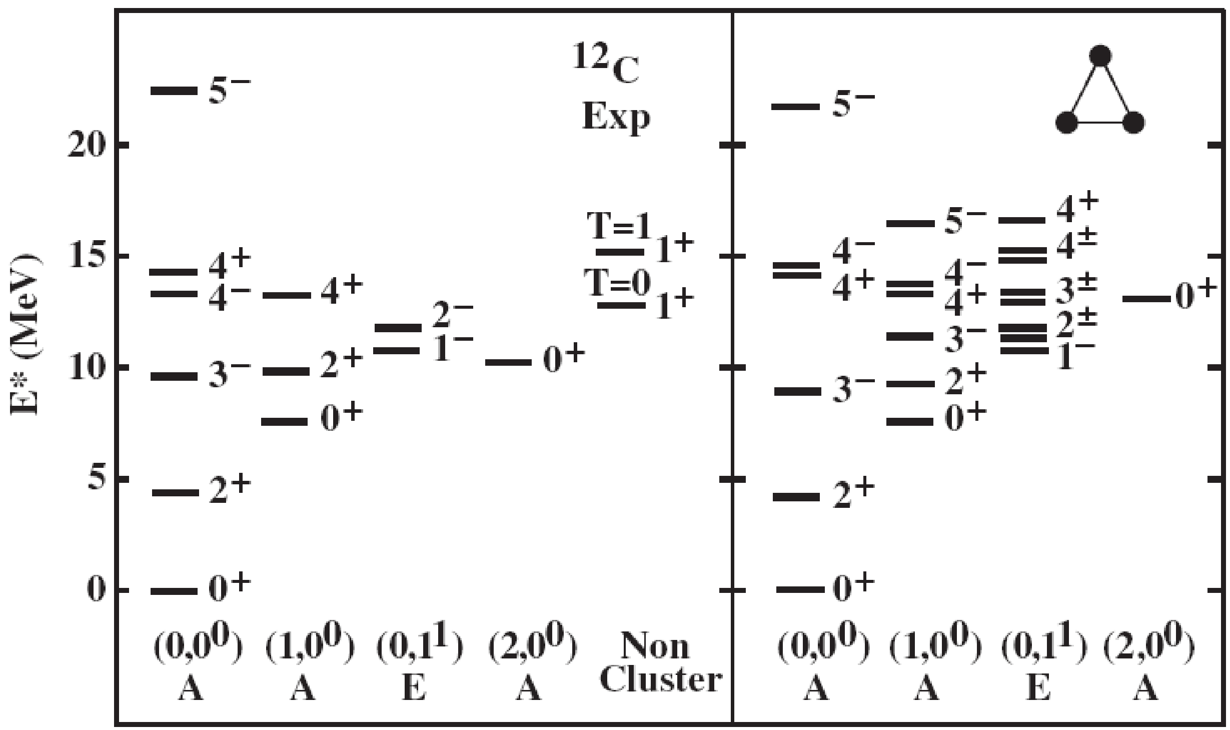
Vibrations

Rotations

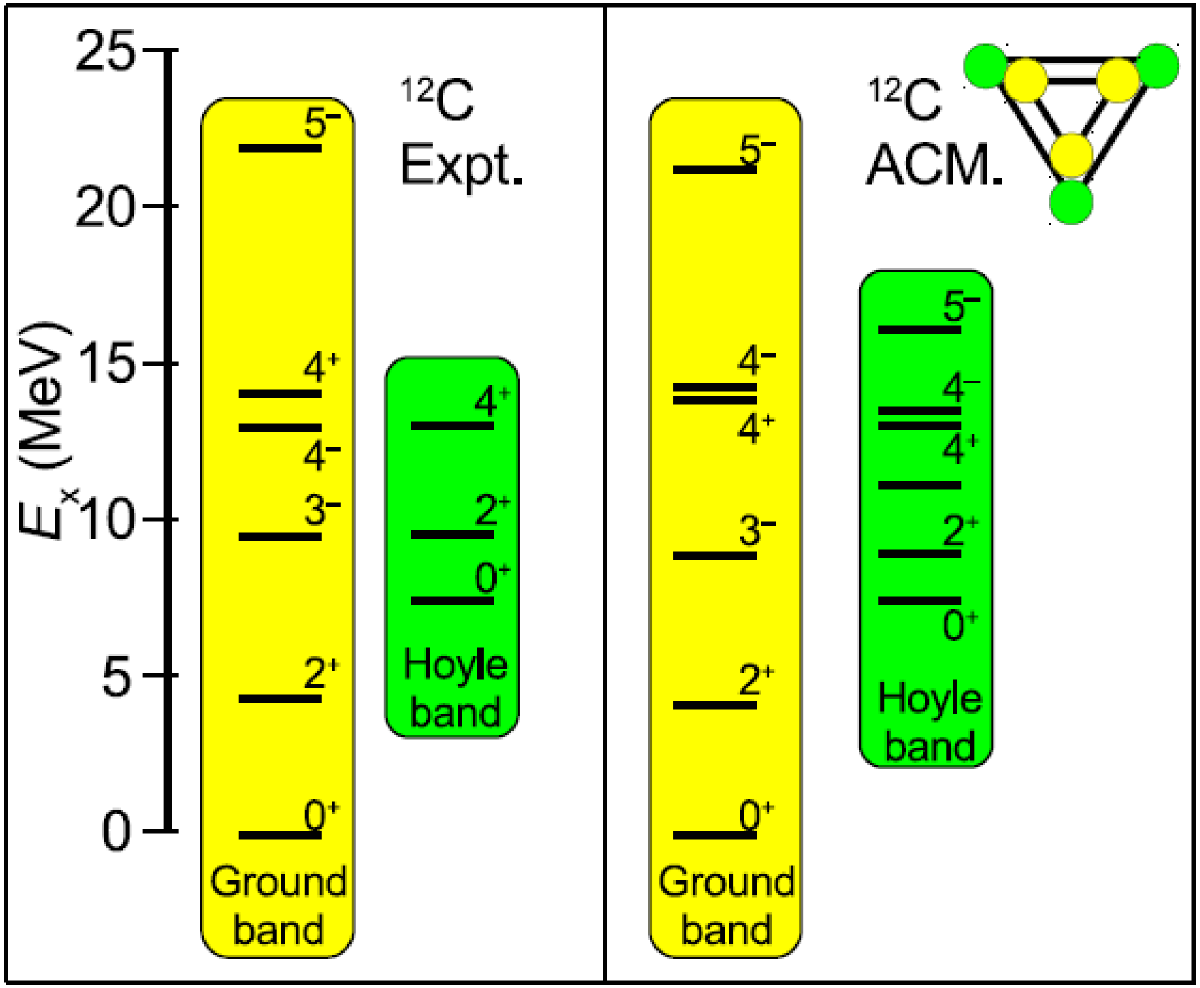
Vibration-rotation couplings

Marín-Lámbarri et al, PRL 113, 012502 (2014)

ACM for ^{12}C



Marín-Lámbarri, Bijker, Freer, Gai, Kokalova, Parker, Wheldon, PRL 113, 012502 (2014)



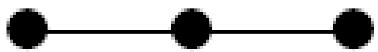
$$\left\langle r^2 \right\rangle_{gs}^{1/2} = 2.47 \text{ fm}$$

$$\left\langle r^2 \right\rangle_{H}^{1/2} = 3.45 \text{ fm}$$

t: experimentally observed states currently assigned to the grou

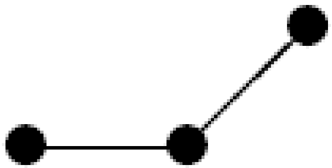
Tzany Kokalova, JPCS 569, 012010 (2014)

Alpha-Cluster Configurations



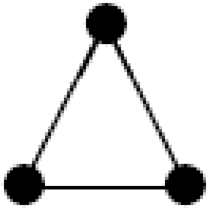
$\mathcal{D}_{\infty h}$

Linear



$\mathcal{C}_{2v}$

Bent



$\mathcal{D}_{3h}$

Triangular

Examine rotational structure!

Morinaga, PR 101, 254 (1956)
Epelbaum et al, PRL 109, 252501 (2012)
Wheeler (1937), Robson (1978)
Bijker & Iachello, AP 298, 334 (2002)

Electric Transitions

Form factor

$$F(0^+ \to L^P) = a_L\, j_L(q\beta)\, \mathrm{e}^{-q^2/4\alpha}$$

$$a_L^2 = \frac{2L+1}{3}\left[1 + 2P_L(-\tfrac{1}{2})\right]$$

Long-wavelenght limit

$$B(EL;0^+ \to L^P) = \left(\frac{Ze}{3}\right)^2 \frac{2L+1}{4\pi}\left[3 + 6P_L(-\tfrac{1}{2})\right]\beta^{2L}$$

