# Pairing fluctuations and giant dipole resonance

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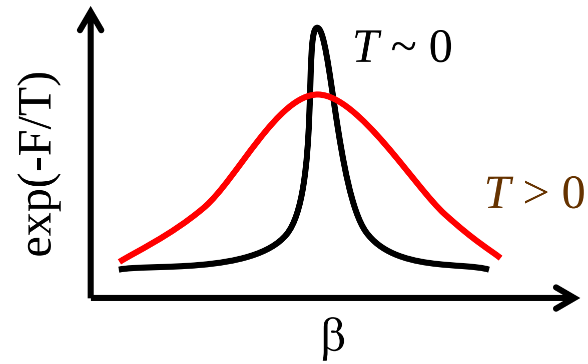
# Theoretical framework – hot nuclei

Fluctuations exist in finite systems




**Microscopic-Macroscopic approach**

- model for nuclear deformation energy calculations at any given temperature and spin

**F (β, γ)**

$$\langle \sigma \rangle_{\beta,\gamma} = \frac{\int \sigma(\beta,\gamma)\, e^{-F(\beta,\gamma)/T}\, d\tau}{\int e^{-F(\beta,\gamma)/T}\, d\tau}$$

**HO potential with separable dipole-dipole int.**

- model that relates the nuclear shape to the GDR cross section

**σ (β, γ)**

**Fluctuations**

- formalism that takes care of thermal fluctuations and modifies GDR cross sections accordingly

Y. Alhassid and B. Bush, PRL 63, 2452 (1989).
W. E. Ormand, P. G. Bortignon, and R. A. Broglia, NPA 614, 217 (1997).
P. Arumugam, G. Shanmugam and S.K. Patra, PRC 69, 054313 (2004)

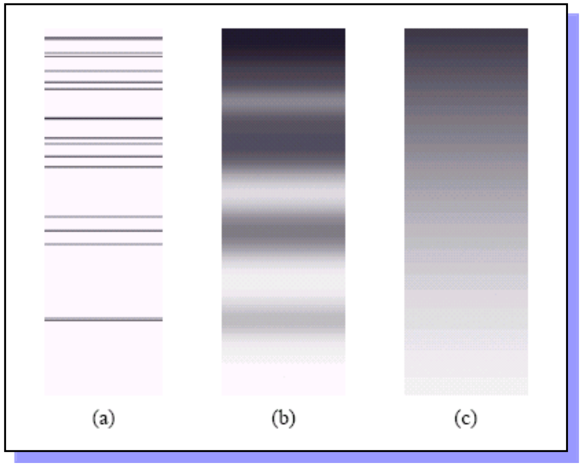
# Microscopic-Macroscopic method

**Strutinsky theorem :** $F_{\mathrm{TOT}} = E_{\mathrm{LDM}} + \sum_{p,n} \delta F$

$$\delta F = F - \widetilde{F}$$

$$F = E - TS = \sum_{i=1}^{\infty} e_i n_i - T \sum_{i=1}^{\infty} s_i$$

$$\widetilde{F} = 2\sum_i e_i \widetilde{n}_i^T - 2T\sum_i \widetilde{s}_i + 2\gamma_s \int_{-\infty}^{\infty} \widetilde{f}(x) x \sum_i n_i(x) dx$$




$$g(e) = \sum_{i=1}^{\infty} \frac{1}{4T \cosh^2[(e - e_i)/2T]}$$

At finite spin : $F_{\mathrm{TOT}} = E_{\mathrm{RLDM}} + \sum_{p,n} \delta F$

$$F_{\mathrm{TOT}} = E_{\mathrm{LDM}} + \sum_{p,n} \delta F^{\omega} + \frac{1}{2}\omega\left(I_{\mathrm{TOT}} + \sum_{p,n} \delta I\right)$$

$$\delta I = I - \widetilde{I}. \qquad I_{\mathrm{TOT}} = \Im_{rig}\omega + \delta I$$

P. Arumugam, G. Shanmugam and S.K. Patra, **PRC 69**, 054313 (2004)
P. Arumugam, A.G. Deb and S.K. Patra, **EPJA 25**, 199 (2005)

# Model for GDR

Rotating anisotropic harmonic oscillator potential
with a separable dipole-dipole residual interaction

$$H = H_{\mathrm{av}} + H_{\mathrm{int}}$$

$$H_{\mathrm{av}}(\Omega) = \sum_{\nu=1}^{A} h_{\nu}(\Omega)$$

$$h(\Omega) = \frac{p^2}{2m} + \frac{m}{2}(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2) - \Omega\, l_z$$

$$L_z = \sum_{\nu=1}^{A} l_z(\nu)$$

$$H_{int} = \eta \sum_{i=x,y,z} \frac{m\omega_i^2}{2A} \left[ \sum_{\nu=1}^{A} \tau_3^{(\nu)} x_i(\nu) \right]^2$$

$$\tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

R.R. Hilton, **ZPA 309**, 233 (1983)
G. Shanmugam and M.Thiagasundaram, **PRC 37**, 853 (1988); **PRC 39**, 1623 (1989)

# Model for GDR – contd.

In lab frame

$$\tilde{\omega}_z = (1+\eta)^{1/2}\omega_z \ ,$$

$$\tilde{\omega}_2 \mp \Omega = \left\{(1+\eta)\frac{\omega_y^2+\omega_x^2}{2} + \Omega^2 + \frac{1}{2}\left[(1+\eta)^2(\omega_y^2-\omega_x^2)^2 + 8\Omega^2(1+\eta)(\omega_y^2+\omega_x^2)\right]^{\frac{1}{2}}\right\}^{\frac{1}{2}} \mp \Omega \ ,$$

$$\tilde{\omega}_3 \mp \Omega = \left\{(1+\eta)\frac{\omega_y^2+\omega_x^2}{2} + \Omega^2 - \frac{1}{2}\left[(1+\eta)^2(\omega_y^2-\omega_x^2)^2 + 8\Omega^2(1+\eta)(\omega_y^2+\omega_x^2)\right]^{\frac{1}{2}}\right\}^{\frac{1}{2}} \mp \Omega \ ,$$

For collective rotation

All 5 frequencies exist

For s.p. rotation

$\tilde{\omega}_1$ and $\tilde{\omega}_2 - \Omega = \tilde{\omega}_3 + \Omega$

For sphere

$$\tilde{\omega}_1 = \tilde{\omega}_2 - \Omega = \tilde{\omega}_3 + \Omega$$

$$\sigma(E_\gamma) = \sum_i \frac{\sigma_{mi}}{1 + (E_\gamma^2 - E_{mi}^2)^2 / E_\gamma^2 \Gamma_i^2}$$

$$\sigma_m = 60\,\frac{2NZ}{\pi A \Gamma}(1+\alpha)$$

$$\Gamma_i = \Gamma_0 (E_i / E_0)^{\delta}$$

$$\Gamma_i \approx 0.026\, E_i^{1.9}$$

PAIRING FLUCTUATICNS IN EXCITED NUCLEI AND THE ABSENCE OF A SECOND ORDER PHASE TRANSITION *

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Volume 40B, number 1 PHYSICS LETTERS 12 June 1972

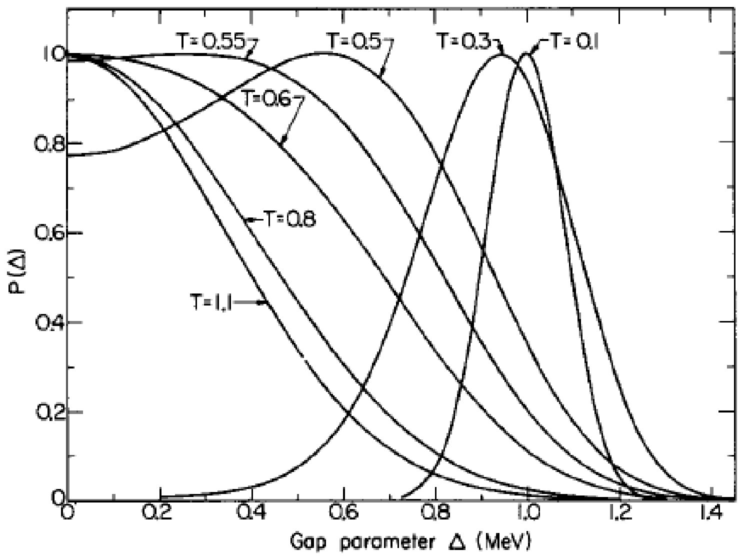



Fig.1. Probability distributions for the gap parameter Δ at different termperatures. The value of Δ at the maximum corresponds to the solution of the gap equation. The critical temperature is $T$=0.57.

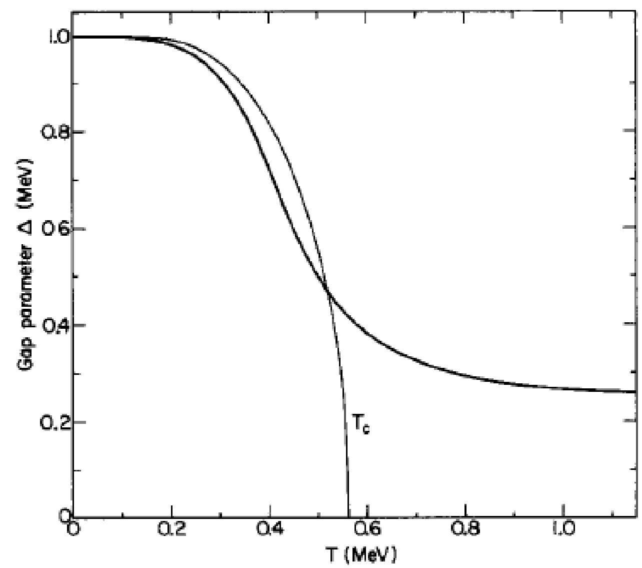
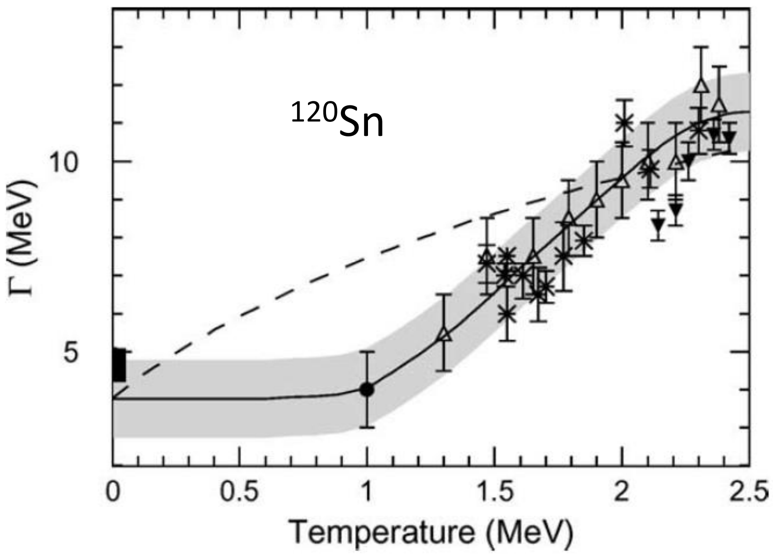
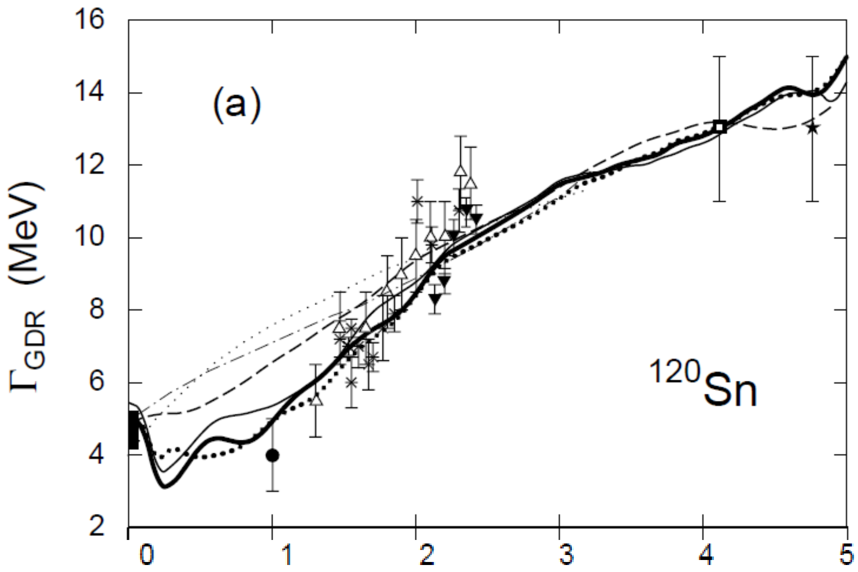



Fig.2. The average gap parameter (thick line) and the most probable gap parameter is a function of temperature.

**Importance in GDR** proposed by N.D. Dang, K. Tanabe, A. Arima, NPA675 (2000) 531

# Low-T GDR




P. Heckman, et al., Phys. Lett. B **555** (2003) 43

N. Dinh Dang, A. Arima, PRC **68**, 044303 (2003)

$^{179}$Au at T = 0.7 MeV, Γ is quenched: F. Camera et al., PLB **560**, 155 (2003)

Shell effects increase Γ: P. Arumugam, A.G. Deb and S.K. Patra, *Europhys. Lett.* **70**, 313 (2005)

TSFM with pairing: P. Arumugam, N. Dinh Dang, RIKEN Accel. Prog. Rep. **39** (2006) 28

Low-T GDR measurements at VECC, Kolkata

PLB **709,** 9 (2012); PLB **713,** 434 (2012); PLB **731,** 92 (2014)

# Model for GDR with pairing

$$H = H_{osc} + \eta D^{\dagger} D + \chi P^{\dagger} P$$

$$\omega_{\nu} = \omega_{\nu}^{OSC} - \chi \omega^{P}$$

$$\omega^{P} = \left( \frac{Z\Delta_{P} + N\Delta_{N}}{Z+N} \right)^{2}$$

$$\eta = \eta_{0} - \chi_{0} \sqrt{T} \omega^{P}$$

## GDR frequencies with pairing

$$\tilde{\omega}_{1} = (1+\eta)^{1/2} \omega_{z} \ ,$$

$$\tilde{\omega}_{2} = \left\{ (1+\eta) \frac{\omega_{y}^{2} + \omega_{x}^{2}}{2} + \frac{1}{2} \left\{ (1+\eta)^{2} (\omega_{y}^{2} - \omega_{x}^{2})^{2} \right\}^{1/2} \right\}^{1/2}$$

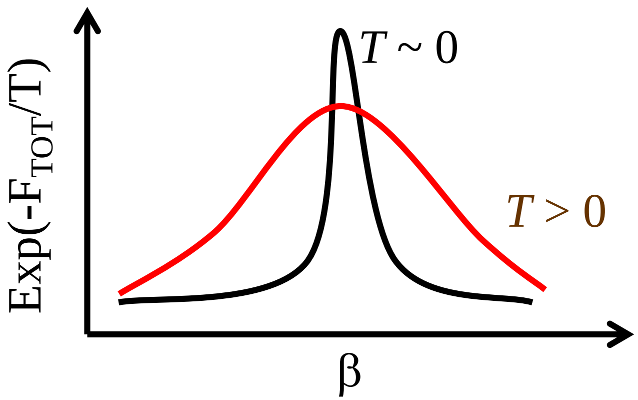
$$\tilde{\omega}_{3} = \left\{ (1+\eta) \frac{\omega_{y}^{2} + \omega_{x}^{2}}{2} - \frac{1}{2} \left\{ (1+\eta)^{2} (\omega_{y}^{2} - \omega_{x}^{2})^{2} \right\}^{1/2} \right\}^{1/2}$$

A.K. Rhine Kumar, P. Arumugam and N.D. Dang, PRC **91**, 044305 (2015); PRC **90**, 044308 (2014).

# Role of fluctuations

$$\langle \vartheta \rangle_{\beta,\gamma} = \frac{\int\limits_{\beta}\int\limits_{\gamma} D[\alpha] e^{-F_{TOT}(T;\beta,\gamma)/T} \vartheta}{\int\limits_{\beta}\int\limits_{\gamma} D[\alpha] e^{-F_{TOT}(T;\beta,\gamma)/T}}$$

$$D[\alpha] = \beta^4 |\sin 3\gamma| d\beta d\gamma$$




Free energy ($F_{TOT}$), calculated at abscissas using FTNS with exact $T$ dependence

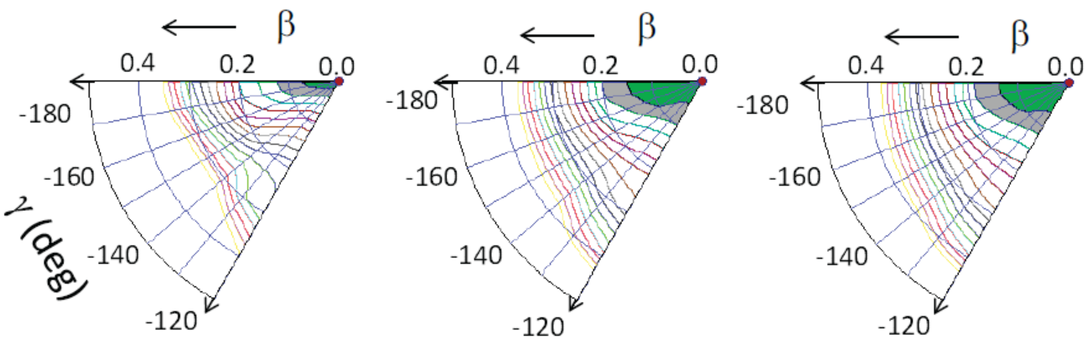
Fluctuations in pairing field**:** λ is fixed at its BCS value and Δ is allowed to vary

$$\langle \vartheta \rangle_{\beta,\gamma,\Delta_P,\Delta_N} = \frac{\int\limits_{\beta}\int\limits_{\gamma}\int\limits_{\Delta_P}\int\limits_{\Delta_N} D[\alpha] e^{-F_{TOT}(T;\beta,\gamma,\Delta_P,\Delta_N)/T} \vartheta}{\int\limits_{\beta}\int\limits_{\gamma}\int\limits_{\Delta_P}\int\limits_{\Delta_N} D[\alpha] e^{-F_{TOT}(T;\beta,\gamma,\Delta_P,\Delta_N)/T}}$$

$$D[\alpha] = \beta^4 |\sin 3\gamma| \Delta_P \Delta_N d\beta \; d\gamma \; d\Delta_P \; d\Delta_N$$