	Th.	Exp.	
$B(E2;2_1^+ \to 0_1^+)$	7.1	7.6 ± 0.4	$e^2\mathrm{fm}^4$
$B(E3;3_1^- \to 0_1^+)$	45	103 ± 17	$e^2\mathrm{fm}^6$
$B(E4;4_1^+ \to 0_1^+)$	48		$e^2\mathrm{fm}^8$
$B(E2;0_2^+ \to 2_1^+)$	0.10	13.1 ± 1.8	$e^2\mathrm{fm}^4$
$M(E0;0_2^+ \to 0_1^+)$	0.14	5.5 ± 0.2	fm^2
$\langle r^2 \rangle^{1/2}$	2.468	2.468 ± 0.12	fm

$$B(EL;0_1^+ \to 1_1^-) = 0$$

$$B(EL;0_1^+ \to 2_1^+) = (Ze)^2 \frac{5}{4\pi}\frac{1}{4}\beta^4$$

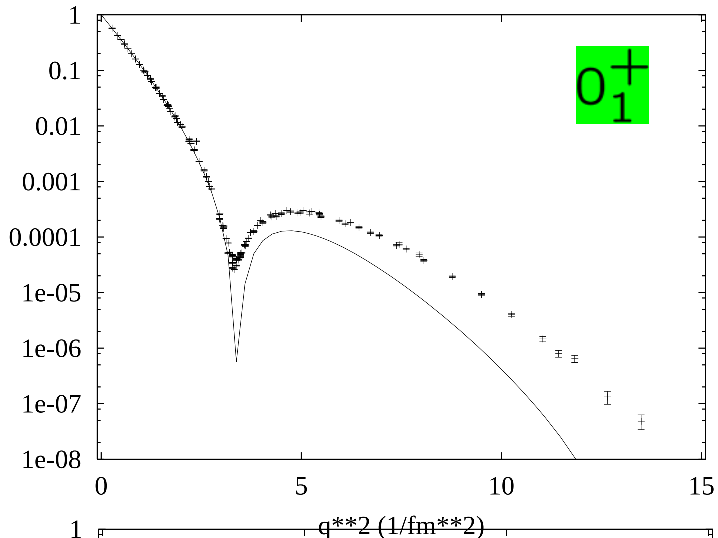
$$B(EL;0_1^+ \to 3_1^-) = (Ze)^2 \frac{7}{4\pi}\frac{5}{8}\beta^6$$

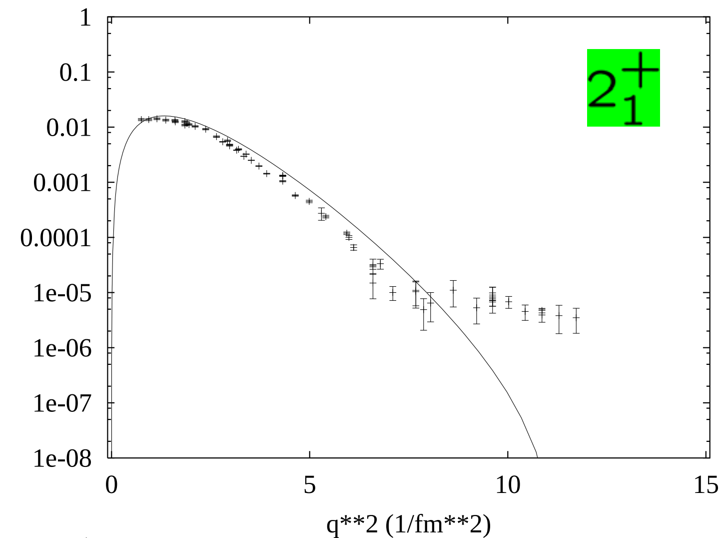
$$B(EL;0_1^+ \to 4_1^+) = (Ze)^2 \frac{9}{4\pi}\frac{9}{64}\beta^8$$