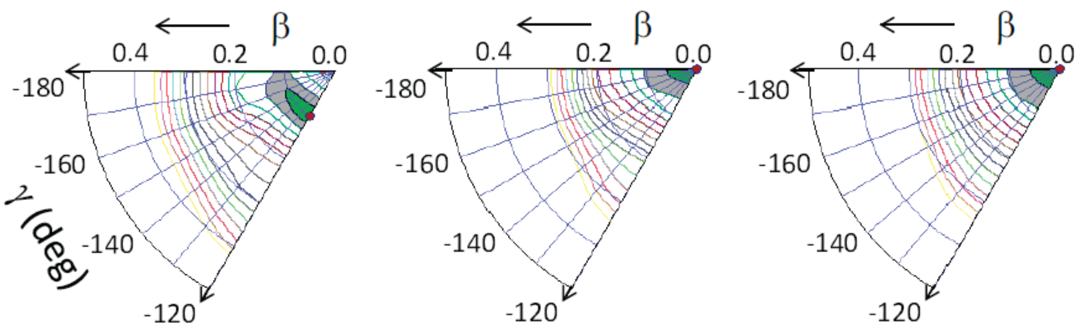
A.K. Rhine Kumar, P. Arumugam and N.D. Dang, PRC **91**, 044305 (2015)

# Role of pairing in $^{120}$Sn



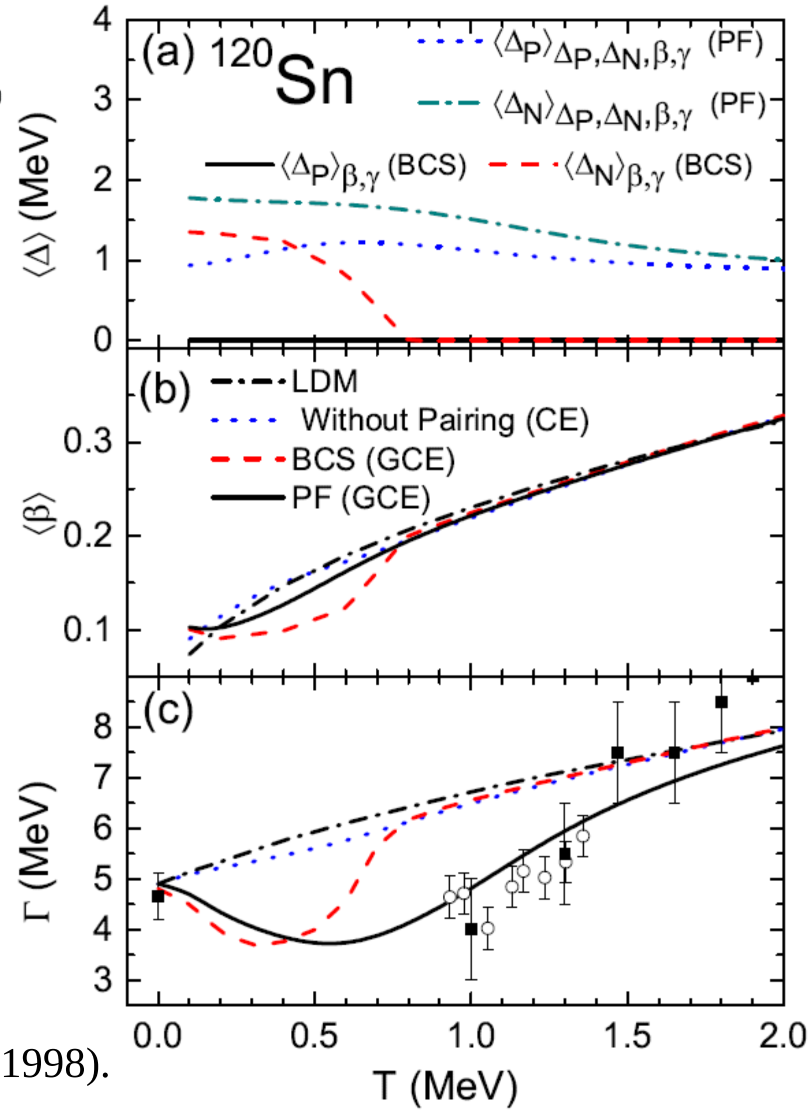

T = 0.1 MeV T = 0.4 MeV T = 0.6 MeV

$^{120}$Sn Without pairing




T = 0.1 MeV T = 0.4 MeV T = 0.6 MeV

$^{120}$Sn With pairing




Expt. Data: $^{120}$Sn: Phys. Lett. B **555**, 43 (2003); NPA **635**, 428 (1998).
$^{119}$Sb: Phys. Lett. B **709**, 9 (2012).

A.K. Rhine Kumar, P. Arumugam and N.D. Dang, PRC **91**, 044305 (2015); PRC **90**, 044308 (2014).

# Testing prescriptions for G

*G* from Strutinsky calculations

$$\frac{1}{G}=\tilde{\rho}\left\{\left[\left(\frac{N_p}{2\tilde{\rho}\tilde{\Delta}}\right)^2+1\right]^{(1/2)}\frac{N_p}{2\tilde{\rho}\tilde{\Delta}}\right\}$$

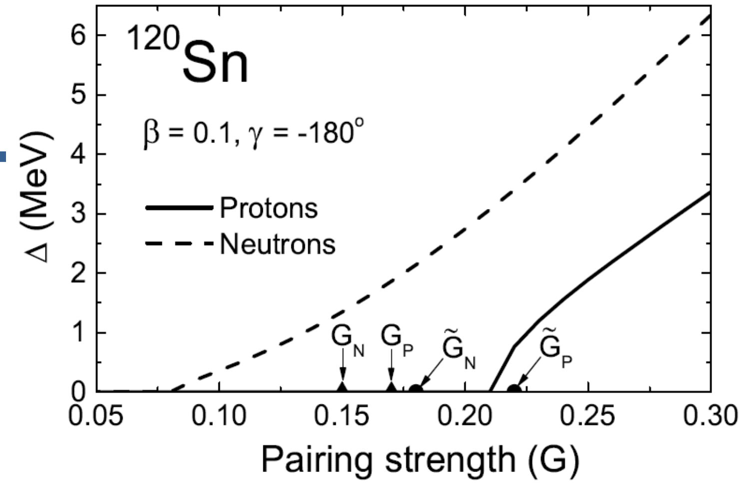
$$\tilde{\rho}=\frac{1}{2}\tilde{g}(\tilde{\lambda}) \qquad \tilde{\Delta}=\frac{12}{\sqrt{A}}$$

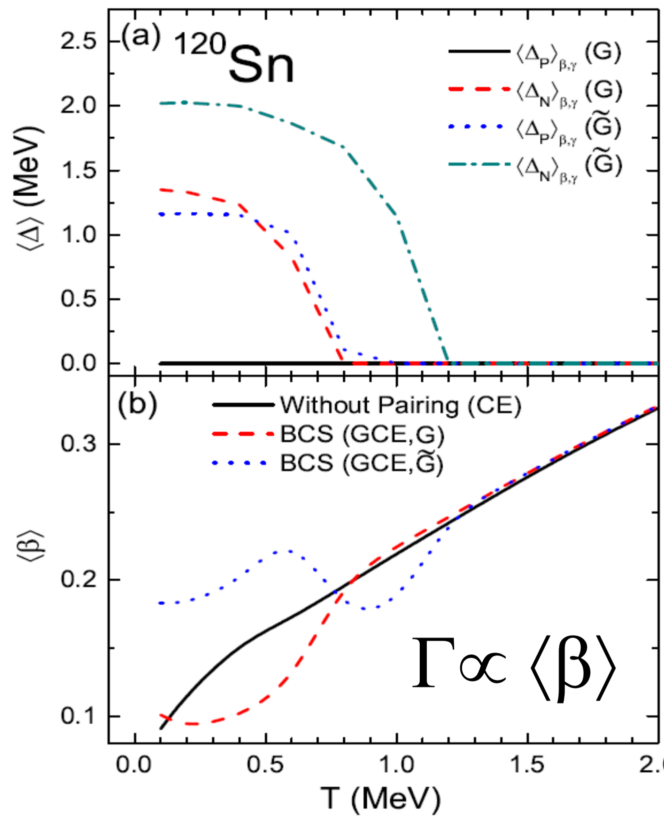
$$\tilde{g}(e)=\frac{1}{\gamma\sqrt{\pi}}\sum_i \exp(-x^2)\sum_{m=0}^{q}C_m H_m(x)$$

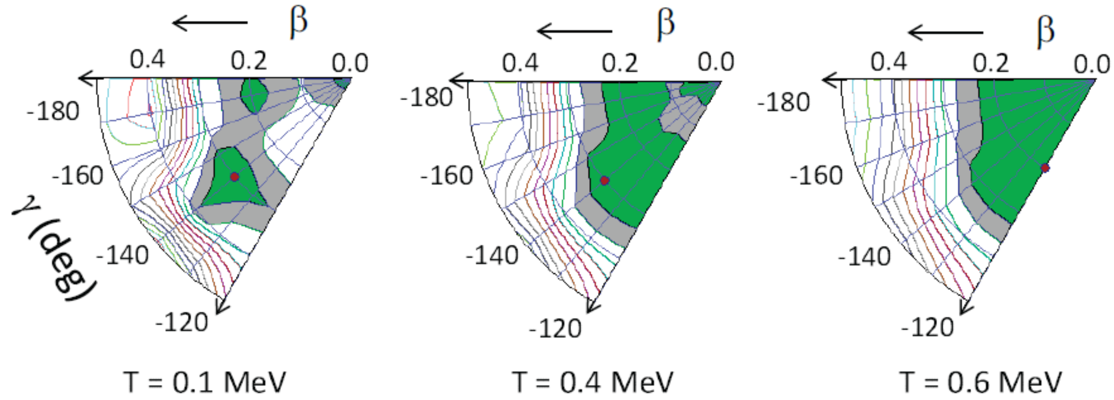
*G* from empirical data

$$G_{P,N}=[19.2\pm 7.4(N-Z)]\big/A^2$$

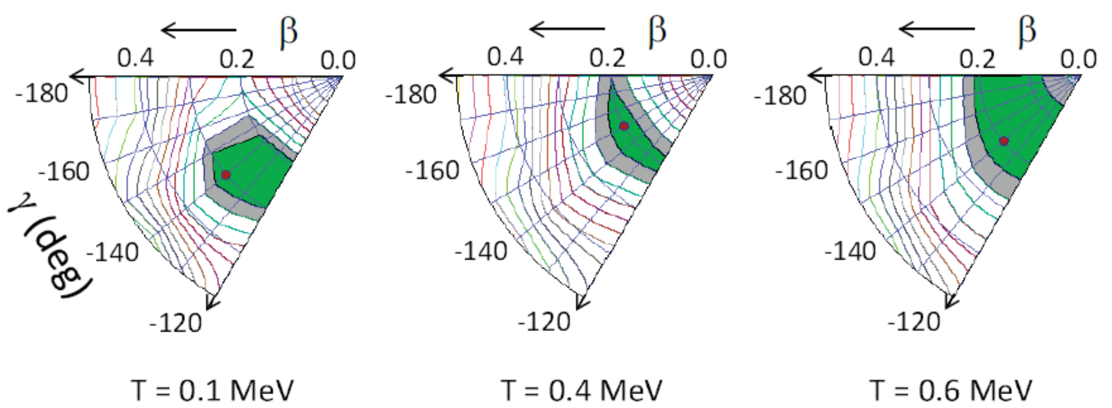
P. Arumugam, N. Dinh Dang, RIKEN Accel. Prog. Rep. **39** (2006) 28



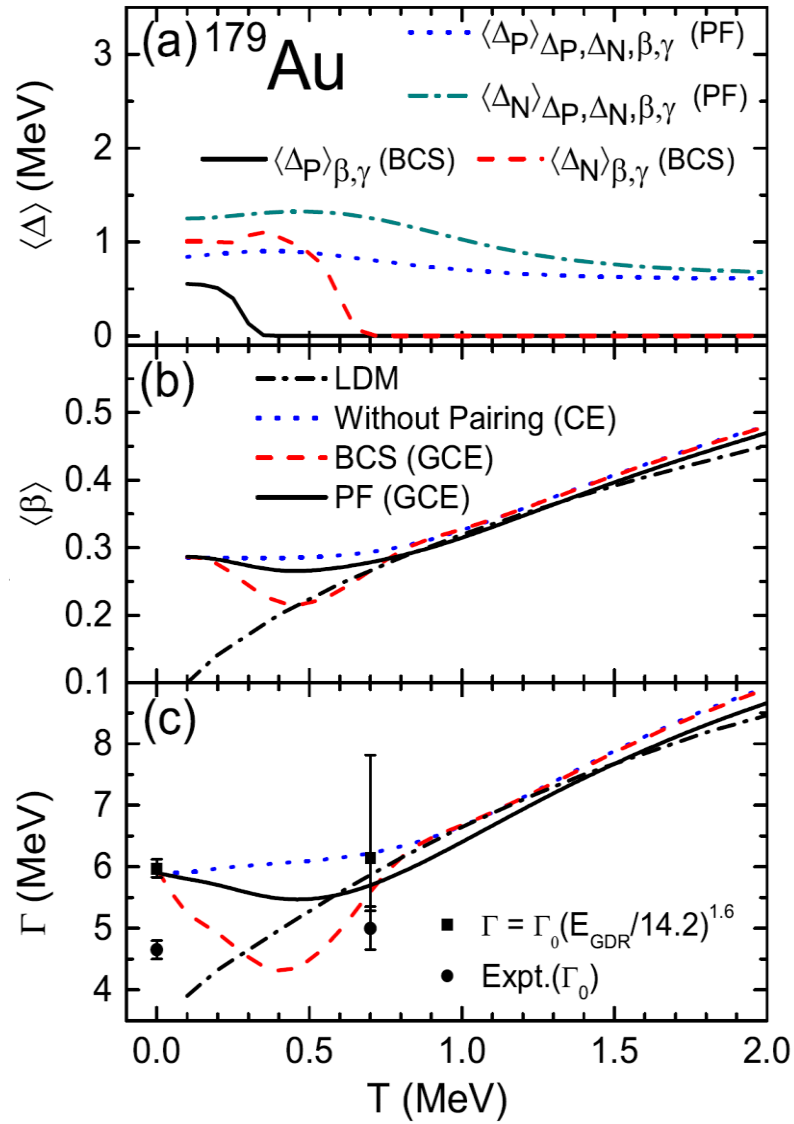






## $^{179}$Au Without pairing

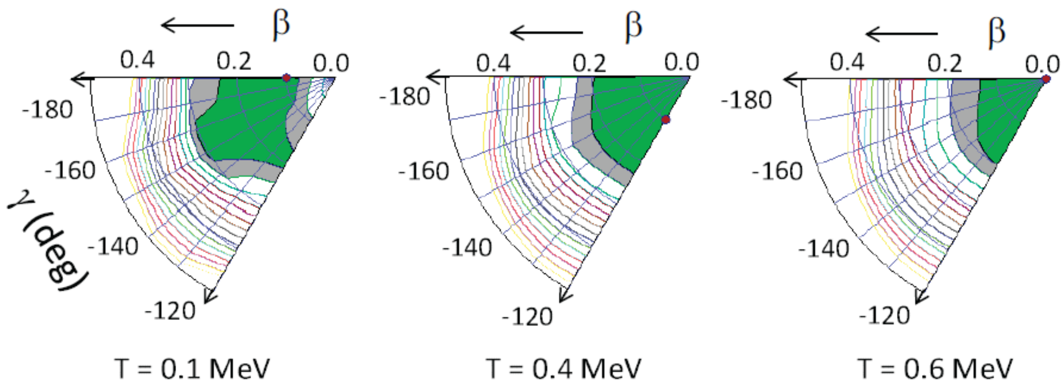



## $^{179}$Au With pairing

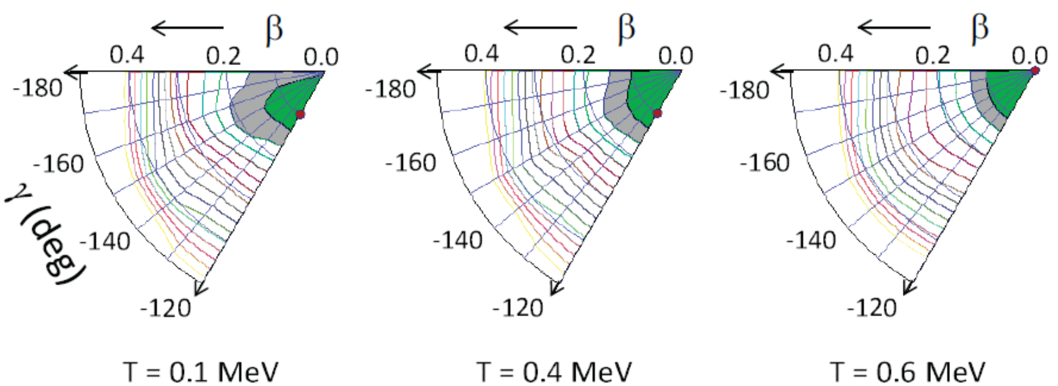



Expt. Data: Phys. Lett. B **560**, 155 (2003).