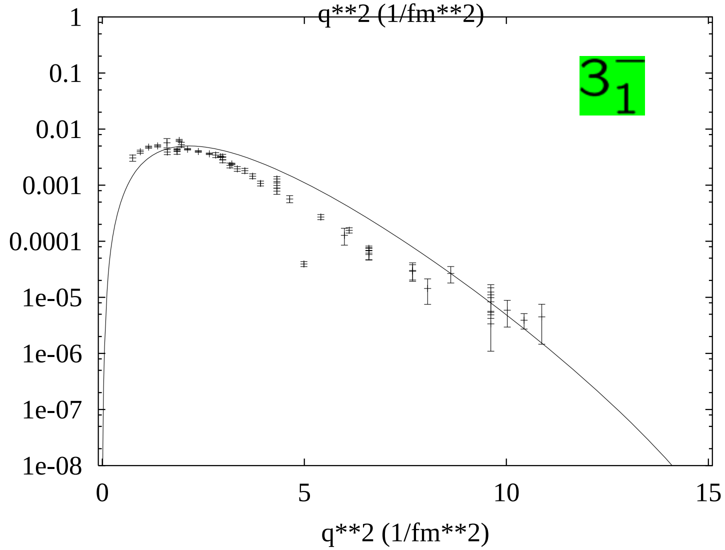
$$B(EL;0_1^+ \to 4_1^-) = 0$$

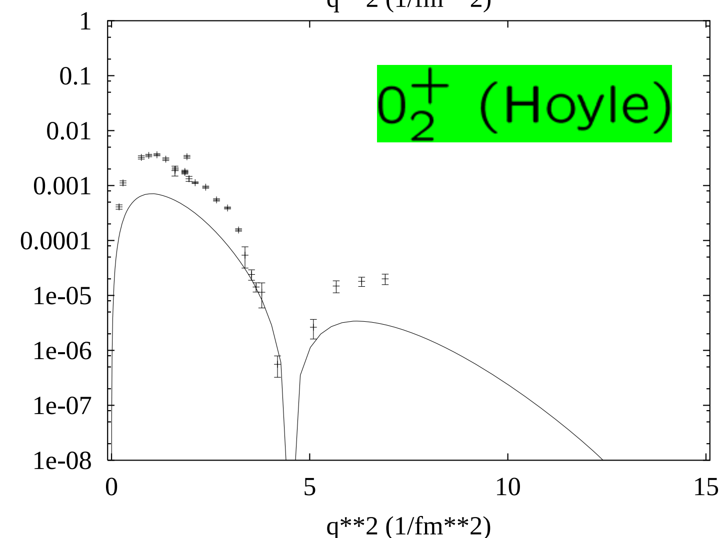
$$B(EL;0_1^+ \to 5_1^-) = (Ze)^2 \frac{11}{4\pi}\frac{35}{128}\beta^{10}$$