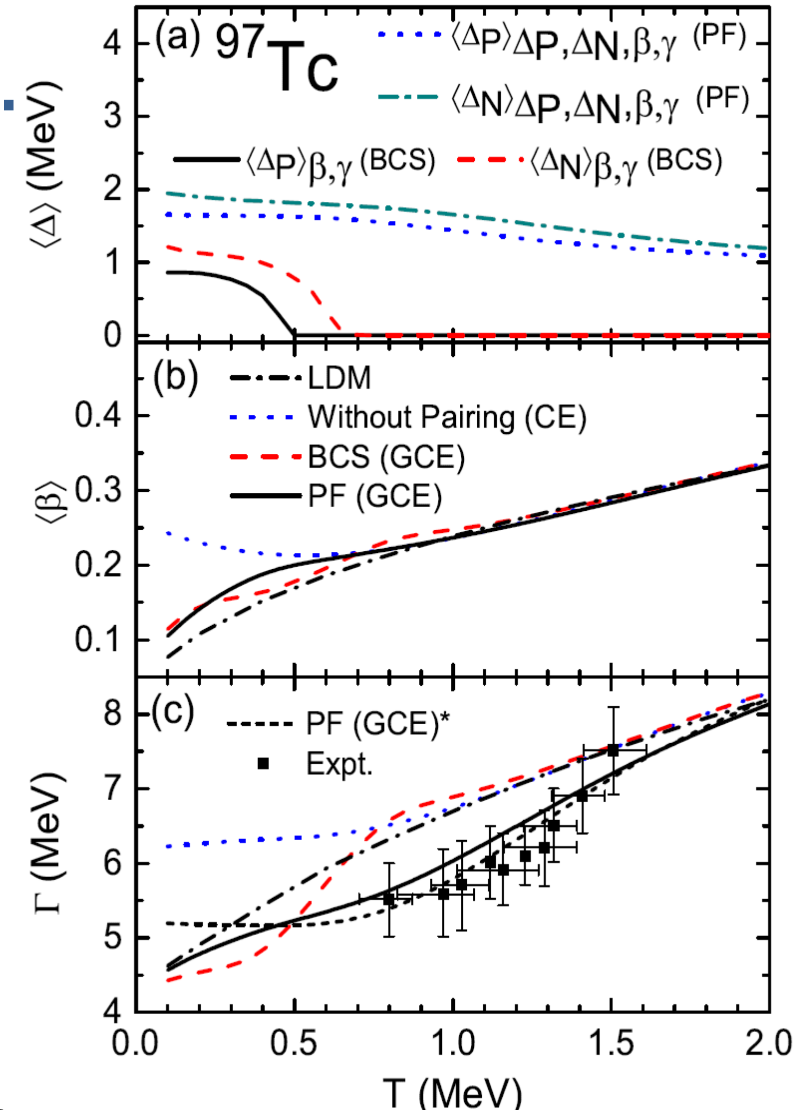
A.K. Rhine Kumar, P. Arumugam and N.D. Dang, PRC **91**, 044305 (2015); PRC **90**, 044308 (2014).




## $^{97}$Tc Without pairing




## $^{97}$Tc With pairing




Expt. Data: Phys. Lett. B **731**, 92 (2014).

PF(GCE)* correspond to adjusted energy dependence of Γ

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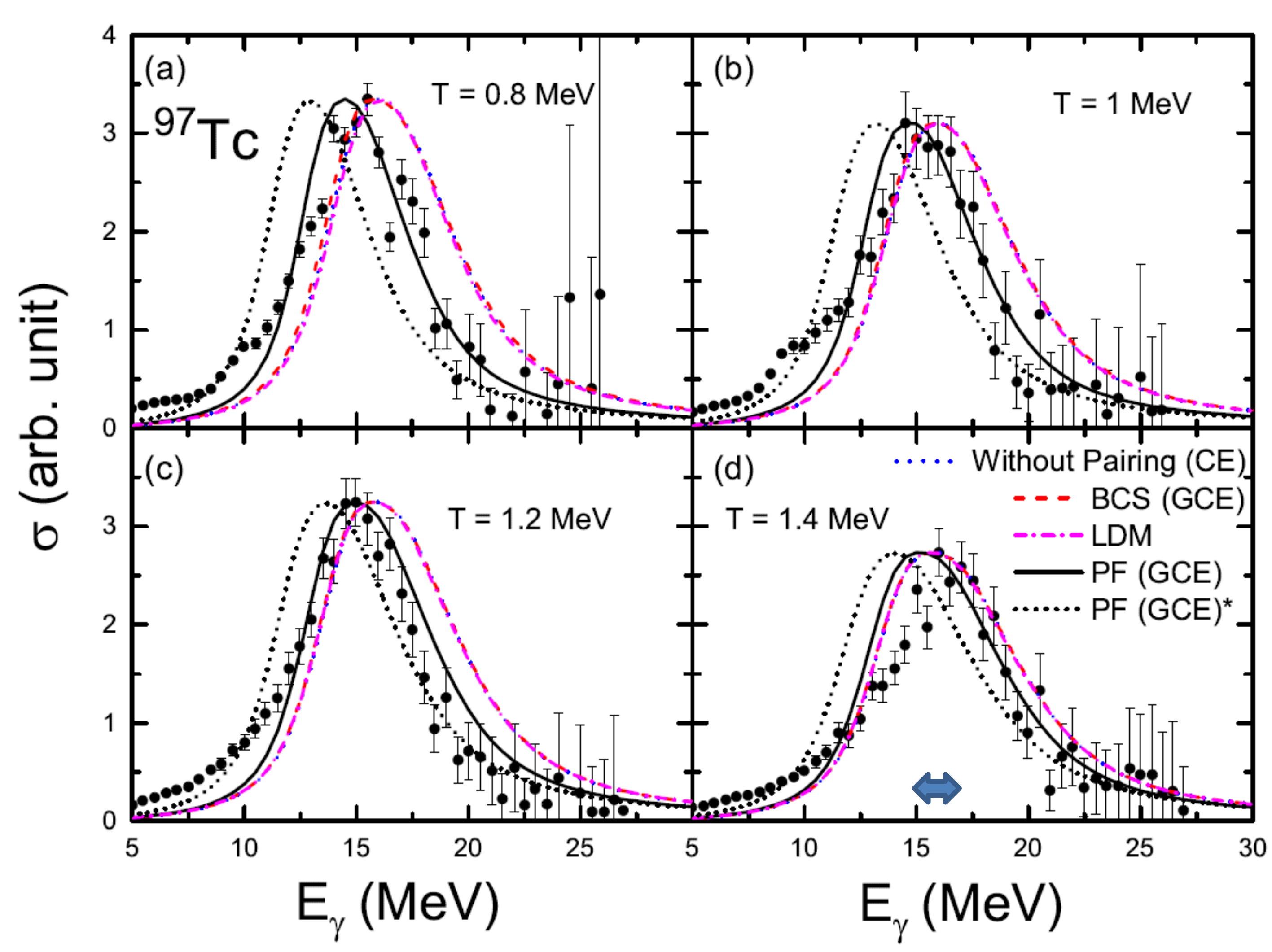


- Precisely measured shift in $E_{GDR}$ could reveal more about $\chi$
- Only a qualitative fit is possible
- Analysis with BW form of cross sections is in progress

Expt. Data: PLB **731**, 92 (2014)

A.K. Rhine Kumar, P. Arumugam and N.D. Dang, PRC **91**, 044305 (2015); PRC **90**, 044308 (2014).

# Interplay of various effects

| Nucleus | (i) Shell effects | (ii) Pairing effects through $F_{\mathrm{TOT}}$ | | | (iii)Pairing effects through $\chi\mathcal{P}^{\dagger}\mathcal{P}$ | Net effect |
|---|---|---|---|---|---|---|
| | | $T = 0.1$ MeV | $T = 0.4$ MeV | $T = 0.6$ MeV | | |
| $^{120}$Sn | — | ↑ | ↓ | ↓ | ↓ | ↓ |
| $^{179}$Au | ↑ | ↓ | ↓ | ↓ | ↓ | ↓ |
| $^{208}$Pb | ↓ | — | — | — | ↓ | ↓ |
| $^{97}$Tc | ↑ | ↓ | ↓ | — | ↓ | ↓ |