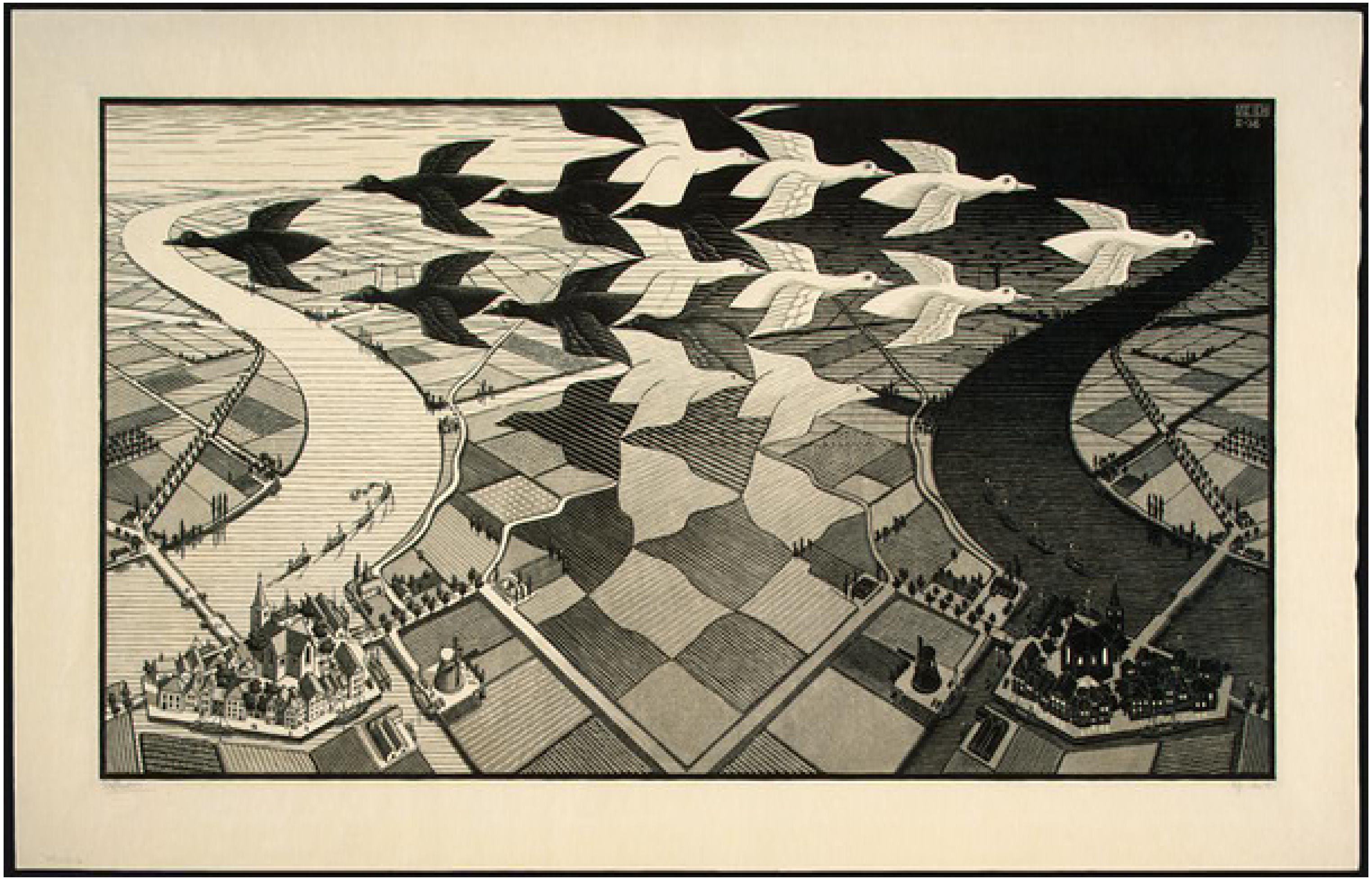
Form Factors











Shape-Phase Transitions

$$
\begin{aligned}
H \;=\; & (1-\chi)\sum_m (b^\dagger_{\rho,m} b_{\rho,m} + b^\dagger_{\lambda,m} b_{\lambda,m}) \\
& + \frac{\chi}{4(N-1)} (R^2\, s^\dagger s^\dagger - b^\dagger_\rho \cdot b^\dagger_\rho - b^\dagger_\lambda \cdot b^\dagger_\lambda)\,(\text{h.c.}) \\
& + \frac{\xi}{N-1} \left[(b^\dagger_\rho \cdot b^\dagger_\rho - b^\dagger_\lambda \cdot b^\dagger_\lambda)\,(\text{h.c.}) + 4(b^\dagger_\rho \cdot b^\dagger_\lambda)(\text{h.c.}) \right]
\end{aligned}
$$

Parameters	:	$0 \leq \chi \leq 1$, $\xi \geq 0$	
$\chi = 0$, $\xi \geq 0$	:	(an)harm. osc.	$U(6)$ limit
$\chi = 1$, $R^2 = 1$, $\xi = 0$	:	def. osc.	$SO(7)$ limit
$\chi = 1$, $\xi > 0$	:	oblate top	

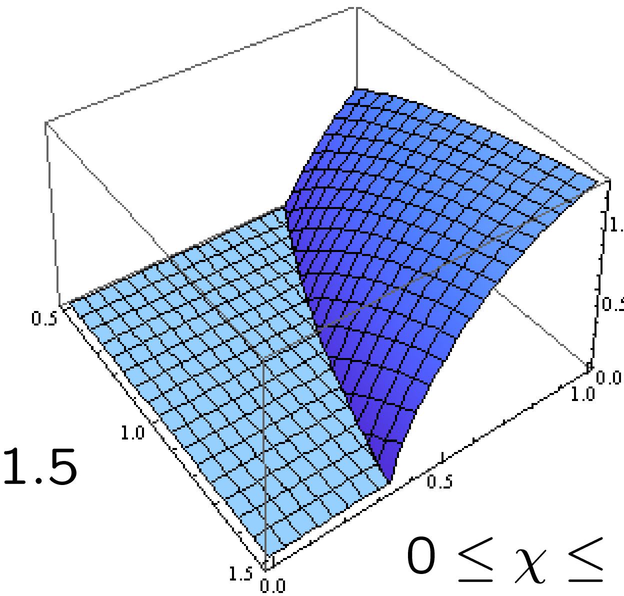
Equilibrium Shapes

$$
\begin{aligned}
V_{\mathrm{cl}} &= \frac{1-\chi}{2} q^2 + \frac{\chi}{4}\left[R^2 - \frac{1}{2} q^2 (1+R^2)\right]^2 \\
&\quad + \frac{\xi q^4}{4}\left(\cos^2 2\eta + \sin^2 2\eta \cos^2 2\zeta\right)
\end{aligned}
$$

$$
\begin{aligned}
\rho &= q \sin\eta \\
\lambda &= q \cos\eta \\
\vec{\rho}\cdot\vec{\lambda} &= \rho\lambda \cos 2\zeta
\end{aligned}
$$

$$
q_0^2 = \begin{cases} 0 & \chi \le \chi_c \\ \frac{2R^2}{1+R^2} + \frac{4(\chi-1)}{\chi(1+R^2)^2} & \chi \ge \chi_c \end{cases}
$$

$$
\chi_c = \frac{1}{1 + R^2(1+R^2)/2}
$$

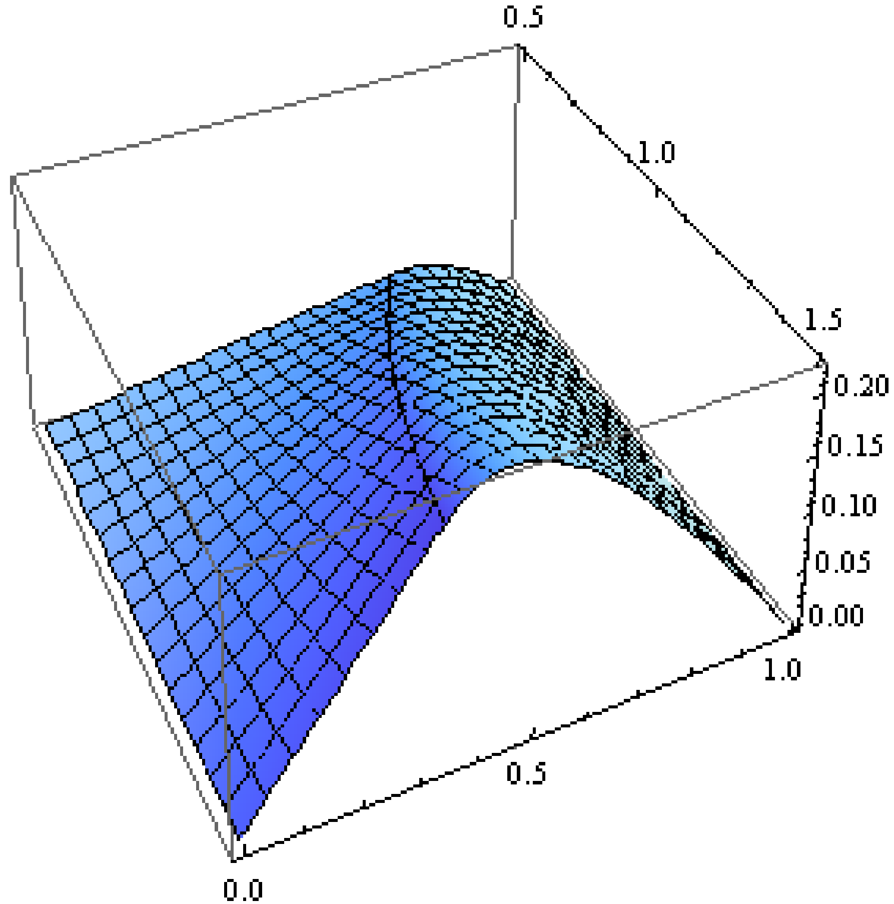


$0.5 \le R^2 \le 1.5$