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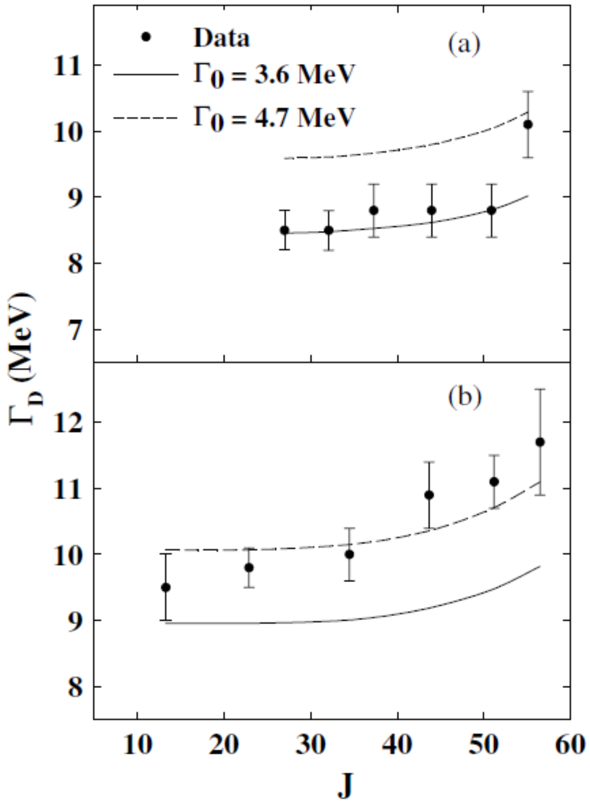
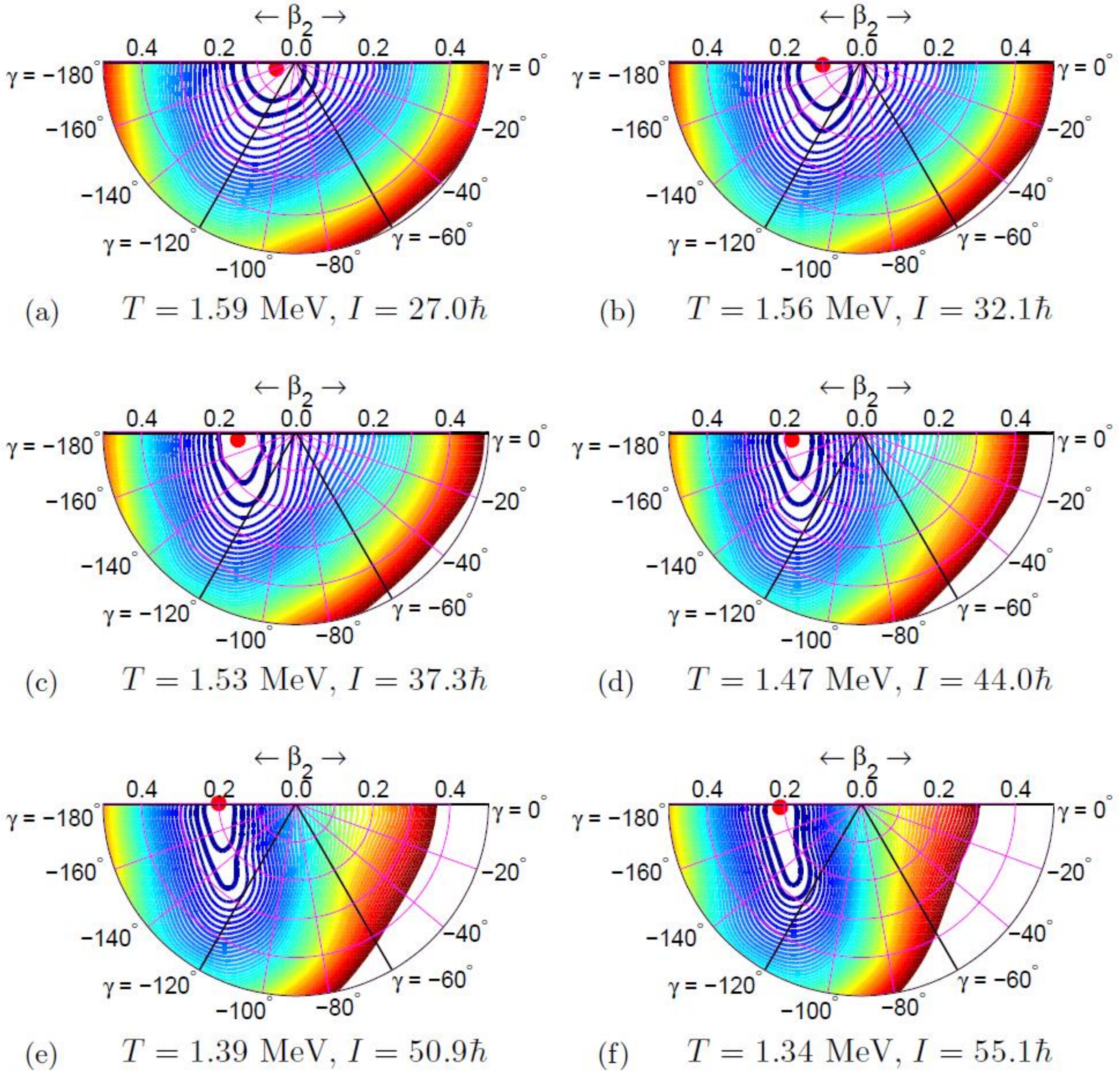
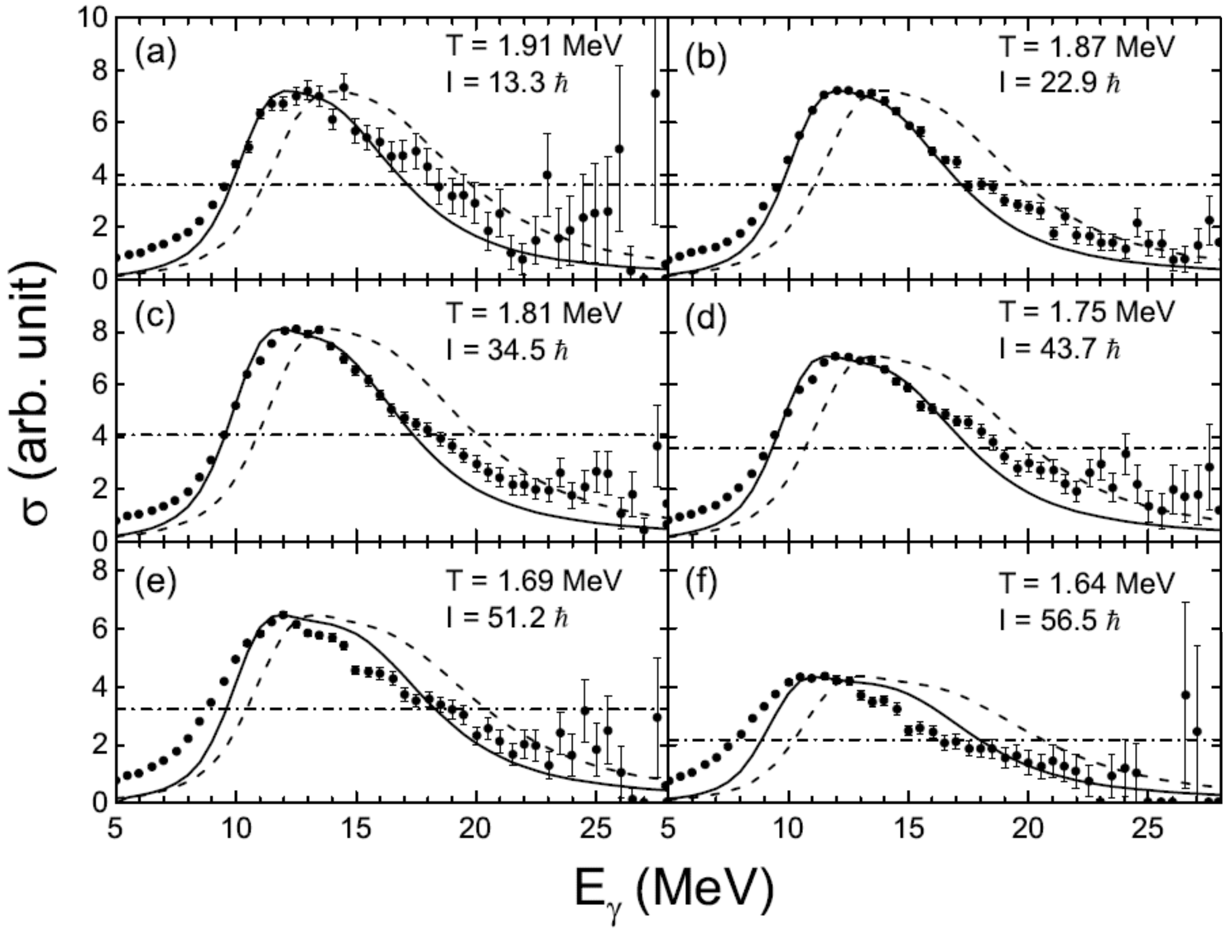
# Hot and rotating $^{152}$Gd




D R Chakrabarty *et al*,
JPG 37 (2010) 055105;
NPA 770, 126 (2006).



# $^{152}$Gd ($^{28}$Si+$^{124}$Sn) at beam energy 185 MeV



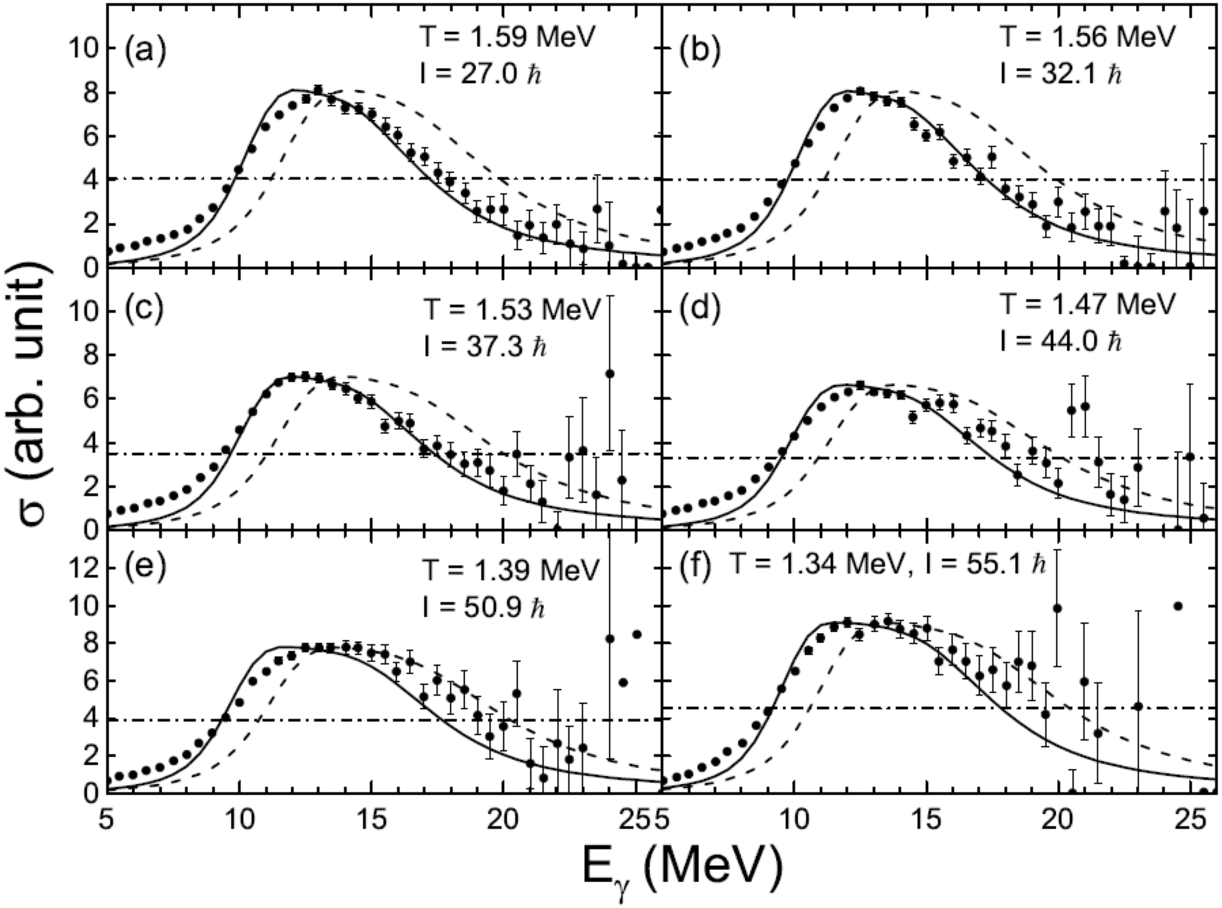

| $T$ (MeV) | $I$ ($\hbar$) | $\Gamma_{\rm Expt}$ (MeV) | $\Gamma_{\rm TSFM}$(MeV) $\eta = 2.3$ | $\eta = 3.35$ |
|---|---|---|---|---|
| 1.91 | 13.3 | 9.5±0.5 | 7.4 | 8.7 |
| 1.87 | 22.9 | 9.8±0.3 | 7.5 | 8.8 |
| 1.81 | 34.5 | 10.0±0.4 | 7.9 | 9.2 |
| 1.75 | 43.7 | 10.9±0.5 | 8.3 | 9.6 |
| 1.69 | 51.2 | 11.1±0.4 | 8.7 | 9.9 |
| 1.64 | 56.5 | 11.7±0.8 | 9.1 | 10.4 |

Data: JPG 37 (2010) 055105

Tuning parameters to fit Γ could be misleading. Comparison with cross-sections is more robust.

A.K. Rhine Kumar and P. Arumugam, to appear in PRC

# $^{152}$Gd at beam energy 149 MeV




| $T$ | $I$ | $\Gamma_{\rm Expt}$ | $\Gamma_{\rm TSFM}$(MeV) | |
|---|---|---|---|---|
| (MeV) | ($\hbar$) | (MeV ) | $\eta = 2.3$ | $\eta = 3.35$ |
| 1.59 | 27.0 | 8.5±0.3 | 7.4 | 8.7 |
| 1.56 | 32.1 | 8.5±0.3 | 7.5 | 8.8 |
| 1.53 | 37.3 | 8.8±0.4 | 7.7 | 9.0 |
| 1.47 | 44.0 | 8.8±0.4 | 8.0 | 9.2 |
| 1.39 | 50.9 | 8.8±0.4 | 8.4 | 9.6 |
| 1.34 | 55.1 | 10.1±0.5 | 8.7 | 9.8 |

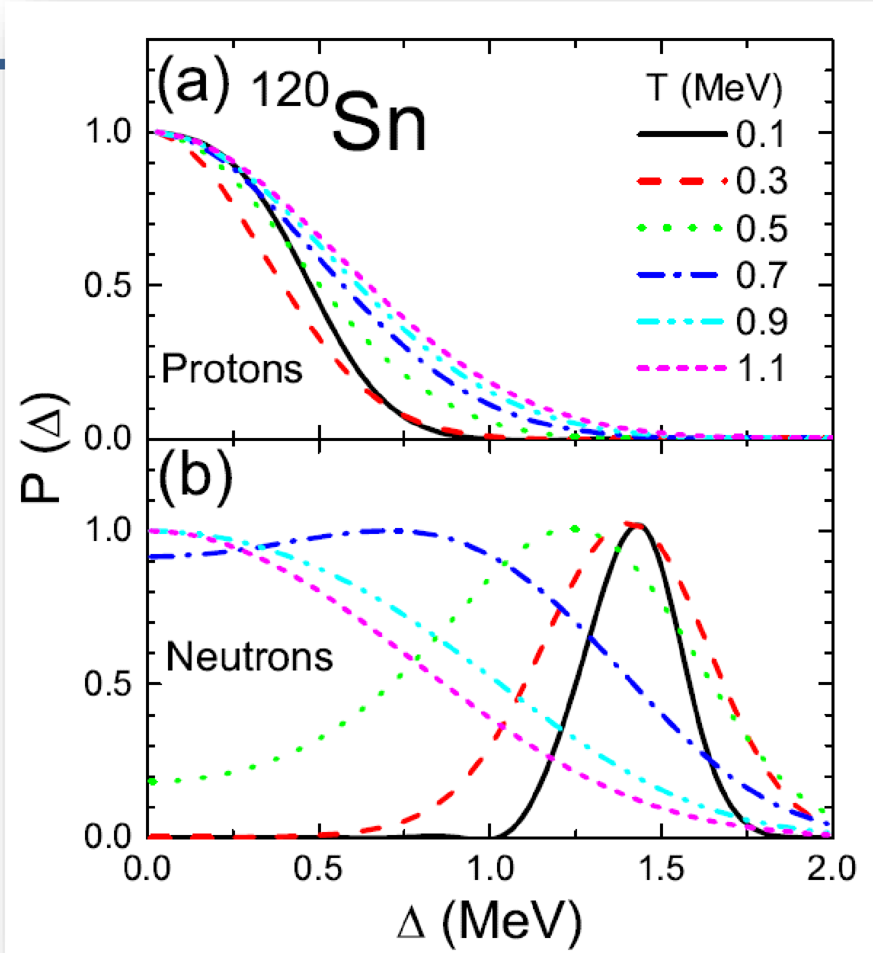
Data: JPG 37 (2010) 055105

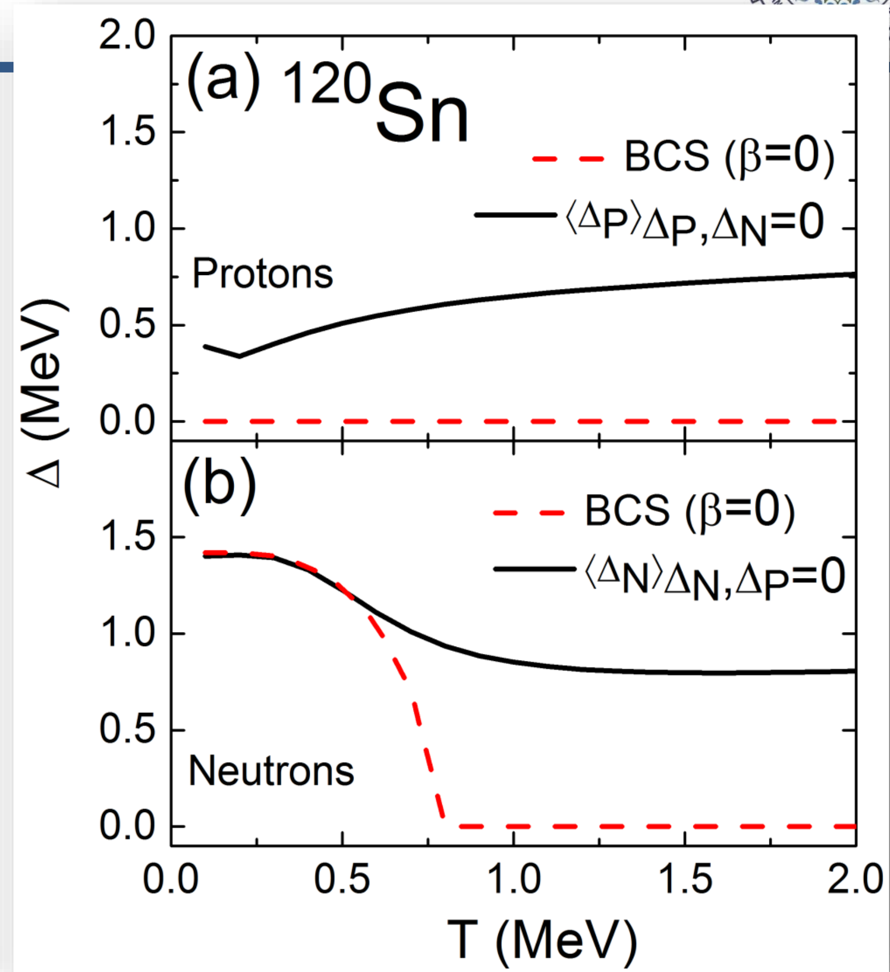
A.K. Rhine Kumar and P. Arumugam, to appear in PRC