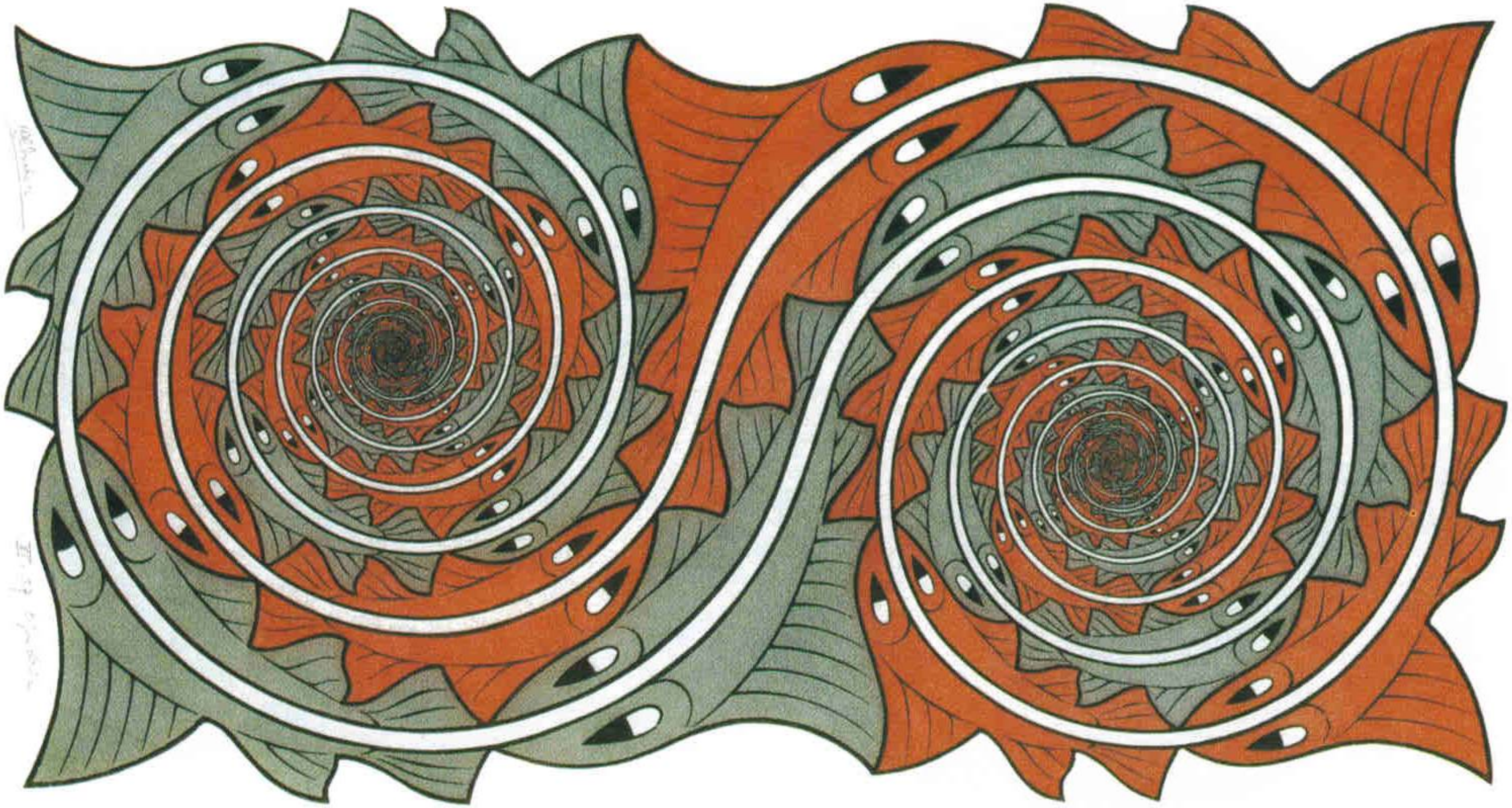
$0 \le \chi \le 1$

2nd Order Phase Transition

$E_0\|_{\chi=\chi_c}$	continuous
$\left.\frac{dE_0}{d\chi}\right\|_{\chi=\chi_c}$	continuous
$\left.\frac{d^2E_0}{d\chi^2}\right\|_{\chi=\chi_c}$	discontinuous



In progress ….