# Summary

- Thermal fluctuations in pairing (PF) leads to a quenching of GDR width in agreement with the experimental observations.
- For small systems like the atomic nuclei, there are no sharp superfluid-normal phase transitions.
- For a precise match with the $^{120}$Sn and $^{97}$Tc data, the consideration of PF is crucial.
- In $^{208}$Pb, the shell effects are so dominating as to favour a spherical (closed-shell/unpaired) configuration and hence the role of PF is negligible.
- One may consider the precise low-*T* GDR measurements as a benchmark for pairing prescriptions.
- $^{152}$Gd: Tuning parameters to fit $\Gamma$ could be misleading. Comparison with cross-sections is more robust.


Thank You




A.K. Rhine Kumar, P. Arumugam and N.D. Dang, Phys. Rev. C **91**, 044305 (2015);
Phys. Rev. C **90**, 044308 (2014).

# Energy dependence of width

- This is a profound question and most of the answers are tentative
  - The feasible approach with predictive power is only empirical

$$\Gamma_i \approx 0.026 E_i^{1.9}$$

P. Carlos *et al*, **NPA219**, 61 (1974)

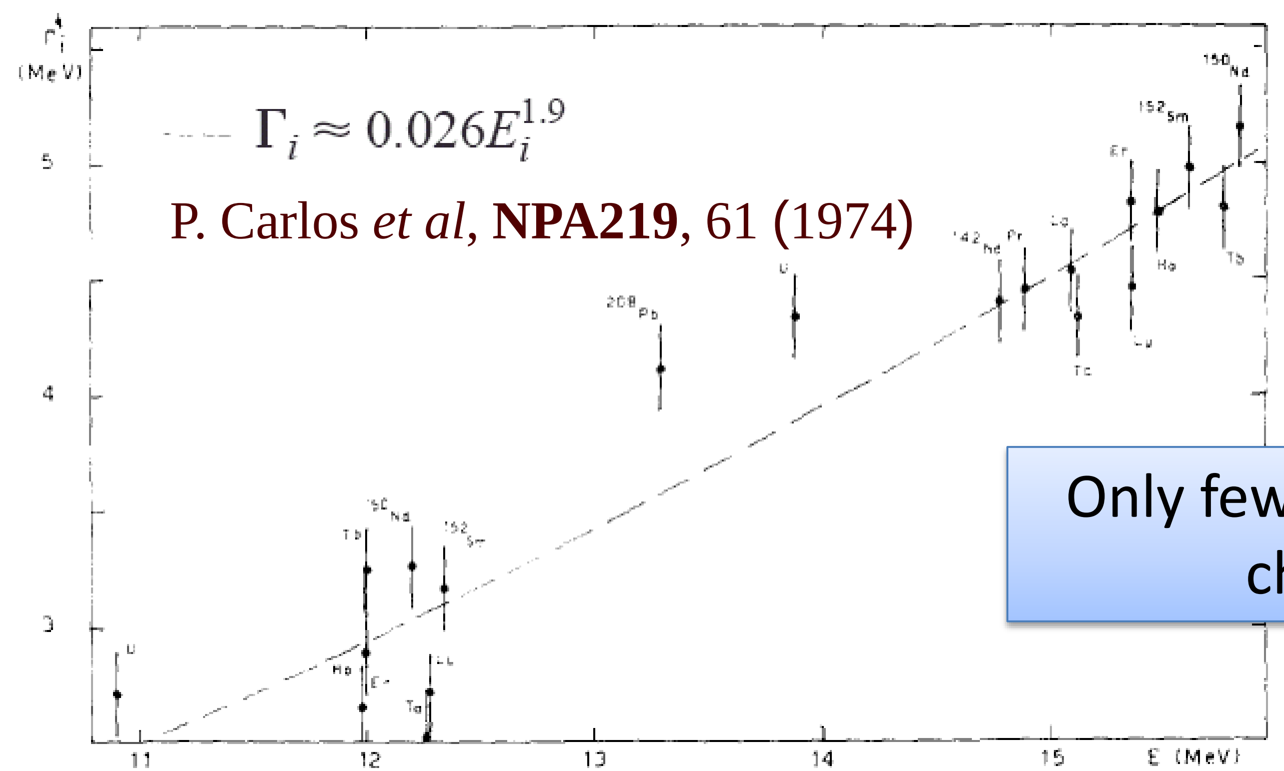
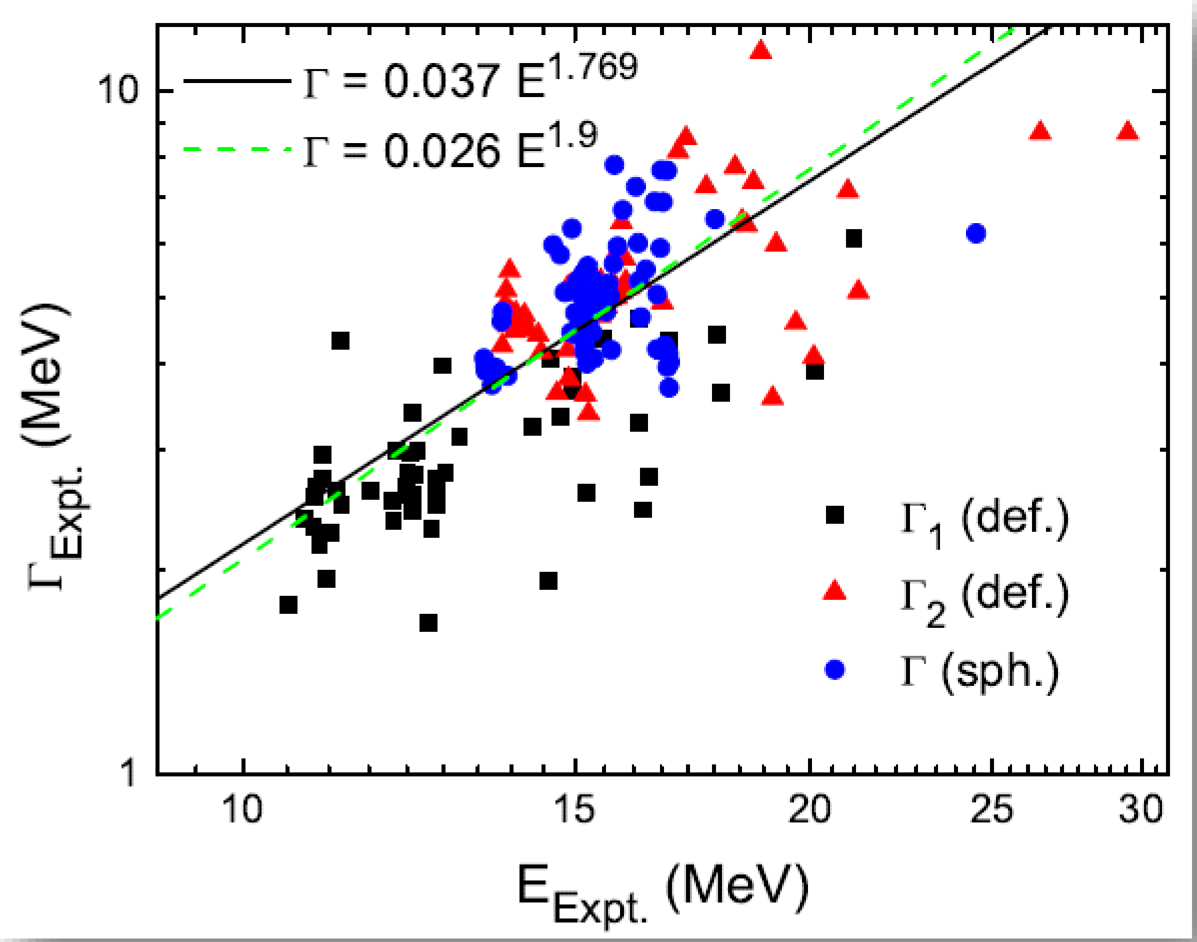



Fig. 9. Experimental $\Gamma_i\downarrow$ values (●) and best fit (dotted line) corresponding to $\Gamma\downarrow = 0.026\ E^{1.9}$.

Only few nuclei were chosen

# $\Gamma(E)$ – Our attempt




No improvement in fit even with new data

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Determination of width continues to be a weak point (applicable for other models also, including microscopic ones)

# Woods-Saxon Strutinsky results