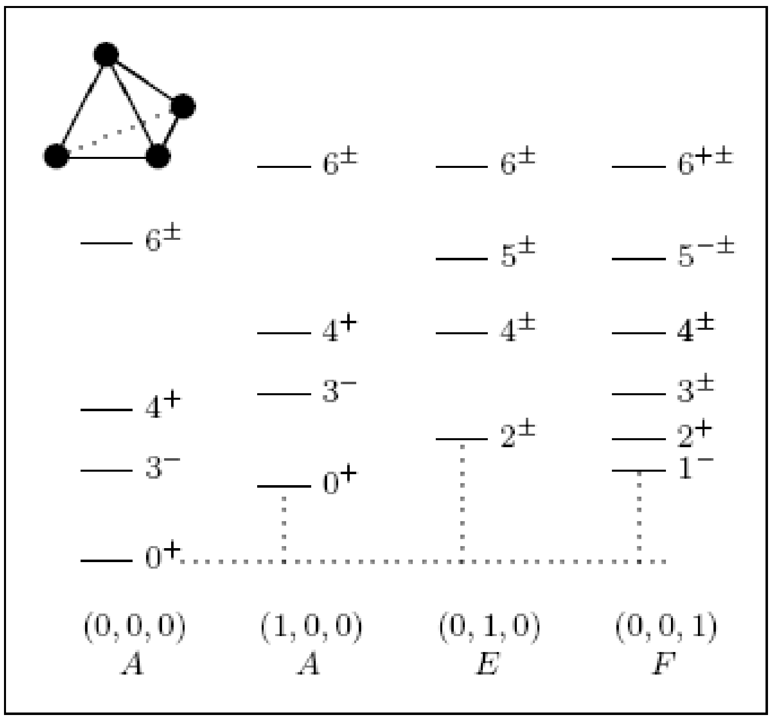


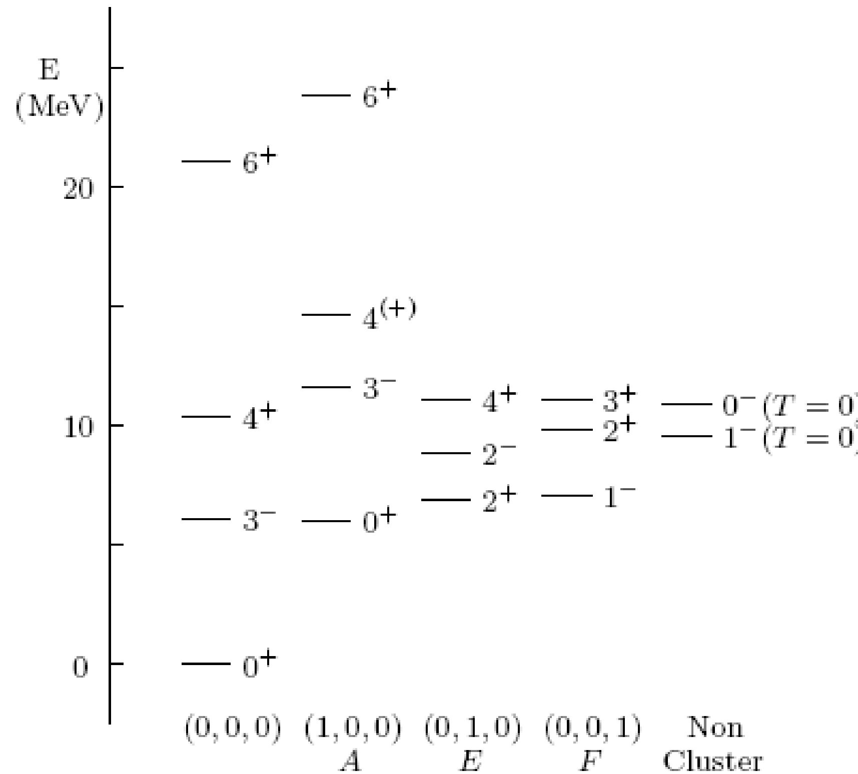
Algebraic Cluster Model

	2α	3α	4α
ACM	$U(4)$	$U(7)$	$U(10)$
Point group	$\mathcal{Z}_2$	$\mathcal{D}_{3h}$	$\mathcal{T}_d$
Geom. conf.	Linear	Triangle	Tetrahedron
Model	Rotor	Oblate top	Spherical top
Vibrations	1	3	6
Rotations	2	3	3
G.s. band	0^+	0^+	0^+
	2^+	2^+	
		3^-	3^-
	4^+	$4^\pm$	4^+
		5^-	
	6^+	$6^{\pm+}$	$6^\pm$

ACM for ^{16}O

Ground state rotational band $L^P = 0^+, 3^-, 4^+, 6^\pm$





Electric Transitions

$$B(EL;0 \rightarrow L) \; = \; \left(\frac{Ze}{4}\right)^2 \beta^{2L} \, \frac{2L+1}{4\pi} \left[4 + 12 P_L(-\frac{1}{3})\right]$$

$B(EL;L^P \rightarrow 0^+)$	Th	Exp	
$B(E3;3_1^- \rightarrow 0_1^+)$	181	205 ± 10	$e^2\mathrm{fm}^6$
$B(E4;4_1^+ \rightarrow 0_1^+)$	338	378 ± 133	$e^2\mathrm{fm}^8$
$B(E6;6_1^+ \rightarrow 0_1^+)$	8245		$e^2\mathrm{fm}^{12}$

Bijker & Iachello, PRL 112, 152501 (2014)

Summary and Conclusions

- Algebraic Cluster Model
- U(3n-2) SGA for n-body systems
- Discrete and continuous symmetries
- Special solutions: spherical and deformed oscillators, oblate top, spherical top
- Shape-phase transitions
- Rotational bands: fingerprints of point group symmetries
- Applications in molecular, nuclear, hadron physics

Alpha-Cluster Nuclei

- Rotational bands: fingerprints of geometric configuration of alpha clusters
- Oblate top with triangular symmetry for ^{12}C
- Ground state band: triangular
- Hoyle band: bent-arm, triangular?
- Search for negative parity states 3^-, 4^- (Gai et al)
- Spherical top with tetrahedral symmetry for ^{16}O

- Giant Dipole Resonances (PRL 113, 032506, 2014)
- Rel. Nuclear Collisions (PRL 112, 112501, 2014)